More spatial stuff: Public goods games

Public goods games are games where there are a bunch of cooperators + some defectors. If N guys interact, then each cooperator increases the common resource by rc at a cost c. The total is divided by all N. So if there are k cooperators then $P_D(k) = \frac{rkc}{N}$, $P_C(k) = P_D(k) - c$, so always better to defect as usual. If $r<1$ then $P_C(N) \leq 0 = P_D(0)$ so cooperation is doomed. In any interaction group cooperators always fare worse, so on a population wide scale need to look at different sized groups.

We will now explore the following scenario based on placing these on an ecological setting. Let u be the population of cooperators, v, defectors, where we can have both go extinct leaving empty space, w. ($w = 1-u-v$)

$$\frac{du}{dt} = u\left[\underbrace{w}_{\text{available space}}(f_c + b) - \underbrace{d}_{\text{death}}\right], \quad \frac{dv}{dt} = v\left(w(\underbrace{f_d}_{\text{fitness}} + \underbrace{b}_{\text{birth}}) - d\right)$$

If $f_C = f_D = 0$, $u_t = u(b(1-u-v) - d)$, $v_t = v(b(1-u-v) - d)$ and $u+v=1$ is equil.

So, what are f_C, f_D.

u = prob of finding a coop, v = prob of defector, w = failure to find a participant

$(u+v)N$ is average group size. An individual finds itself in a group of size S w probability $\binom{N-1}{S-1}(1-w)^{S-1}w^{N-S}$ (since he is there, there are $N-1$, $S-1$ left)

In this group you face m coop + $S-1-m$ defectors with prob:

$$\binom{S-1}{m}\left(\frac{u}{u+v}\right)^m\left(\frac{v}{u+v}\right)^{S-1-m}$$

Payoff for defectors is $\frac{r}{S}\sum_{m=0}^{S-1}\binom{S-1}{m} m \left(\frac{u}{u+v}\right)^m\left(\frac{v}{u+v}\right)^{S-1-m} = P_D(S)$

$P_C(S) = \underbrace{P_D(S)}_{\text{from others}} + \underbrace{\frac{r}{S} - 1}_{\text{from yourself}}$, where assume $c=1$ WLOG

We average this over all sizes $2 \to N$:

$$f_i = \sum_{S=2}^{N} \binom{N-1}{S-1} (1-w)^{S-1} w^{N-S} P_i(S)$$

$$f_D = r \frac{u}{1-w}\left(1 - \frac{(1-w^N)}{N(1-w)}\right) \qquad f_C = f_D - F(w)$$

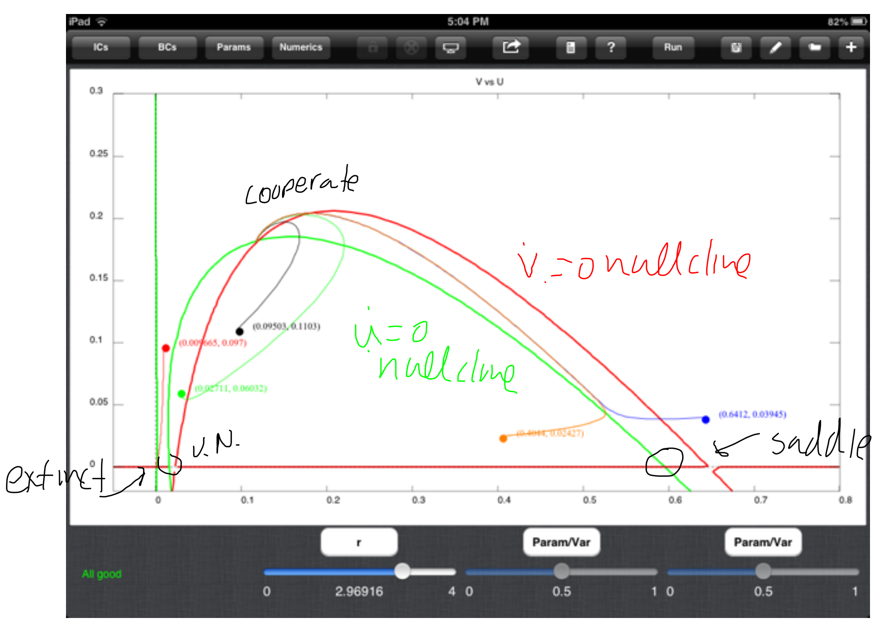
$$F(w) = 1 + (r-1)w^{N-1} - \frac{r}{N}\,\frac{1-w^N}{1-w}$$

So this is pretty cool. First note that if there are no cooperators then $f_D = 0$ + $\dot{v} = v(wb-d)$, thus if we assumed $d > b$, then defectors will die!!

