1. Please prove the following statements from the Press et al paper
   (a) Since the classical adjoint, A satisfies $AM' = det(M')I = 0$ where $M' = M - I$, show that this means that each row of A is proportional to the left eigenvector, $v^T M = v^T$.
   (b) Prove the statement that $v \cdot f = CD(p, q, f)$ where C is some constant (that comes from not normalizing the rows of the adjoint).This requires a little bit of work.
   (c) Prove equations (8,9) in the paper
   (d) Prove equations (12-14)
2. Use simulation to compare the payoff of the extortion strategy (with a small epsilon chance of cooperating when DD) , $(11/13, 1/2.7/26, .01)$ against TFT $(1, 0, 1, 0)$, ALLD $(0, 0, 0, 0)$, ALLC $(1, 1, 1, 1)$, WSLS $(1, 0, 0, 1)$, ZDGTFT-2 $(1, 1/8, 1, 1/4)$, EXTORT-2 $(8/9, 1/2, 1/3, 0)$, TFT with error $(.9, .1, .9, .1)$, and, say, four randomly chosen strategies , (q_1, q_2, q_3, q_4) (that is, pick each q_i using a random number from your calculator or something). You should simulate for 2000 rounds and except for ALLD, play C first as each player. Make a table showing the expected payoff of X and Y. If you are Y, which is the best strategy for you? Here is my version in XPP:

```
# simulate payoff from two players
par p1=.846,p2=.5,p3=.269,p4=0.01
# for x xy cc cd dc dd
# for y yx cc cd dc dd
# play 1 to cooperate and 0 to defect
par q1=.5,q2=.5,q3=.5,q4=.5
init x=1,y=1
par rr=3,tt=5,ss=0,pp=1
# choose which probability to use based on last move
px=p1*x*y+p2*x*(1-y)+p3*(1-x)*y+p4*(1-x)*(1-y)
py=q1*x*y+q2*(1-x)*y+q3*(1-y)*x+q4*(1-x)*(1-y)
# choose new strategy
xp=ran(1)<px
yp=ran(1)<py
# reward (cc)
co=xp*yp
# punishment (dd)
de=(1-xp)*(1-yp)
# sucker - from y's pt of view
su=(1-xp)*yp
# temptation from y's pt of view
te=(1-yp)*xp
# update
x'=xp
```

```
y'=yp
# compute running total
sxt'=sxt+rr*co+de*pp+su*tt+te*ss
syt'=syt+rr*co+de*pp+su*ss+te*tt
# and running average
aux sx=sxt/max(t,1)
aux sy=syt/max(t,1)
@ meth=discrete,total=2000,bound=10000000
@ xp=t,yp=sx
@ nplot=2,xp2=t,yp2=sy
@ xhi=2000,ylo=-0.5,yhi=5.5
done
```

3. In class, I proposed an alternative to replicator dynamics based on the following idea. Given a payoff matrix, A for N strategies, and a distribution, $\vec{u}$ of players (so that u_i is the fraction playing strategy i), the fitness $f_i = (A\vec{u})_i$. Given a function $H(x)$ that is nonnegative and increasing, then my model has the following equations:

$$u_i' = \sum_j H(f_i - f_j)u_j - H(f_j - f_i)u_i \quad (*)$$

In the remainder of this exercise, let $H(u) = 1/(1 + \exp(-10u))$.

- Take $N = 2$ and $A = [0 - c; -d, 0]$ where c, d are positive. Write a single ODE for u_1, say, $u_1' = g(u_1)$. Fix $c = 1$ and plot $g(u_1)$ as you vary d between 0 and 3. At what point d does there appear a new stable equilibrium? Suppose we take $H(u)$ to be the step function: $H(u) = 0, u < 0$, $H(u) = 1, u > 0$, $H(0) = 1/2$. For a two player game, show that if a pure strategy is a strict nash equilibrium, then it is also an equilibrium of (*) and that it is stable,
- Now consider the three strategy game with matrix

$$A = \begin{bmatrix} 0 & 1 & -b \\ -b & 0 & 1 \\ 1 & -b & 0 \end{bmatrix}$$

When $b = 1$ this is a rock-paper-scissors model and we know that it creates heteroclinic cycles in replicator dynamics. For dynamics of the form (*) with $H(u)$ as specified, it is easy to show that $u_j = 1/3$ is an equilibrium point. Either analytically or numerically, study the stability and behavior as b increases from 0 to 5, Here is an XPP file if you want:

```
# three player game that is kind of RPS
h(u)=1/(1+exp(-gamma*u))
par gamma=10
```

```
par a=1,b=1
init u1=.6,u2=.1
u3=1-u1-u2
f1=a*u2-b*u3
f2=-b*u1+a*u3
f3=a*u1-b*u2
u1'=h(f1-f2)*u2+h(f1-f3)*u3-(h(f2-f1)+h(f3-f1))*u1
u2'=h(f2-f1)*u1+h(f2-f3)*u3-(h(f3-f2)+h(f1-f2))*u2
aux u3=u3
@ xp=u1,yp=u2,xlo=0,ylo=0,xhi=1,yhi=1
@ nmesh=80,total=40
done
```

4. Prove that if

$$z(x) = \frac{1}{2}\int_{-\infty}^{\infty} e^{-|x-y|} u(y)\ dy$$

then

$$\frac{d^2 z}{dx^2} = z(x) - u(x)$$