THE FULL RECESSION: PRIVATE VERSUS SOCIAL COSTS OF COVID-19

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Abstract

Official recession figures ignore the costs associated with the loss of human life due to COVID-19. This paper constructs full recession measures that take into account the death toll. Our model features tractable heterogeneity, constant relative risk aversion to mortality risk, and age-specific survival rates. Using an estimated one-year death toll of 500 thousand people and a 3.5% recession, we find that the corresponding full recession is 24% on average across individuals, 13% for a median voter, and 7% for planner with moderate inequality aversion.

Key words: social welfare, value of statistical life, mortality risk aversion, consumption risk, Epstein-Zin-Weil preferences

JEL codes: I14, I31, J17

1 Introduction

As the COVID-19 pandemic spreads around the world, societies face a trade-off between the loss of human life and economic stagnation. In a situations like this, traditional measures of economic activity, such as GDP, may be misleading indicators of society’s well-being since they do not typically incorporate any adjustment for the loss of human life. A more reliable indicator of well-being during pandemics, one that could be used to guide policy, would need to include a sensible adjustment for the increased risk at the individual level, and the death toll at the aggregate level.

This paper provides a flexible framework to evaluate individual, social, and distributional welfare effects of COVID-19 and other maladies. This framework is also useful to compute the full recession, a measure of well-being in the presence of economic and mortality shocks. The paper builds on Cordoba and Ripoll (2017, CR henceforth) by introducing incomplete markets, incomplete annuities, age-dependant survival rates, and tractable heterogeneity. A distinct feature of this new framework is the disentangling of three key preference parameters that are otherwise embodied in a single parameter in the commonly used expected utility model (EU). The three parameters are: (i) the elasticity of intertemporal substitution; (ii) the coefficient of relative risk aversion to non-mortality risks; and (iii) the coefficient of relative risk aversion to mortality risk. This disentangling,

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an extension of the original Epstein-Zin-Weil (EZW) preferences, allows to preserve a constant relative aversion to mortality risk for any value of the elasticity of intertemporal substitution.\(^1\) As a result, the valuation of life is proportional to consumption for both low and high-income individuals. This stands in sharp contrast with the commonly used non-homothetic EU model, which implies that relative to consumption, the value of a year of life for a poorer individual is significantly lower than that of a richer one (Hall \textit{et al.}, 2020).\(^2\) Such a strongly decreasing valuation of life is not supported by the existing evidence, as argued by CR.\(^3\) More importantly, since the mortality risks involved in COVID-19 are heterogeneous, increasing with age and affecting the poor at significantly higher rates, our EZW model has distinct predictions for the distributional welfare effects of COVID-19 by age and consumption level.

We calibrate the model to recent US data. The age-varying parameters of the consumption process are estimated from 1999-2017 PSID data. The key parameter, the coefficient of relative aversion to mortality risk, is calibrated to match plausible measures of the value of statistical life commonly used in the health literature. The age-specific decreases in survival probabilities due to COVID-19 are calculated from the pre-lockdown data in Menachemi \textit{et al.} (2020) and Ferguson \textit{et al.} (2020). We use the model to compute the willingness to pay (WTP) to avoid the pandemic for individuals and the social planner, to measure the full recession, and to characterize the trade-off between lives and consumption by tracing a utility frontier for the average individual, the median voter and the planner.

We find that the welfare costs of a one-year COVID-19 pandemic are high: on average, individuals would be willing to reduce one-year consumption by 41.2% to avoid the pre-lockdown deaths of 1.9 million people. We also find a large dispersion of welfare costs across ages: while the average 25-year old would only pay 5% of one-year consumption, the 60-year old would pay 80%. In fact, the standard deviation of the age-specific welfare costs is 41.2%, capturing the substantial dispersion in fatality rates across individuals in different age groups. Third, there is also disagreement between the average welfare cost and the 46-year old median voter, whose welfare cost is 22.7% of one-year consumption. Last, the extent to which the social planner cares about inequality has sizable effects on the social WTP to avoid the pandemic: while the welfare cost for a planner with moderate inequality aversion is 10.8%, it is 72.7% for a planner with higher aversion, who places a higher weight on the utility of the elderly.