So we assume $d > b$, thus the only way defectors will survive is if there are cooperators around!

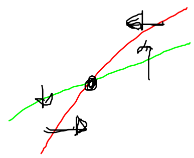
With this assumption $(u = v = 0)$ is always A.S.

Of course there is also a state with no defectors, $v = 0$ + as it turns out a state of coexistence. We fix $N = 8$, $b = 1$, $d = 1.2$ and vary r, this is the reward for cooperation



As r decreases there is a HB but it is subcritical + unstable

This is an example of an activator/inhibitor system

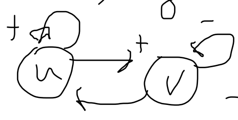


$$\begin{bmatrix} \alpha & -\beta \\ \gamma & -\delta \end{bmatrix} \quad \begin{aligned} \dot u &= \alpha u - \beta v \\ \dot v &= \gamma u - \delta v \end{aligned}$$

Slope of v-nullcline $\left(v = \frac{\gamma}{\delta} u\right)$ is $\frac{\gamma}{\delta}$

Slope of u-nullcline $\left(v = \frac{\alpha}{\beta} u\right)$ is $\frac{\alpha}{\beta}$

from picture, $\frac{\gamma}{\delta} > \frac{\alpha}{\beta} \Rightarrow \gamma\beta > \alpha\delta \Rightarrow \det > 0$

Note  $\Rightarrow$ Activator/inhibitor

SPATIAL MODEL

$$\frac{\partial u}{\partial t} = f(u,v) + D_u \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial v}{\partial t} = g(u,v) + D_v \frac{\partial^2 v}{\partial x^2}$$

Periodic BC, eg or No Flux, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0$

Note $u(x,t) = \bar u$, $v(x,t) = \bar v$ are equilibria of the full spatial system. Linearize about $(\bar u, \bar v)$ get:

$$\frac{\partial u}{\partial t} = \alpha u - \beta v + D_u u_{xx}$$

$$\frac{\partial v}{\partial t} = \gamma u - \delta v + D_v v_{xx}$$

Let L = length of the domain & assume No flux

Guess $u(x,t) = u^* e^{\lambda t} \cos \frac{\pi n}{L} x$, $v(x,t) = v^* e^{\lambda t} \cos \frac{\pi n}{L} x$

Note $u_x(0,t) = u_x(L,t) = 0$ as required. Define $k = \frac{\pi n}{\ell}$ $u_{xx} = -k^2 u$, $v_{xx} = -k^2 v$ so

$$\lambda u^* = \alpha u^* - \beta v^* - D_u k^2 u^*$$

$$\lambda v^* = \gamma u^* - \delta v^* - D_v k^2 v^*$$

$$\lambda \begin{pmatrix} u^* \\ v^* \end{pmatrix} = \begin{bmatrix} \alpha - D_u k^2 & -\beta \\ \gamma & -\delta - D_v k^2 \end{bmatrix} \begin{pmatrix} u^* \\ v^* \end{pmatrix}$$

$\mathrm{Re}\,\lambda < 0$ iff $\det > 0$, $\mathrm{tr} < 0$

$\mathrm{Tr} = \alpha - \delta - (D_u + D_v) k^2$

Since $(\bar{u}, \bar{v})$ is stable solution of the space-independent model ($k = 0$), we have $\alpha - \delta < 0$ and $\beta\gamma > \alpha\delta$

$\Rightarrow \mathrm{Tr}(k) < 0 \ \forall k$ since D_u, D_v are positive

$\det = D_u D_v k^4 - (\alpha D_v - \delta D_u) k^2 + \beta\gamma - \alpha\delta$

