HOMEWORK # 1

1. Analyze the fixed points and their stability for the growth model:

$$x_{n+1} = r x_n e^{-\beta x_n}$$

2. Determine the existence and stability of all the fixed points for the discrete predator prey model

$$\begin{aligned} x_{n+1} &= r x_n (1 - x_n - y_n) \\ y_{n+1} &= y_n (d + f x_n) \end{aligned}$$

where all parameters are positive. One is only interested in nonnegative fixed points.

3. Simulate the trillium model without density dependent effects under the ambient case. Then change the two most important herbivory parameters, p_{al} and p_{ls} the probability of going from the adult reproductive (flowering) state to the large 3 leaf stage and the probability of going from the large to small state. Without (with) herbivory, these are $p_{al} = 0.37$, $p_{ls} = 0.17$ ($p_{al} = 0.61$, $p_{ls} = 0.33$). Here is an ODE file giving all the parameters

```
# trillium model, trillium3.ode
# germination
g'=gamma*sigma*a
# gamma is germination rate, sigma is number seeds per adult
par gamma=.79,sigma=7
# cotyledon stage
c'=pgc*g
par pgc=.51
# one leaf stage
d'=pdd*d+psd*s+pcd*c
par pdd=.5,psd=.04,pcd=.31
# small 3 leaf stage
s'=pds*d+pss*s+pls*l
par pds=.32,pss=.82,pls=.33
# large 3 leaf stage
l'=psl*s+(1-pls-pla)*l+pal*a
par psl=.09,pal=.61
# adult stage
a'=(1-pal)*a+pla*l
par pla=.09
init a=100
# tell XPP to use a discrete method, set total iterations
# set bounds, and plotting parameters
@ meth=discrete,total=400,bound=100000,xhi=400,yhi=100,ylo=0,yp=a
done
```

4. Simulate the trillium model with density dependence on the flowering adults. Show that the model leads to a stable population. Change the parameter k_a which determines the shape of the density dependence. Here again is an ODE file,

```
# trillium model with density dependence trildens.ode
# pls, pal are functions of the adult population
# germination
g'=gamma*sigma*a
# gamma is germination rate, sigma is number seeds per adult
par gamma=.79,sigma=7
# cotyledon stage
c'=pgc*g
par pgc=.51
# one leaf stage
d'=pdd*d+psd*s+pcd*c
par pdd=.5,psd=.04,pcd=.31
# small 3 leaf stage
# pls is saturating function depending on adults
s'=pds*d+pss*s+pls(a)*l
par pds=.32,pss=.82
pls(a)=pls0+pls1*a/(ka+a)
par pls0=.17,pls1=.2,ka=500
# large 3 leaf stage  pal is density dependent
l'=psl*s+(1-pls(a)-pla)*l+pal(a)*a
par psl=.09
pal(a)=pal0+pal1*a/(ka+a)
par pal0=.37,pal1=.4
# adult stage
a'=(1-pal(a))*a+pla*l
par pla=.09
init a=100
# tell XPP to use a discrete method, set total iterations
# set bounds, and plotting parameters
@ meth=discrete,total=2000,bound=100000,xhi=2000,yhi=100,ylo=0,yp=a
done
```

5. Trillium has shown some pollen-related limitations. That is, if there are many adults, then the number of bees that visit the flowers increase so that the pollination is increased and thus the number of seeds produced per plant increases. How would you alter the above model to incorporate this. Note that there is a minimum number of seeds produced per plants even at low densities and the number will increase to some maximum number.

6. Analyze the existence and stability of the endemic ($i_n \neq 0$) states in the

two disease models

$$\begin{aligned} s_{n+1} &= s_n + \alpha(1 - s_n - i_n) - \beta i_n s_n \\ i_{n+1} &= i_n + \beta s_n i_n - \gamma i_n \end{aligned}$$

for a recurring infectious disease (eg spread by bacteria); and

$$\begin{aligned} s_{n+1} &= \delta(1 - s_n) + s_n - \beta i_n s_n \\ i_{n+1} &= i_n + \beta s_n i_n - (\gamma + \delta) i_n \end{aligned}$$

which models a viral disease in the population. Note that all parameters are positive.

7. Simulate the disease models starting with different initial data

8. Develop a discrete model for the following situation. A prey species grows to some maximal density in absence of prey species. There are two predator species which in absence of the prey both die at the same rate. They both affect the prey identically, but the utilization of the prey by the predators is not identical; that is, one predator makes better use (in the model in problem 2, only the parameter f will differ.) What sort of behavior can you find? Does one predator always win? Can you ever get coexistence?