Regarding the full recession, we find that a 3.5% contraction in consumption during the year of the pandemic combined with a loss of 500 thousand lives corresponds to a full recession of 24.2% for the average individual, 13.4% for the median voter, 7.0% for a planner with moderate inequality aversion, and 15.8% for a planner with higher aversion. To interpret these figures, notice that for a planner with moderate aversion, a 3.5% economic recession corresponds to a 7.0% full recession, with the additional 3.5% accounting for the planner’s valuation of the lives lost. Assuming a US

\(^1\)Epstein and Zin (1989) and Weil (1990).
\(^2\)As we show in Section 2, non-homotheticity in the form of a minimum consumption is required in the EU model for the value of life to be positive when the elasticity of intertemporal subsitution is lower than one with CRRA utility.
\(^3\)See Section 2.3 in Cordoba and Ripoll (2017).
GDP of $22 trillion, this roughly corresponds to a planner’s valuation of $1.5 million per life lost. The planner with moderate inequality aversion places a lower weight on the utility of the elderly. In contrast, the implied marginal valuation of life for the average individual is $9 million, reflecting the fact that those above age 60 would be willing to pay more to reduce their risk exposure.

We also find that the consumption-lives frontier varies substantially across the average individual and the median voter. While the average frontier is more concave and reflects a higher willingness to sacrifice consumption to save lives, the frontier for the median voter tends to be less concave and reflects lower willingness to give up consumption.

Last, relative to the EU model, we find that our EZW framework has distinct implications for the distribution of welfare costs by age and level of consumption. First, the average welfare costs are increasing with age in both models, but they are relatively higher in our EZW model as people age. In our model the disentangling between intertemporal substitution and mortality risk aversion introduces nonlinear effects on how the change in survival due to COVID-19 affects the welfare costs, with these effects being larger for the elderly since they are affected by the pandemic the most. Second, for any given age, the distribution of welfare costs by consumption level is log normal in our EZW model, while in the EU model it has a significant mass close to zero and a longer upper tail. Since utility is homothetic in our EZW model, welfare costs inherit the log normal distribution of consumption at every age. In contrast, the non-homotheticity of the EU model results in particularly low life valuations for individuals whose consumption is closer to a minimum level, and very high valuations for those with consumption draws from at the upper tail.

Our paper complements a number of other recent papers on COVID-19, but it is most closely related to Hall et al. (2020). Hall et al. (2020) also compute the welfare costs of COVID-19. Relative to them our model includes incomplete markets, inequality, and a non-expected utility representation. In this respect, we develop a rich framework that merges the traditional analysis of the welfare costs of economic shocks, consumption inequality, and the literature on the value of life over the life cycle. Although rich, our framework is tractable. While the aggregate welfare costs of COVID-19 are similar in both models, the distribution of the costs by age and level of consumption are quite different. The same distinction applies to other recent COVID-19 papers using the EU framework (Hur, 2020; Glover et al., 2020; Eichenbaum et al., 2020, among others).

The remainder of the paper is organized as follows. In a preliminary analysis, Section 2 starts with an intuitive computation of the welfare costs of the pandemic and compares the predictions of the EU and EZW models for an economy with age-dependent mortality rates and deterministic consumption inequality. Section 3 presents our general EZW model with age-dependent mortality rates, consumption risk and stochastic inequality. It also derives the social welfare function, and the private and social welfare costs of COVID-19. The calibration procedure in explained in Section 4. Numerical results and the relationship with the literature are presented in Section 5. Section 6 concludes.
2 Preliminaries

