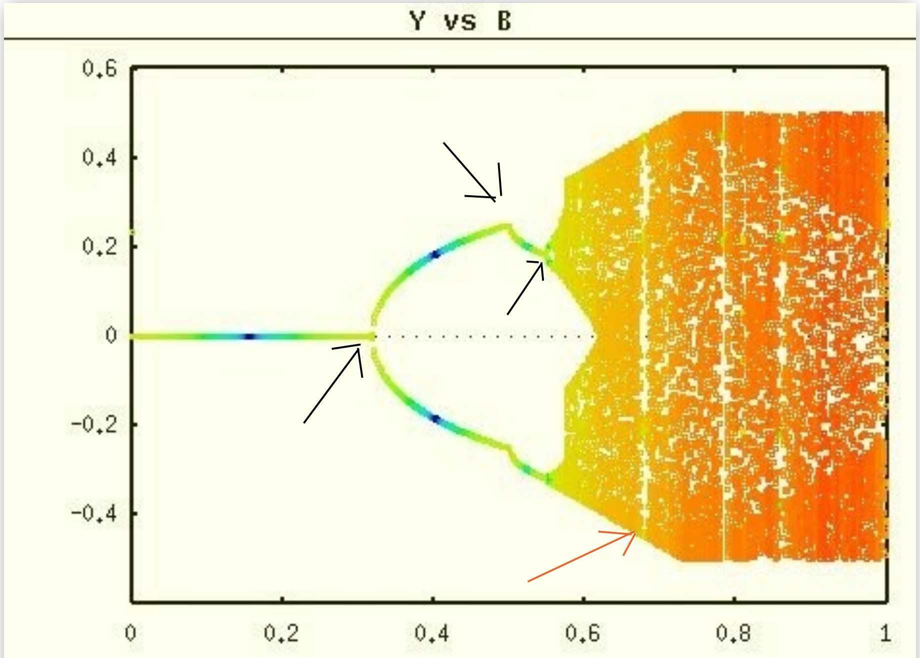

orbit diagram & liapunov exponent

The standard map is what arises when we set tau=0 and vary b obtaining the map
x'=x - b sin(2 pi x)== S(x)
The figure on the left shows

(x+1/2) mod 1 - 1/2

so that you can see the dynamics better. I plot 50 points from the orbit after waiting for 700 points as a transient. I start with x_0=0.1234
The color code is the Lyapunov exponent, with blue being most negative and orange the most positive. Near the bifurcation points (where there are branches, shown with arrows) the LE goes to zero. There are regions of chaos with small windows of periodic orbits, shown by the orange arrow, as an example.

HW. Find the stability of fixed points for S(x) as a function of the parameter, b. The first arrow in the diagram shows the first period doubling bifurcation. Find the value of this point. Try to reproduce this diagram (don't bother with the color code). Also compute the diagram for -1 < b < 0.

Here is XPP code:

```
# initconds range OK
@ meth=discrete,total=250,trans=200
x'=x-b*sin(2*pi*x)
b'=b
init x=.2345
@ xp=b,yp=y
@ xlo=0,xhi=1
@ rangeover=b
@ rangelow=0,rangehigh=1,rangestep=500
@ rangereset=0
@ ylo=-.6,yhi=.6
@ maxstor=1000000
@ lt=0
# this is for pretty plotting purposes
aux y=mod(x+.5,1)-.5
done
```

Here are some more HW problems

1. For the forced oscillator (note x' means x(t+1) here since we are talking maps)

$$x' = x + \tau - b \sin(x) = f(x) \quad (*)$$

Set b=0.5 and compute the range of tau such that there is 1:2 locking. That is, x + 2 pi = f(f(x)) has a solution. For a particular value of tau (you can choose it how you like) find the exact values of the fixed points and their stability.

2. Compute the Devil's staircase for the map

$$x(t+1) = F(x(t)) + \tau$$

for F(x) = x + 0.75 (1-cos(x))

(a) First find values of tau for which there is 1:1 locking

(b) Compute range of tau for 1:2 locking (x(t+2)=x(t)+2 pi)

(c) Now compute the Devil's staircase by plotting the rotation number as a function of tau.

x(0)=0. Iterate to get x(2000). Plot x(2000)/(2000 * 2 pi) vs tau for tau between 0 and 10 in increments of say 200 steps.