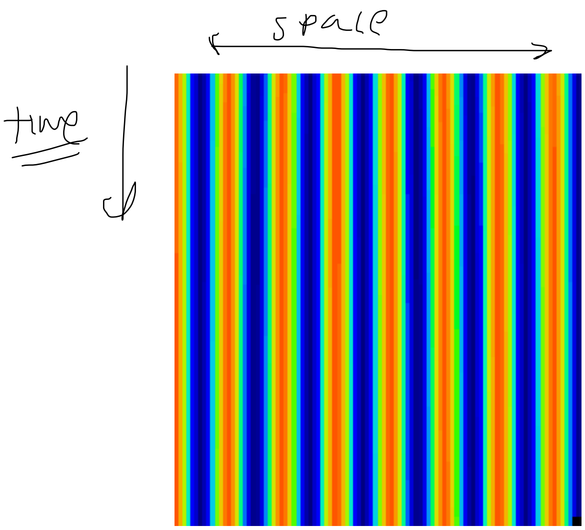
For $k \sim 0$ + k large, $\det(k) > 0 \Rightarrow$ stable but what about intermediate values of k?

If $\alpha D_v - \delta D_u$ is large + positive, then can get k^2 st $\det < 0$ so those modes will grow.

$$\alpha D_v - \delta D_u >> 0 \Rightarrow \frac{D_v}{D_u} >> \frac{\delta}{\alpha} > 1$$

So this says to get spatial growth (patt form) need v to wander more than u

"cooperators stick together" defectors wander around to find more suckers to exploit!



$D_v = 10 D_u$

$b=1, d=1.2, N=8, r=2.5$

Front

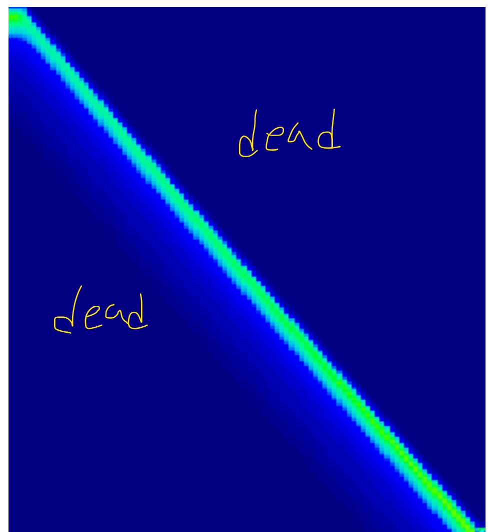
$r=3$

$D_u = 0.25$

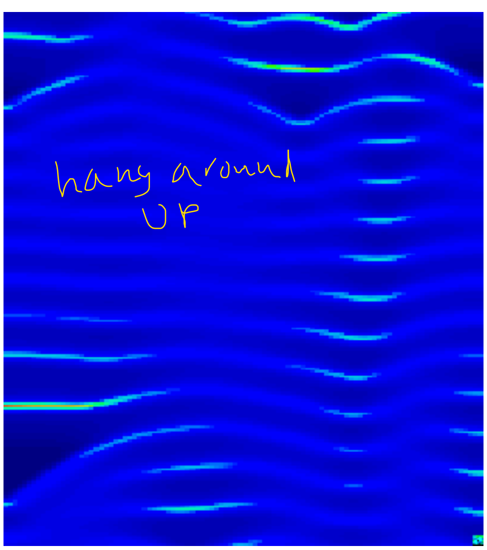
$D_V = 0.25$

bistable

pulse $r=2.3$, $D_U = .25$, $D_V = .125$



monostable

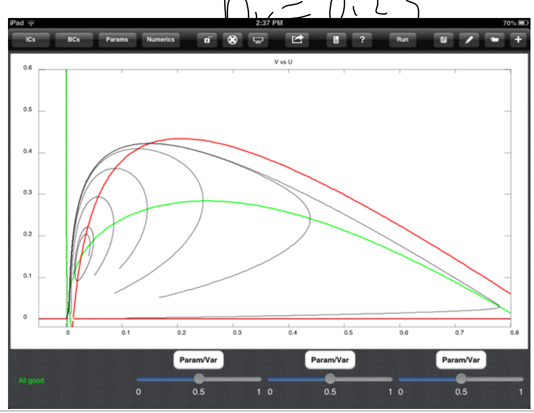


kuramoto/Shiv chaos ?