In this preliminary section we discuss the importance of having a model to compute the welfare costs of COVID-19 and the full recession, and highlight the main features of our EZW model. Consider the case of an economy in which individuals face age-dependent mortality risk. Assume that consumption inequality is deterministic, so there is no social mobility, and that the distribution of consumption at any age is log normal. Individual consumption grows at an exogenous common rate to reflect the life cycle profile from the data. Section 3 considers the more general case with stochastic consumption inequality. The welfare cost of a one-year pandemic corresponds to the percentage of one-year consumption that individuals would be willing to give up to avoid the increased mortality risk. An intuitive calculation of the welfare cost for individual $i$ at age $a$ is the value of the lost years of life relative to one-year consumption, $c_{i,a}$. The value of statistical life (VSL) measures the value of the remaining life in terms of consumption. Therefore, the intuitive calculation of the welfare cost of a pandemic that decreases the probability of survival by $\Delta \pi_a < 0$ is given by

$$\alpha_{i,a} \approx -\Delta \pi_a \frac{VSL_{i,a}}{c_{i,a}},$$

a formula also derived in Hall et al. (2020). $\alpha_{i,a}$ also corresponds to the WTP to avoid death as a percentage of one-year consumption. Although in principle this intuitive calculation might be practical, having a model of the value of life is essential for at least three reasons. First, while there are estimates of $\Delta \pi_a$ and $\pi_a$ for all ages, VSL estimates are available for only some ages. As a result, tracing the full life-cycle profile of the VSL requires a model of the value of life. Second, modelling individuals’ attitudes towards mortality risk fundamentally affects how the VSL varies by age and by level of consumption. A model of the value of life has implications for the distribution of VSL across age and consumption states, as well as the distribution of the welfare costs of COVID-19. Third, the distribution of the VSL has potentially important implications for the aggregate welfare costs of the pandemic.

The most commonly used model of the value of life, including the recent COVID-19 literature, features the EU framework (Hall et al., 2020; Hur, 2020; Glover et al., 2020; Eichenbaum et al., 2020, among others). As we now show, the EU model tends to understate the VSL and welfare costs for the elderly and the poor, precisely the two groups that have been most affected by COVID-19. We now highlight two key differences between the EU and our EZW models. First, the VSL for individual $i$ at age $a$ in the EU model, which is given by

$$VSL_{i,a}^{EU} = \frac{1}{\pi_a} \left[ \sum_{s=a}^{\infty} \frac{1}{(1+r)^{s-a}} \frac{(c/c_{i,s})^{1-\sigma} - 1}{\sigma - 1} c_{i,s} \right],$$

See Cordoba and Ripoll (2017, p. 1487) for related equations. All derivations are included in the online appendix. The equations shown in this section assume $\sigma \neq 1$, no annuity markets, and no consumption risk.


while in the EZW model is

$$VSL_{i,a}^{EZW} = \frac{1}{\pi_a} \left[ \sum_{s=a}^{\infty} \frac{1}{(1+r)^{s-a}} \frac{1}{1 - \gamma c_{i,s}} \right],$$

where $\pi_a$ is the probability of survival between ages $a - 1$ and $a$, $c$ is a level of minimum consumption below which individuals do not value life, $1/\sigma$ is the elasticity of intertemporal substitution in the standard CRRA utility, $r$ is the interest rate, and $\gamma$ is the coefficient of mortality aversion. Notice that if $\sigma > 1$, the most relevant case for quantitative work, the EU model must be non-homothetic since $c_{i,a} > c > 0$ is necessary for $VSL_{i,a} > 0$ in (2). In contrast, the EZW model is homothetic since life is valued for all $c_{i,a} > 0$ and for $0 < \gamma < 1$. Both equations above show that the VSL corresponds to a present discounted sum of future consumption flows, where these flows are weighted by a measure of mortality risk aversion that depends on the utility function. In the case of EU, aversion to mortality risk is given by $((c_{i,a})^{1-\sigma} - 1)/(\sigma - 1)$, while for EZW preferences it is driven by $1/(1 - \gamma)$.

