Assignment 1:
Quasi-Species

Due Thursday, Feb 8,2007

The quasispecies equation is of the form

$$\dot{x}_i = \sum_{j=1}^{n} f_j x_j q_{ji} - \phi x_i.$$

The average fitness is given by $\phi = \sum_i x_i f_i$. The mutation matrix is given by

$$q_{ij} = (1-u)^{L-h_{ij}} u^{h_{ij}}.$$

Here L denotes the sequence length and u the mutation rate per bit. The Hamming distance, h_{ij}, counts the number of point mutations between sequences i and j.

Take a binary quasispecies of length $L = 5$. There are 32 different sequences. Consider the following fitness landscape:
00000 ... 10.0
01111 ... 8.5
10111 ... 8.8
11110 ... 8.9
11111 ... 9.0
All other sequences have fitness 1.

How can you calculate the equilibrium distribution of the quasi-species equation?

One paragraph with a "formula-based" argument in which you rewrite the quasi-species equation in matrix form. This is very easy and also in the textbook. But pay attention to the indexes i and j and from which side you multiply $\vec{x}$.

If $\vec{x}$ a vector, which properties have to be satisfied to make it a vector of frequency distributions?

two conditions

How do you calculate the average fitness at equilibrium?

one sentence

If you increase u beginning from zero, at which point ($=: u_{\max}$) have organisms lost their ability to replicate? That is, when are the bits completely randomized

one value for $u_{\max}$ and a one-sentence explanation.

What does the equilibrium distribution look like for $u = u_{\max}$.

One expression with a one-sentence explanation

Plot the equilibrium frequencies for all x_i versus the mutation rate $u \in [0, u_{\max}]$.

You can do this with matlab or using the XPP code which I have placed on the web - fitness.ode fbook.tab. You can plot all the equilibria or just the important ones whose fitness is greater than 1.

Plot the average fitness at equilibrium versus u.

one plot with one line in it

How many error thresholds do you observe?

Max 5 sentences describing how may thresholds you observe and what happens at each of them

What does 'survival of the fittest' mean and how is it related to what you observe here?

one paragraph