$b = 0.2$, $d = 0.3$

$r = 2.45$, $D_u = 0.25$

$D_v = 0.25$



Return to simple replicator plus space:
Hutson + Vickers. Need to be a little careful since with diffusion, cannot be sure that at each spatial location, the total is conserved. Thus we replace the usual replicator dynamics by:

$$u_i\left(\frac{(Au)_i}{N}-\frac{u^T A u}{N^2}\right)$$

Where $N=\sum u_i$. In absence of space $N=1$ is conserved.

In keeping with Vickers paper, $A=\begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix}$
so in absence of space. We let $u_1=u$, $u_2=v$
$f_u=\alpha u$, $f_v=\beta v$, $\phi=\alpha u^2+\beta v^2$
so get:

$$u\left(\frac{\alpha u}{u+v}-\frac{\alpha u^2+\beta v^2}{(u+v)^2}\right)=u\frac{\alpha u^2+\alpha uv-\alpha u^2-\beta v^2}{(u+v)^2}$$

$$=\frac{uv}{(u+v)^2}(\alpha u-\beta v)$$

for v, $-\frac{uv}{(u+v)^2}(\alpha u-\beta v)$

$$\frac{\partial u}{\partial t}=\frac{uv}{(u+v)^2}(\alpha u-\beta v)+D_u u_{xx}$$

$$\frac{\partial v}{\partial t}=\frac{-uv}{(u+v)^2}(\alpha u-\beta v)+D_v v_{xx}$$

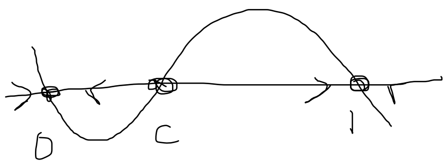
Suppose $D_u=D_v$. Then $(u+v)_t=D(u+v)_{xx}$ $\Rightarrow u+v\to$ constant, say 1

$$\frac{\partial u}{\partial t} = u(1-u)\left[\alpha u - \beta(1-u)\right] + D\frac{\partial^2 u}{\partial x^2}$$

$$= u(1-u)\left((\alpha+\beta)u - \beta\right) = (\alpha+\beta)u(1-u)(u-c), \quad c = \frac{\beta}{\alpha+\beta} \in (0,1)$$

By rescaling time & space, we can get rid of $\alpha+\beta$, D

$$u_t = u_{xx} + u(1-u)(u-c)$$



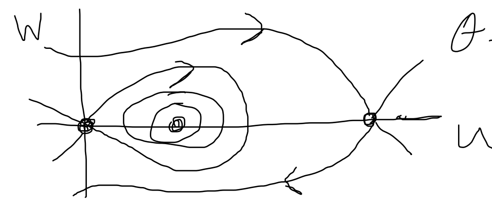
bistable (assume WLOG $c < \frac{1}{2}$) ($\beta < \alpha$))

Look for Traveling Wave

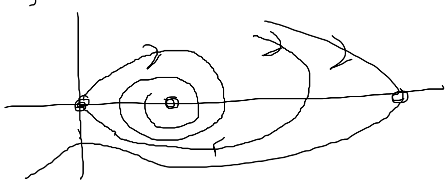
$$u(x,t) = U(x - \theta t)$$

$$-\theta U' = U'' + f(u)$$

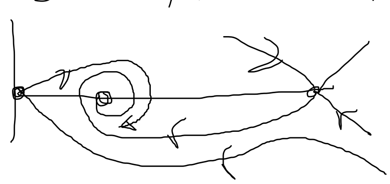
$$U' = W, \quad W' = -\theta W - f(u)$$



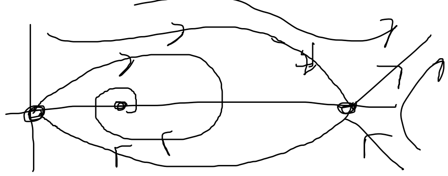
θ small + positive



θ large + positive



θ "just right"



Extortion Strategies & Zero Determinant (ZD) strategies

Here, we return to the iterated PD models with $\begin{pmatrix} R & S \\ T & P \end{pmatrix}$ as usual, e.g. $\begin{pmatrix} 3 & 0 \\ 5 & 1 \end{pmatrix}$

Press + Dyson show first, an interesting result. If a short memory player plays a long memory player, the short memory players score is the same as if the long memory guy played a short memory strategy. Because of this result, we can derive strategies for X (short mem) assuming each player has memory 1!