The most important distinction between equations (2) and (3) is that for a given age, the distribution of $VSL_{i,a}$ across individuals with different consumption levels varies, particularly for those at the tails. The bottom panels of Figure 1 illustrate these distributions for ages 40 and 70. At the left tail, the VSL mass is particularly large for the EU relative to the EZW model. In the EU model individuals with low levels of consumption value life little, or not at all, because $c_{i,a}$ is closer to the minimum consumption $c$. In the case of Figure 1, which assumes $\sigma = 2$, the mortality risk aversion term in (2) reduces $(c_{i,a}/c) - 1$, which tends to zero as $c_{i,a} \to c$. In contrast, as can be seen from equation (3), the non-homotheticity introduced by $c$ is not present in the EZW case, implying a log normal distribution of the VSL. In this sense, the EU model understates the VSL for the poor. Similarly, at the right tail the VSL mass is larger for the EU model. In this case, for consumption far enough from the minimum level $c$, mortality aversion becomes particularly large, and the VSL can be well above $15$ million. In contrast with the predictions of the EU model, evidence from meta-analysis suggests that life is not a luxury good, implying that the non-homotheticity from $c$ induces income effects on the VSL that are inconsistent with the data (Masterman and Viscusi; 2018; and Viscusi and Masterman, 2017). In fact, as shown in the top-right panel of Figure 1, while ratio $VSL/c$ at age 40 is 150 in the EZW model for all levels of consumption, it varies between zero and 1,300 in the EU model.

The second key difference between the EU and EZW models pertains to the welfare cost of COVID-19 and its distribution by age and consumption levels. The welfare cost of a one-year pandemic for individual $i$ at age $a$ in the EU model is given by

$$\alpha^{EU}_{i,a} \approx -\Delta \pi_a \frac{VSL_{i,a}^{EU}}{c_{i,a}},$$

where $\Delta$ is the change in consumption and $\pi_a$ is the survival probability.
while in the EZW model is

$$
\alpha_{i,a}^{EZW} \approx - \left[ \left(1 + \frac{\Delta \pi_a}{\pi_a} \right)^{\frac{1-\alpha}{1-\gamma}} - 1 \right] \frac{1 - \gamma}{1 - \sigma} \frac{VSL_{i,a}^{EZW}}{c_{i,a}}. \tag{5}
$$

Two main insights emerge from the equations above. First, the welfare costs of the pandemic for the EU model in (4) coincides with the intuitive computation in (1), while expression (5) for the EZW model is more general. In fact, only when $\sigma = \gamma$ the EZW welfare costs are equivalent to the intuitive calculation. The main message here is that multiplying the VSL times $\Delta \pi_a$ is the correct approximation of the welfare cost only in the EU model, where utility is linear in survival probabilities. In contrast, this does not hold in the EZW model because the nonlinearity in survival, which stems from $\sigma \neq \gamma$, requires an additional adjustment. In fact, as seen in (5), a key feature of the EZW model is that it disentangles intertemporal substitution ($1/\sigma$) from mortality risk aversion ($\gamma$), departing from the standard EU model assumption that agents are indifferent to the timing of resolution of mortality uncertainty, or that $\sigma = \gamma$. The second insight from the equations above is that having $\sigma \neq \gamma$ makes a difference for the distribution of welfare costs by age. As we illustrate in the top-left panel of Figure 2, when $\sigma > 1 > \gamma$ the average welfare costs are similar for the young in both the EU and EZW models, but are larger for the elderly in the EZW model. The difference between the two models is not driven by the average $VSL_{i,a}/c_{i,a}$ by age, since these are similar (top-left panel of Figure 1). It stems from the fact that $\Delta \pi_a$ for COVID-19 is larger for the elderly, case in which the nonlinear effect of $\Delta \pi_a$ in the welfare costs becomes significant in the EZW model.

