MATH 3375 - Assignment 4 – Due Nov 2

1. In this exercise, you simulate driving an integrate and fire neuron with a train of alpha functions that are generated from a Poisson process. Here is what you want to model:

$$\tau \frac{dV}{dt} = -(V - V_l) - As(V - V_{syn})$$

where s is modeled as an alpha function:

$$s'' + 2rs' + r^2 s = 0$$

and each time a spike comes, $ds/dt \to ds/dt + r^2$. The Poisson rate is 1000 Hz which is 1/msec. Choose $r = 0.25$ (a 4 msec time constant), $A = .5$, $V_{syn} = 0$, $V_l = -65$, $V_{spike} = -50$, $V_{reset} = -60$, $\tau = 20$ msec. You are free to do this using whatever software you'd like. Simulate it for 1000 msec. If you need help, I can show you how to do it in XPP. If you manage to get it working, try the following:

- Roughly, what is the average value of $s(t)$. Can you show this analytically? What if the rate of the Poisson process is 250 Hz (0.25/msec)? Does the mean of s depend on r?
- Do say, 100-500 trials and save the total spike count of the integrate and fire model for each trial. Compute the Fano factor of the spike count as well as the mean spike rate.
- Repeat the above for $A = .35$ and $A = .25$. What do you notice about the Fano factor? At low firing rates, the neuron is more sensitive to fluctuations so that you expect noisier statistics.

2. CV for balanced and unbalanced networks. Consider a LIF model that is bombarded with synaptic input from excitatory and inhibitory neurons which are firing at a constant Poisson generated rate.

$$\tau_m \frac{dV}{dt} = -V + I_e s_e - I_i s_i$$

where I_e, I_i are current synapses (instead of conductance-based), and $\tau_m = 10$. Units are mV and msec. When $V = 10$, V is reset to -1. s_e, s_i are driven by N_e, N_i Poisson generated spikes, with rates, r_e, r_i. That is

$$\begin{aligned}\frac{ds_e}{dt} &= -s_e/\tau_e + \sum_{jk} \delta(t - t^e_{jk}) \\ \frac{ds_i}{dt} &= -s_i/\tau_i + \sum_{jk} \delta(t - t^i_{jk}).\end{aligned}$$

Here $t^{e,i}_{jk}$ are the k^{th} spike times of the j^{th}. Your task is to simulate this model as you vary the strength of inhibition, I_i between 0 to 1. Here is a simple way to simulate a synapse bombarded with Poisson generated spikes. Suppose that N is the number of presynaptic neurons and r is the rate of each one. Let h be the time step of interest. Then we can compute $\rho = Nrh$ as the impact to a Poisson random number generator, such as `poissrnd()` in MatLab, or `poisson()` in XPP. This function returns a non-negative integer corresponding to a random number of spikes in the dimensionless interval ρ. Thus the effective model is

$$\frac{ds}{dt} = -s/\tau + \frac{1}{h}\text{poissrnd}(Nrh)$$

Division by h is necessary to approximate the delta function. The expected value of s is just $\tau r N$.

So take $N_e = 4000, N_i = 1000$, $\tau_e = 5$, $\tau_i = 8$, $r_e = 5$ Hz, $r_i = 12$ Hz, and $I_e = 1$ mV. Vary I_i from 0 to 0.8 mV and compute the CV of the ISI for the above LIF model as a function of I_i. With low values of I_i observe that the CV is very low but rises above 1 as I_i increases. Why is this? Simulate for 30 seconds. If you are stuck, I can supply an XPP file. What is the expected value of s_e, s_i and $I_e s_e - I_i s_i$? At what value of I_i does the expected synaptic drive take you to threshold. What happens if I_i is too high? (say, 1)

Consider now, a related problem. Set $I_i = 0$ and $I_e = 0.15$ Simulate for 30 seconds and observe that the mean firing rate is roughly 50 Hz. What is the CV? Note that the expected value of $I_e s_e$ is 12 mV, which is 2 mV above threshold. Now choose $I_e = 0.4, I_e = 1$ and choose I_i so that $I_e s_e - I_e s_i$ remains about 12 mV. Compute the CV in both cases and note the mean firing rate (which should still be about 50 Hz). This shows that scaling the E and I inputs together can maintain a constant firing rate but cause a big change in variability.

3. Consider a renewal process with a hazard function given by the following. $H(\tau) = h$ for $\tau < 1$ and $H(\tau) = 1$ for $tau > 1$. If $h < 1$, then right after a spike the probability of firing is lower – you can think of this as some kind of outward current like a potassium AHP. If $h > 1$, then this can be though of as an afterdepolarization (ADP) for example due to a calcium channel in the dendrite which causes the soma to be more depolarized after a spike. Simulate this process for $h = 0, h = 1, h = 2$ up to $t = 2000$ to get a nice number of interspike intervals. Compute the CV. Use the formula:

$$P(\tau) = H(\tau)\exp(-\int_0^\tau H(s)\ ds)$$

to analytically compute the mean spike time:

$$\tau_1 = \int_0^\infty \tau P(\tau)\ d\tau$$

and the second moment:

$$\tau_2 = \int_0^\infty \tau^2 P(\tau)\ d\tau$$

and thus the CV:

$$\mathrm{CV} = \frac{\sqrt{\tau_1^2 - \tau_2}}{\tau_1}$$

and plot the CV as a function of h. Note that you can get a CV greater than 1!

Simulate the process for even longer (say $T = 10000$) and collect the spike times. Plot the autocorrelation of the spike times in the window $[-5, 5]$ divided into 100 bins. Do this for $h = 0, 1, 2$.

4. Simulate the random-dot discrimination experiment. Denote the stimulus by plus or minus, corresponding to the two directions of motion. On each trial, choose the stimulus randomly with equal probability for the two cases. When the minus stimulus is chosen, generate the responses of the neuron as 20 Hz plus a random Gaussian term with a standard deviation of 10 Hz (set any rates that come out negative to zero). When the plus stimulus is chosen, generate the responses as $20 + 10d$ Hz plus a random Gaussian term with a standard deviation of 10 Hz, where d is the discriminability (again, set any rates that come out negative to zero). First, choose a threshold $z = 20+5d$, which is half-way between the means of the two response distributions. Whenever $r > z$ guess plus, otherwise guess minus. Over a large number of trials (1000, for example) determine how often you get the right answer for different d values. Plot the percent correct as a function of d over the range $0 \leq d \leq 10$.

   Next, by allowing z to vary over a range, plot ROC curves for $d = 0, 1, 2, 3, 4$. To do this, determine how frequently the guess is plus when the stimulus is, in fact, plus , ($beta$) and how often the guess is plus when the real stimulus is minus (this is α). Then, plot α versus β for z over the range 0 to 140.