Let X, Y be random variables with values x, y that are the players respective moves on a given iteration. Suppose player X keeps history H_0 but Y keeps longer history H_1

We want to show that the joint prob distribution of (x, y) given history (H_0, H_1) averaged over (H_0, H_1) is same as joint prob of (x, y) averaged over the shorter history H_0

$$\langle P(x,y \mid H_0, H_1)\rangle_{H_0,H_1} \stackrel{\Delta}{=} \sum_{H_0,H_1} P(x,y \mid H_0,H_1)\, P(H_0,H_1)$$

$$= \sum_{H_0,H_1} P(x \mid H_0)\, P(y \mid H_0,H_1)\, P(H_0,H_1) \quad \leftarrow \text{conditional independ ??}$$

Definition of conditional

$$= \sum_{H_0} P(x \mid H_0) \left[\sum_{H_1} P(y \mid H_0,H_1)\, P(H_1 \mid H_0)\, P(H_0) \right]$$

$$= \sum_{H_0} P(x \mid H_0) \left[\sum_{H_1} P(y, H_1 \mid H_0) \right] P(H_0) \quad \text{sum over } H_1 \text{ gives marginal}$$

$$= \sum_{H_0} P(x \mid H_0)\, P(y \mid H_0)\, P(H_0)$$

$$= \langle P(x,y \mid H_0)\rangle_{H_0}$$

Intuitively - from X's point of view, X views Y's long strategy as a peculiar random number generator. Thus the player with the shortest memory sets the rules of the game

ZD Strategies:

As usual let $xy \in (cc, cd, dc, dd)$, where c, d are cooperate, defect

X's strategy is $\vec{p} = (p_1, p_2, p_3, p_4)$ are probabilities to cooperate given the outcome & $\vec{q} = (q_1, q_2, q_3, q_4)$ are Y's strategies, seen from his perspective, ie $yx = \{cc, cd, dc, dd\}$ (Note this is a little diff than we used before. So payoff is

$$\begin{bmatrix} p_i q_j & p_i(1-q_j) \\ (1-p_i) q_j & (1-p_i)(1-q_j) \end{bmatrix}$$

As before we get a Markov transition matrix:

$$M = \begin{bmatrix} p_1 q_1 & p_1(1-q_1) & (1-p_1) q_1 & (1-p_1)(1-q_1) \\ p_2 q_3 & p_2(1-q_3) & (1-p_2) q_3 & (1-p_2)(1-q_3) \\ p_3 q_2 & p_3(1-q_2) & (1-p_3) q_2 & (1-p_3)(1-q_2) \\ p_4 q_4 & p_4(1-q_4) & (1-p_4) q_4 & (1-p_4)(1-q_4) \end{bmatrix}$$

$\vec{V}$ steady states left eigenvector: $\vec{V}^T M = \vec{V}^T \Rightarrow \vec{V}^T M' = 0$

$M' = M - I \Rightarrow \det M' = 0$

Recall Cramers Rule: $\underset{(4\times4)}{Adj(M')}\, M' = \det(M')\, I = 0$

$\begin{pmatrix} a_{11} & \cdots & a_{14} \\ \vdots & & \end{pmatrix} M' = 0$

$Adj(M')$ = matrix of minors (3x3) determinants, since $V^T M' = 0$

This means every row of $Adj(M')$ is proportional to $\vec{V}$

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \quad C_{ij} \text{ cofactor } \quad Adj(M') = C^T$$

so, for example, up to sign, the components of $\vec{V}$ are the fourth row of $Adj(M')$, which are the cofactors of the 4^{th} column of M' which are just the determinants of the first three columns of M' leaving out the relevant row. These determinants are unchanged by adding the first column of M' to the second & third columns

Let $\vec{f}$ be arbitrary vector. Then $\vec{V}\cdot\vec{f}$ is easy to compute using the above manipulations:

$$\vec{V}\cdot\vec{f} = \det \begin{bmatrix} -1+p_1q_1 & -1+p_1 & -1+q_1 & f_1 \\ p_2q_3 & -1+p_2 & q_3 & f_2 \\ p_3q_2 & p_3 & -1+q_2 & f_3 \\ p_4q_4 & p_4 & q_4 & f_4 \end{bmatrix} = D(p,q,f)$$

$$\qquad\qquad\qquad = \tilde{p} \qquad \tilde{q}$$

Note that two columns depend only on one player's strategy!

