Homework # 2 Math 3380 DUE FEB 18

1. For the two strategy game with payoff $A = (a_{ij})$, prove that there is an interior equilibrium, $x^* \in (0,1)$ if and only if $(a_{11} - a_{21})(a_{12} - a_{22}) < 0$.

2. Given an $n \times n$ matrix, A, consider a new matrix B where $b_{ij} = a_{ij} - c_j$ for some c_j. Prove that the replicator equations

$$x_i' = x_i(f_i^A - \phi^A)$$

where $f_i^A = (Ax)_i$ and $\phi^A = x \cdot Ax$ are the same if A is replaced by B.

3. Let A be the payoff matrix for a game with n strategies. Suppose the pure strategy, R_i is a strict Nash equilibrium. Prove that $x_i = 1$, $x_j = 0$, $j \neq i$ is an asymptotically stable equilibrium for the replicator dynamics. (Hint, just prove it for $i = 1$.)

4. For the rock paper scissors game, the payoff is

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

Prove that $V = x_1 x_2 x_3$ is a constant of motion for the corresponding replicator dynamics

5. In the Hawk Dove game there exists a mixed strategy which is evolutionarily stable. This strategy is given by playing Hawk with probability b/c and playing Dove with probability $1 - (b/c)$. Consider the interaction between three strategies: (i) pure Hawk, (ii) pure Dove, and (iii) the above mixed strategy.

Calculate the missing values in the payoff matrix

$$A = \begin{array}{c} \\ \text{Hawk} \\ \text{Dove} \\ \text{Mixed} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Hawk} & \text{Dove} & \text{Mixed} \end{array} \\ \begin{pmatrix} \frac{b-c}{2} & b & a_{1,3} \\ 0 & \frac{b}{2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} \end{array}$$

6. Consider the game with payoff matrix:

$$A = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

Find the Nash equilbria and show that they are asymptotically stable but not evolutionarily stable, Sketch the phase-portrait for the three strategy game. (Here, you should use the replicator equations to find the mixed probability strategy.)

7. Consider the game with payoff matrix:

$$A = \begin{pmatrix} 0 & 10 & 0 \\ 10 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Show that strategy 3 is stable against invasion by strategy 1 alone and against strategy 2 alone, but not both. Sketch the phase-portrait for the three strategy game using the replicator equations.

8. Construct a payoff matrix for four strategies: (i) Hawk always escalates the battle, (ii) Dove which displays etc, but eventually retreats (iii) retaliators which stick to displays unless the opponent escalates and then they escalate too; (iv) bullies who fake escalation but run off if their opponent escalates too. Simulate your model using the replicator equations.