Received probabilities

- Book uses C for transmitted, Y for received
- Here, we use a_i for transmitted and b_j for received

$$\text{Channel}: P_{Y|C} = P_{B|A} = \begin{pmatrix} p(b=0 \mid a=0) & p(b=0 \mid a=1) \\ p(b=1 \mid a=0) & p(b=1 \mid a=1) \end{pmatrix}$$

$$p(b=0) = p(a=0,b=0) + p(a=1,b=0)$$

$$\Rightarrow p(b=0) = p(b=0 \mid a=0)p(a=0) + p(b=0 \mid a=1)p(a=1)$$

$$\Rightarrow p(b) = P_{B|A}\,p(a)$$

Backward probabilities

$$p(a=0 \mid b=0) = \frac{p(a=0,b=0)}{p(b=0)} = \frac{p(b=0 \mid a=0)p(a=0)}{p(b=0 \mid a=0)p(a=0) + p(b=0 \mid a=1)p(a=1)}$$

Backward probabilities are used to interpret received signals

Channel Example

$$\text{Channel}: \begin{pmatrix} .9 & .1 \\ .1 & .9 \end{pmatrix}$$

Transmission probabilities: $p(a{=}0) = 0.95 \quad p(a{=}1) = 0.05$

Received probabilities: $p(b{=}0) = (0.9)(0.95)+(0.1)(0.05) = 0.86$
$p(b{=}1) = (0.1)(0.95)+(0.9)(0.05) = 0.14$

Backward probabilities:

$$p(a=0 \mid b=0) = \frac{(0.9)(0.95)}{0.86} = 0.9942 \qquad p(a=1 \mid b=0) = \frac{(0.1)(0.05)}{0.86} = 0.0058$$

$$p(a=0 \mid b=1) = \frac{(0.1)(0.95)}{0.14} = 0.6786 \qquad p(a=1 \mid b=1) = \frac{(0.9)(0.05)}{0.14} = 0.3214$$

Channel Capacity, BSC

- Channel Capacity is the maximum value of the mutual information over all transmission probability distributions

$$C = \max_{p\{a\}} I(A;B)$$