$\vec{S}_X = [R, S, T, P]$, $\vec{S}_Y = [R, T, S, P]$

$$s_X = \frac{V\cdot\vec{S}_X}{V\cdot\mathbb{1}} = \frac{D(p,q,\vec{S}_X)}{D(p,q,\mathbb{1})}, \quad s_Y = \frac{V\cdot\vec{S}_Y}{V\cdot\mathbb{1}} = \frac{D(p,q,\vec{S}_Y)}{D(p,q,\mathbb{1})}$$

$$\alpha s_X + \beta s_Y + \gamma = \frac{D(p,q,\alpha\vec{S}_X+\beta\vec{S}_Y+\gamma\mathbb{1})}{D(p,q,\mathbb{1})} \longleftarrow \text{Really cool}$$

If I choose $\hat{p} = k[\alpha\vec{S}_X + \beta\vec{S}_Y + \gamma\mathbb{1}]$, then this determinant vanishes since two columns are proportional and this means I enforce a linear relationship between the payoffs no matter what q!!!

Of course, they may not be feasible since $\hat{p}_i \in [0,1]$

<u>EXAMPLE</u> X sets Y's score!

set $\alpha = 0$. Use $\tilde{p} = \beta\vec{S}_Y + \gamma\mathbb{1}$ & solve for p_2, p_3 in terms of p_1, p_4

eliminating β, γ. SHOW:

$$p_2 = \frac{p_1(T-P) - (1+p_4)(T-R)}{R-P}, \quad p_3 = \frac{(1-p_1)(P-S) + p_4(R-S)}{R-P}$$

To force $s_Y = \dfrac{(1-p_1)P + p_4 R}{(1-p_1) + p_4}$ in PD $T > R > P > S$

so there is a feasible strategy when $p_1 \lessapprox 1$, $p_4 \gtrapprox 0 \Rightarrow p_2 \lessapprox 1$, $p_3 \approx 0$. Clearly s_Y is weighted average of $1-p_1$ & p_4 so all possible scores between P & R are possible for Y. X can completely ignore Y but set Y's score. X can spoof Y & then play a better strategy

X cannot really set his own score: $\tilde{p} = \alpha \vec{S}_X + \gamma \mathbb{1}$

$$p_2 = \frac{(1+p_4)(R-S) - p_1(P-S)}{R-P} \geq 1 \qquad p_3 = \frac{-(1-p_1)(T-P) - p_4(T-R)}{R-P} \leq 0$$
or never coop

Only one feasible point: $(1,1,0,0)$ which is always cooperate. So X cannot unilaterally set his score.

"Extortion" Suppose $\tilde{p} = \phi[(\vec{S}_X - P\mathbb{1}) - \chi(\vec{S}_Y - P\mathbb{1})]$. Then this means $\phi[(s_X - P) - \chi(s_Y - P)] = 0 \Rightarrow (s_X - P) = \chi(s_Y - P) \Rightarrow$ X can make his gain above mutual defection (P) χ times greater than Y's !!

Solving the equations for p_i:

$$p_1 = 1 - \phi(\chi - 1)\frac{R-P}{P-S}, \quad p_2 = 1 - \phi\left(1 + \chi\frac{T-P}{P-S}\right), \quad p_3 = \phi\left(\chi + \frac{T-P}{P-S}\right), \quad p_4 = 0$$

so feasible strategies exist for any χ + small ϕ, eg

$$0 \leq \phi \leq \frac{(P-S)}{(P-S) + \chi(T-P)}$$

Clearly X's score depends on Y's strategy + both are maximized when Y cooperates! $(1,1,1,1)$ in which case

$$s_X = \frac{P(T-R) + \chi[R(T-S) - P(T-R)]}{(T-R) + \chi(R-S)} \quad (\chi = 1 \Rightarrow s_X = s_Y = R)$$

