Received probabilities

- Book uses C for transmitted, Y for received
- Here, we use \( a \) for transmitted and \( b \) for received

Channel: \( P_{Y|C} = P_{B|A} = \begin{pmatrix} p(b = 0 | a = 0) & p(b = 0 | a = 1) \\ p(b = 1 | a = 0) & p(b = 1 | a = 1) \end{pmatrix} \)

\[ p(b = 0) = p(a = 0, b = 0) + p(a = 1, b = 0) \]

\[ \Rightarrow p(b = 0) = p(b = 0 | a = 0) p(a = 0) + p(b = 0 | a = 1) p(a = 1) \]

\[ \Rightarrow p(b) = P_{B|A} b(a) \]

Backward probabilities

\[ p(a = 0 | b = 0) = \frac{p(a = 0, b = 0)}{p(b = 0)} = \frac{p(b = 0 | a = 0) p(a = 0) + p(b = 0 | a = 1) p(a = 1)}{p(b = 0)} \]

Backward probabilities are used to interpret received signals

Channel Example

Channel: \( \begin{pmatrix} .9 & .1 \\ .1 & .9 \end{pmatrix} \)

Transmission probabilities: \( p(a=0) = 0.95 \quad p(a=1) = 0.05 \)

Received probabilities: \( p(b=0) = (0.9)(0.95)+(0.1)(0.05) = 0.86 \)
\( p(b=1) = (0.1)(0.95)+(0.9)(0.05) = 0.14 \)

Backward probabilities:
\[ p(a = 0 | b = 0) = \frac{(0.9)(0.95)}{0.86} = 0.9942 \quad p(a = 1 | b = 0) = \frac{(0.1)(0.05)}{0.86} = 0.0058 \]
\[ p(a = 0 | b = 1) = \frac{(0.1)(0.95)}{0.14} = 0.6786 \quad p(a = 1 | b = 1) = \frac{(0.9)(0.05)}{0.14} = 0.3214 \]

Channel Capacity, BSC

- Channel Capacity is the maximum value of the mutual information over all transmission probability distributions

\[ C = \max_{p(a)} I(A;B) \]