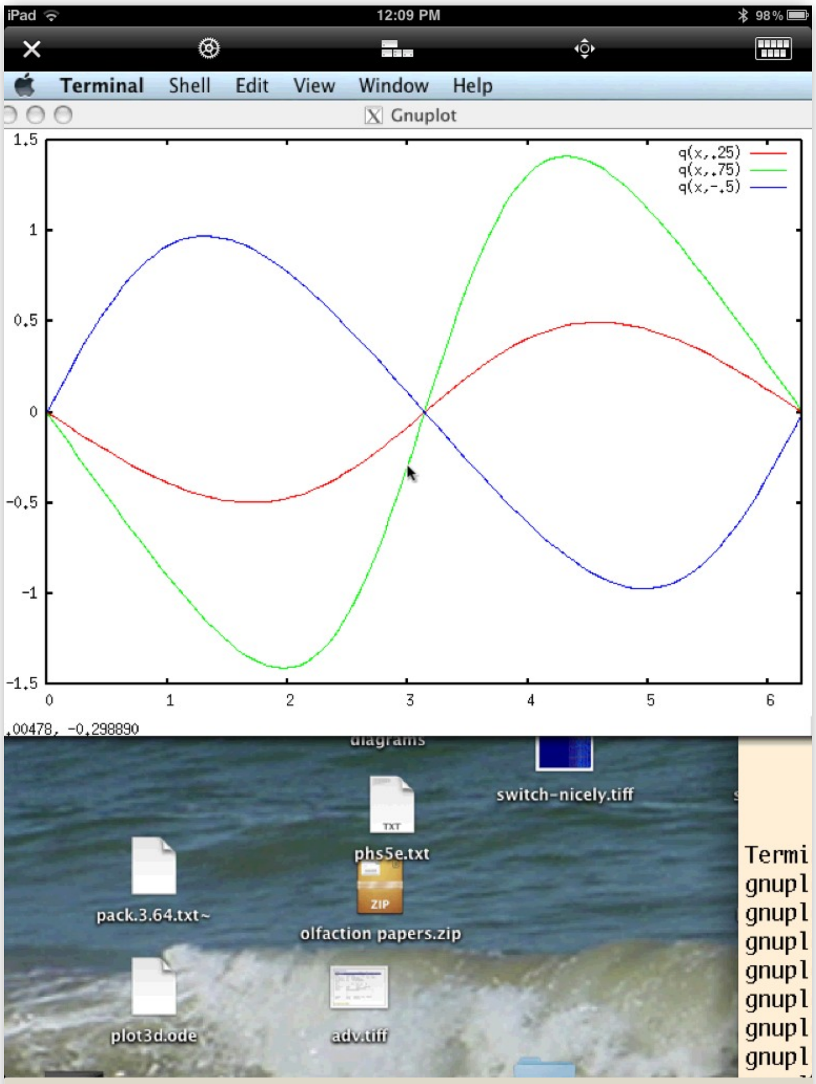
We will study the map for a pair of coupled PRCs. Let F(x) be the PTC and assume that F'(x)>0, F(0)=0,F(T)=T
We will create a Poincare map for the time that oscillator B fires
Let y be the phase of B just as A hits T. Then B-> F(y) and A->0. Since F(y)<T as y<T and F'(x)>0, oscillator B will not yet have fired. It takes T-F(y) time units for B to reach T since it has dynamics X'=1. Thus, A which has the same dynamics advances to T-F(y) and is reset to F(T-F(y)) while B is reset to 0. Thus, A will fire at T-F(T-F(y)) and our map is thus

$$y' = T-F(T-F(y))=G(y)$$

A fixed point, p satisfies p=G(p) and it is stable if |G'(p)|<1. However, note that G'(p)=F'(T-F(p))F'(p) and by hypothesis, F'(*)>0, so stability just means that G'(p)<1. To see the fixed points, we need only plot Q(p)=G(p)-p, and the stable ones will be those for which Q'(p)<0!

For example, let F(x)=x-b sin(x), where |b|<1. Then T=2 pi and it is clear that p=0is a fixed point. G'(0)=(1-b)^2 < 1 if 0 < b, so that we get stable dynamics when 0 < b < 1. (Stability extends beyond b=1, but that would violate our hypothesis on F'(x)>0. Here is a picture of Q for three values



Q(x) for several values

of b. Note that when b<0, the fixed piont at pi is stable (blue curve).

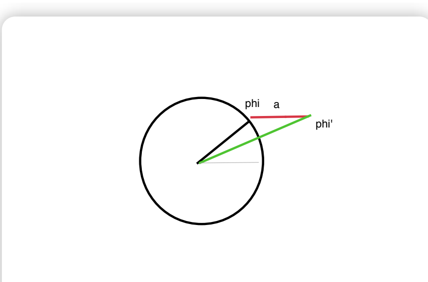
HW.

1. Let the PRC be defined as -a sin(x) - b sin(2x) and the PTC F(x)=x-a sin(x)-b sin(2x) For what range of (a,b) is it true that F'(x)>0 for all x in [0,2 pi]? Plot Q(x) and find the fixed points and their stability for the following pairs (a,b): (0.5,0.2),(0.5,0.4),(0.5,-0.2),(0.5,-0.4) Which cases have multiple stable fixed points?

2. Let F(x)=x+a(1-cos(x)). For what range of a is F'(x)>0? For a=0.5, plot Q(x). Are there any fixed points and are they stable? With this PTC, is possible to stabilize a fixed point interior to (0,2pi)?

3. Suppose that F(x)=x+c x^2(1-x), so that T=1. Study the fixed points of the composite map, G(x)=1-F(1-F(x)) as a function of the parameter c and also figure out the range of c for which the hypothesis F'(x)>0 holds.

4. Recall the PTC for the radial isochron clock that you were asked to compute for homework.



RIC PRC

phi'=atan[sin(phi)/(a+cos(phi))]==F(phi)

Using this map as your PTC, study the behavior of the composite map G(phi) for a=0.5, a=-0.5, and a=1.25. You should be quite careful of how you define atan. You may want to use the numerical function, atan2(y,x)=atan(y/x) where the signs of x and y are also considered.