E.G. $(5,3,1,0)$,

$\tilde{p} = [1 - 2\phi(\chi-1), 1 - \phi(4\chi+1), \phi(\chi+4), 0]$, ok for $0 \leq \phi \leq (4\chi+1)^{-1}$

$s_X = \frac{2+13\chi}{2+3\chi} \quad s_Y = \frac{12+3\chi}{2+3\chi}$. Mutual coop $\Rightarrow 3$ $\quad s_X > 3$ if $\chi > 1$

$s_Y < 3$!!

As $\chi \to \infty$, $s_X \to \frac{13}{3}$ + $s_Y \to 1$ so Y has no reason to cooperate so X should not get too greedy!

Say $\chi = 3$ + ϕ is midpoint of feasibility $\vec{p} = \left(\frac{11}{13}, \frac{1}{2}, \frac{7}{26}, 0\right)$

$s_X \simeq 3.73$, $s_Y \simeq 1.91$. Note $\chi = 1$, $\phi = 1/5 \rightarrow$ TFT
$(1,0,1,0)$

Spatial games in a continuum

This is my version of a spatial game. We will later look at some others. Lets consider a two strategy game with a 2x2 payoff matrix $\begin{pmatrix} R & S \\ T & P \end{pmatrix}$ (not necessarily PD)

Let $u(x,t)$ be density of players using strategy C so that $v(x,t) = 1 - u(x,t)$ is density of players using strategy D. Now here is how I do the spatial game. Let me look at a weighted nbhd around x + compute the fitness if I played C:

$$f_C(x) = R \int w(y)\, u(x-y,t)\, dy \;+\; S \int w(y)\, v(x-y,t)\, dy$$

and fitness if I played D:

$$f_D(x) = T \int w(y)\, u(x-y,t)\, dy \;+\; P \int w(y)\, v(x-y,t)\, dy$$

Here $w(y)$ is a symmetric weighting function, say exponential or Gaussian.

I will go from $C \to D$ at a rate β + from $D \to C$ at a rate α where

$$\alpha = H(f_C - f_D), \quad \beta = H(f_D - f_C)$$ where H is some monotonic non negative function, eg $1/(1+e^{-\gamma x})$

so, eg if $f_C > f_D$, then α will be bigger than β + there will be a loss of D + gain of C at x. $\therefore \dfrac{\partial u}{\partial t} = \alpha v - \beta u = \alpha(1-u) - \beta u$

$$\alpha = H(f_C(x) - f_D(x)), \quad \beta = H(f_D(x) - f_C(x))$$

It is now clear how to do this for m strategy games

Let $A = (a_{ij})$ be payoff matrix. Let $u_i(x,t)$ be density playing strategy i. Then

$$f_i(x,t) = \int w(y) \sum_{j=1}^{n} a_{ij} u_j(x-y,t)\, dy$$

$$\alpha_{ij} = H(f_i - f_j) = \text{rate } j \text{ to } i$$

$$\frac{\partial u_i}{\partial t} = \sum_{j=1}^{n} \alpha_{ij} u_j \qquad \alpha_{ij} = H(f_i(x,t) - f_j(x,t))$$

This is not like replicator dynamics, which we will look at later

It is a <u>new</u> model. Lets look at equilibria for $n=2$

$$\frac{du}{dt} = \alpha(1-u) - \beta u = 0 \Rightarrow u = \frac{\alpha}{\alpha+\beta}$$

$$\alpha = H(a_{11}u + a_{12}(1-u) - a_{21}u - a_{22}(1-u))$$

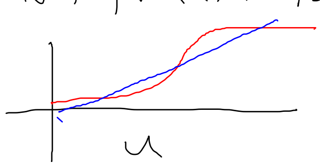
$$= H(a_{12} - a_{22} + (a_{11} - a_{21} + a_{22} - a_{12})u]$$

$$= H(\, c - (d-c)u\,)$$

$\beta = H(-c + (d-c)u)$ so as expected all that matters is the off diagonal terms, So WLOG we can Assume $A = \begin{bmatrix} 0 & c \\ d & 0 \end{bmatrix}$