In sum, while the intuitive welfare cost computation in (1) works if either $\Delta \pi_a$ is close to zero, or if $\sigma = \gamma$ as in the EU model, it does not work in the case in which individuals are not EU maximizers. As we discuss later, when $\sigma > 1 > \gamma$ individuals exhibit a preference for late resolution of uncertainty regarding mortality, a feature consistent with evidence from medical research (see CR). In addition, when $\sigma > 1 > \gamma$, the EZW model also exhibits diminishing returns on the level of survival, making the elderly value life extensions relatively more. This feature of the EZW model is consistent with the empirical evidence that the elderly are willing to pay large sums to prolong their lives (see CR).

Finally, the top-right panel shows $\alpha_{i}^{EZW}$ and $\alpha_{i}^{EU}$ at age 40 as functions of initial (age-18) consumption, mirroring the behavior of $VSL/c$ in the top-right panel of Figure 1. The bottom panels in Figure 2 display the distribution of welfare costs at ages 40 and 70, which resembles the properties of the corresponding distributions of the VSL in Figure 1. Last, Table 1 reports statistics on the welfare costs underlying Figure 2 for the EU and EZW models. As shown, the average and the standard deviation of the costs are higher in the EZW model: the average cost is 34.1% of one-year consumption in our EZW model, and 31.3% in the EU model. The standard deviations are 35.2% and 32.5% respectively. These aggregate differences are driven by the underlying VSL distributions by age and consumption.
3 Model

This section generalizes our EZW model to include both mortality and consumption risks. Different from Section 2, inequality is now stochastic, allowing for social mobility as it is standard in the inequality literature. The exercise of this paper is similar in spirit to Lucas (1987), Tallarini (2000), Cordoba and Verdier (2008), and CR among others. For a given path of individual consumption and mortality rates over the life cycle, individual and social welfare costs are calculated for a benchmark in which a pandemic is occurring.

3.1 Consumption of the alive

Time is discrete. There is an initial age-distribution of the denoted by $M_{a,0}$. Let $\pi_{a,t}$ be the probability of surviving to age $a$ at time $t$ conditional on having survived to age $a - 1$. The distribution of population at $t$ satisfies $M_{a,t} = \pi_{a,t}M_{a-1,t-1}$ for $a > 0$ and $M_{0,t} = n^t$, where $n$ is the birth rate. Let $I(a;t)$ be the set of individuals of age $a$ alive at time $t$ and $c_{i,a,t}$ be the consumption of individual $i$ at age $a$ and time $t$. Assume that (log) consumption follows a random walk, $\ln c_{i,a,t} = \ln c_{i,a-1,t-1} + \epsilon_a$, where $\epsilon_a \sim N(\mu_a - \eta_a/2, \eta_a^2)$. The age-dependent drift and variance would allow us to replicate a realistic life-cycle pattern of consumption.

Assume that age-0 consumption is drawn from a log-normal distribution, $\ln c_{i,0,t} \sim N(\ln c_{0,0} - \Omega^2/2, \Omega^2)$ for $i \in I(0,t)$, where $\Omega^2$ represents age-0 consumption inequality. The average consumption of cohort $t$ at age 0 is then $E[c_{i,0,t} | t] = e^{\ln c_{0,t} - \Omega^2/2 + \Omega^2/2} = c_{0,t}$. A constant rate of aggregate economic growth is introduced by assuming that $c_{0,t} = c_{0,0}e^{gt}$, where $g$ is the growth rate. We normalize $c_{0,0} = 1$. Under these assumptions it follows that the unconditional distribution of consumption at any point in time is log-normal,

$$\ln c_{i,a,t} \sim N\left( (t-a) + v_a - \Omega^2/2 - \Phi_a^2/2, \Omega^2 + \Phi_a^2 \right) \text{ for } i \in I(a,t),$$  \hspace{1cm} (6)

where $v_a = \sum_{j=1}^{a} \mu_j$ and $\Phi_a^2 = \sum_{j=1}^{a} \eta_j^2$ for $a \geq 1$. Intuitively, $(t-a)$ reflects economic growth, $v_a$ life-cycle growth, $\Omega^2$ initial inequality and $\Phi_a^2$ reflects life-cycle inequality. Given that (log) consumption is a random walk, inequality increases with age. Using the law of large numbers, (6) also describes the distribution of consumption, or inequality, at any point in time.

