Homework # 1, Comp Neuro Fall 2011

1. **Computing FI curves for various models.**

   (a) LIF. We have already analytically derived the firing rate for th leaky integrate and fire model. It is not possible to derive it for the LIF with adaptation. Thus, I want to you do this neumerically. Use the following version

$$\begin{aligned} C\frac{dV}{dt} &= -gl(V-E_L) - g_a w(V-E_K) + I(t) \\ \tau_a \frac{dw}{dt} &= -w \end{aligned}$$

   with $V \to V_{reset}, w \to w+1$ when $V$ hits $V_{thr}$ and

$$I(t) = I_0$$

   for $t_{on} < t < t_{on} + t_{dur}$ and $I(t) = 0$ otherwise. Use $g_l = 0.05$, $C = 1$, $E_L = V_{reset} = -65$, $V_{thr} = -50$, $E_K = -90$ and $\tau_a = 80$. First set $g_a = 0$ and plot $V(t)$ for $I_0 = 1$, $t_{on} = 20$, and $t_{dur} = 800$ for a total of 1 second. (Note all units of time are in msec; $C$ is in $\mu F/cm^2$, $I$ is $\mu A/cm^2$, $V$ is mV, and $g$ is $mS/cm^2$. Unless otherwise noted, these will be my standard units.) I am afraid I don't know any MatLab, but will put some MatLab code up when I can find it. I will also post some XPP code. For $I_0 = 5, g_a = .05$, plot the voltage as a function of time. Describe the differences both during and after the stimulus.

   - For the same simulation plot the interspike interval of successive spikes as a function of the spike number.
   - Plot The ISI vs spike number for $I_0 = 5$ (this is a real high frequency oscillation). Plot the ISI for $g_a = 0.01, 0.02, 0.03, 0.05$, if possible on the same curve. (I provide lifa.ode)
   - Finally, plot the frequency as a function of current as follows. Solve the equations for the given applied current, for say 1 second to get rid of transients and then integrte long enough to compute one interspike interval. Plot 1000 divided by this to get the frequency in Hz. (I have provided lifa_FI.ode)

   (b) **Quadratic I and F** The QIF with no adaptation has the form

$$C\frac{dV}{dt} = I + g_L \frac{(V-V_L)(V-V_T)}{(V_T - V_L)}$$

   along with the rule that $V \to V_{reset}$ when $V = V_{spike}$. Assume the following $V_{reset} \le V_L < V_T < V_{spike}$. Find the minimum current for this neuron model to begin to spike. Then compute the FI curve analytically as follows:

$$T_{ISI} = C \int_{V_{reset}}^{V_{spike}} \frac{dV}{I + g_L \frac{(V-V_L)(V-V_T)}{(V_T-V_L)}}$$

With adaptation, we have the equation

$$\begin{aligned} C\frac{dV}{dt} &= I + g_L\frac{(V-V_L)(V-V_T)}{(V_T-V_L)} - w \\ \tau_a\frac{dw}{dt} &= b(V-V_L) - w \end{aligned}$$

with $V \to V_{reset}$ and $w \to w + d$ when $V = V_{spike}$. When $b = 0$, then this is analogous to the QIFA, with no subthreshold adaptation. However, if $b > 0$, there is additional impedence and the threshold to spiking will be changed. Find a value for $I$ that guarantees there will be spiking. (Hint, set $w = b(V - V_L)$ and then find $I$ so that there are *no* points where $dV/dt$ vanish.) Note that $b$ has a big effect on the dynamics at rest. Compute the FI curve for this model where $I_0$ ranges between 0.25 and 3.25 for $b = 0, d = 0$, $b = 0.15, d = 0$, $b = 0, d = 1$, and $b = .15, d = 1$. Use $E_L = -65$, $C = 1$, $g_L = 0.05$, $V_{th} = -50$, and $V_{spike} = 20$.

(c) **Exponential I& F.** The EIFA (with adaptation) has the form

$$\begin{aligned} C\frac{dV}{dt} &= I - g_L(V-V_L) + g_L\Delta e^{(V-V_{th})/Delta} - w \\ \tau_a\frac{dw}{dt} &= b(V-V_L) - w \end{aligned}$$

with $V \to V_{reset}$ and $w \to w + d$ when $V = V_{spike}$. Set $g_L = 0.1, \Delta = 2, V_{th} = -50, V_{spike} = 20$, $E_L = -65 = V_{reset}$, $\tau_a = 80$ and compute the FI curve for $I_0$ between 1.7 and 4.7, and $(b, d) = (0, 0), (0.1, 0), (0, 0.5), (0.1, 0.5)$.

2. **Hodgkin-Huxley Equations** There is a Matlab file *HHPulse.m* and an XPP file *HHpulse.ode* which allow you to inject pulses etc into the HH equations.

   (a) Starting at rest, inject a step of current lasting 200 msec of varying amplitudes between 0 and 15. Describe what happens as the magnitude goes up.

   (b) Inject a pulse of the form $I(t) = I_0H(t-t_{on})H(t_{on}+t_{dur}-t)$ (where $H$ is the step function) with $I_0$ negative. Find $t_{dur}$ such that a spike is elicited. Plot $m, n, h$ to see what is going on.

   (c) Repeat the above, but use a double pulse

   $$I(t; I_1, I_2) = I_1H(t-t_1)H(t_2-t) + I_2H(t-t_2)H(t_3-t)$$

   where $I_1 < 0$ and $I_2 > 0$ and $t_1 < t_2 < t_3$ such that neither $I(t; 0, I_2)$ nor $I(t; I_1, 0)$ elicit a pulse, but $I(t; I_1, I_2)$ does.

   (d) Inject a triangle current of the form

   $$I(t) = I_R(t/a)H(a-t)H(t) + I_R(1-(t-a)/a)H(t-a)H(2a-t)$$

for $2a$ msec. Choose $a = 2000$ and $I_R = 20$. (This is a very slow ramp.) At what value of $I(t)$ does the neuron start to fire and at what value to it stop firing? This is a phenomena known as *hysteresis* and demonstrates that the HH model is bistable.

(e) What is threshold? Threshold for firing is not a real clear concept as the following will illustrate. Inject a simple triangle current:

$$I(t) = I_R t/aH(a-t)H(t)$$

choosing $I_R = 15$. Vary $a$ from 5 to 100 and take note of the current value at which a spike is emitted; note that it depends on the slope or velocity of the stimulus.

(f) Compute the FI curve for the HH equations for $I$ between 0 and 30.

3. Compute the strength duration curev for the LIF analytically and plot it for $g_L = 0.05, V_{th} = -50, V_{reset} = E_L = -65, C = 1$ as follows. Solve

$$C\frac{dV}{dt} = I(t) - g_L(V - E_L), \quad V(0) = E_L,$$

where $I(t) = I_0 H(t)H(t_D - t)$ and $H(t)$ is the Heaviside step function (as usual). You really have to only solve it up to $t = t_D$. For each $t_d$, find the minimum value of $I_0$ so that the neuron reaches threshold to firing. Plot $I_0$ vs $t_D$.