For PD $d>0$, $c<0$. For bistable game: $c, d < 0$

So lets try that: Take $c=-1$, $d=-4$ for example + $\gamma = 10$



<u>3 roots</u> bistable

Suppose $W(x)$ is exponential + There are just two strategies as in our bistable game

$$f_1 = -c\, W(x) * (1-u), \quad f_2 = -d\, W(x) * u$$

$$= -c + c\, W(x) * u,$$

Let $z(x,t) = \int_{-\infty}^{\infty} w(x-y)\, u(y,t)\, dy$ + suppose $w(x) = \frac{1}{2} e^{-|x|}$

Then it is easy to show that:

$$z(x,t) - \frac{\partial^2 z}{\partial x^2} = u(x,t)$$

so we have

$$f_1 - f_2 = -c + c\, w*u + d\, w*u = -c + (d+c) z$$

$$f_2 - f_1 = c - (c+d) z$$

$$\frac{\partial u}{\partial t} = H(-c + (c+d) z)(1-u) - H(c - (c+d) z) u$$

$$\frac{\partial^2 z}{\partial x^2} = z - u$$

Fixed points ind of x, t are $z = u$

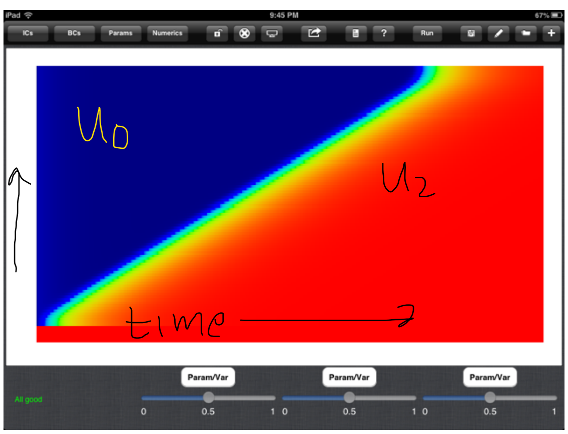
which has 3 equil $u_0 < u_1 < u_2$

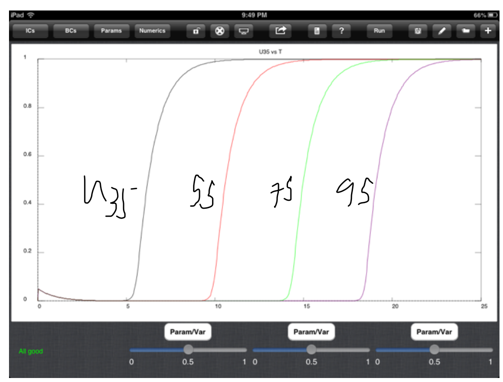
$$H(-c + (c+d) u)(1-u) = H(c - (c+d) u) u$$

Numerical solution

$\frac{e^{-|x|/5}}{10}$ $\Delta x = 0.2$

space





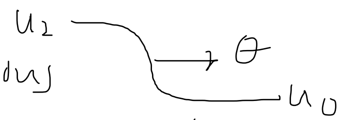
$\frac{20 \times \Delta x}{4.5} \simeq 0.88$ is velocity

Looks like traveling wave joining u_0 to u_2

Let $u(x,t) = U(x - \theta t)$, $\xi = x - \theta t$

$$-\theta \frac{dU}{d\xi} = H(-c + (d+c) z)(1-u) - H(c - (d+c) z) u$$

$$d^2 z / d\xi^2 = z - u$$

Look for solutions 

This is a heteroclinic orbit in the language of D.S.

$$u' = -\frac{F(u,z)}{\theta}, \quad z' = w, \quad w' = z - u$$

$$(u,z,w) = (u_0,u_0,0) \longleftarrow (u_2,u_2,0)$$

$$\vec{u}(-\infty) = (u_2,u_2,0), \quad \vec{u}(+\infty) = (u_0,u_0,0)$$

Linearize about $(u_0,u_0,0)$, Find 2+, 1− real eigenvalues
and around $(u_2,u_2,0)$, 2+, 1− real eigenvalue

1-d stable, 2D unstable