3.2 Preferences

We model individual utility by adopting an extension of the EZW framework of CR to disentangle intertemporal substitution, mortality risk aversion, and aversion to consumption risk. The utility of individual $i$ at age $a$ and time $t$ is described by

$$\nabla_{i,a,t} = \left[ \pi_{a,t} V_{i,a,t}^{1-\gamma} + (1 - \pi_{a,t}) D_{i,a,t}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \text{ with } \gamma \geq 0, \gamma \neq 1,$$  \hspace{1cm} (7)
where $V_{i,a,t} \geq 0$ is the utility of being alive, $D_{i,a,t} \geq 0$ the perceived utility upon death, and $\gamma$ is the coefficient of mortality aversion, a parameter describing attitudes toward mortality risk. In addition,

$$V_{i,a,t} = \begin{cases} 
(1 - \beta_a) \pi_{i,a,t} - \sigma + \beta_a \left( E_a \left[ V_{i,a+1,t+1}^{1-\theta} \right] \right)^{\frac{1-\sigma}{1-\theta}} & \text{for } \sigma \neq 1, \\
\pi_{i,a,t} - \beta_a \left( E_a \left[ V_{i,a+1,t+1}^{1-\theta} \right] \right)^{\frac{1-\sigma}{1-\theta}} & \text{for } \sigma = 1,
\end{cases}$$

(8)

where $\beta_a$ is an age-specific discount factor, $1/\sigma$ is the standard intertemporal substitution, and $\theta$ is the traditional coefficient of risk aversion in EZW preferences, capturing overall aversion to risk. $\pi_{i,a,t}$ is effective consumption defined as $\pi_{i,a,t} = \lambda_{i,a,t} c_{i,a,t}$, where $\lambda_{i,a,t}$ is a proportional shift parameter that will be used to calculate welfare under various scenarios. In the baseline $\lambda_{i,a,t} = 1$ for all $i$, $a$ and $t$.

In what follows we focus on the case in which $D_{i,a,t} = 0$, which ensures that being alive is always preferred to death. Such normalization requires the restriction $\gamma \in (0, 1)$ as otherwise the strong complementarity between $V$ and $D$ would render $V = 0$ too.\(^8\) In this case

$$V_{i,a,t} = \pi_{i,a,t} \gamma V_{i,a,t}, \text{ with } \gamma \in (0, 1),$$

(9)

and

$$V_{i,a,t} = \begin{cases} 
(1 - \beta_a) \pi_{i,a,t} - \sigma + \beta_a \left( \left( E_a \left[ V_{i,a+1,t+1}^{1-\theta} \right] \right)^{\frac{1-\sigma}{1-\theta}} \right) \gamma^{\frac{1-\sigma}{1-\theta}} & \text{for } \sigma \neq 1, \\
\pi_{i,a,t} - \beta_a \left( \pi_{i,a+1,t+1} E_a \left[ V_{i,a+1,t+1}^{1-\theta} \right] \right)^{\frac{1-\sigma}{1-\theta}} & \text{for } \sigma = 1.
\end{cases}$$

(10)

Equation (10) is a Bellman equation with an effective discount factor on future utility of $\beta_a \pi_{i,a+1,t+1}^{(1-\sigma)/(1-\gamma)}$ rather than the standard $\beta_a \pi_{a+1,t+1}$. Restriction $\gamma \in [0, 1]$ guarantees that higher survival results in higher utility. When $\pi = 1$ for all $a$, these preferences correspond to the standard EZW preferences with $\theta$ as the only coefficient of relative risk aversion.

Utility function (10) is flexible and general, capturing individuals’ attitudes toward mortality and consumption risk. When $\sigma > \gamma$, individuals exhibit a preference for late resolution of mortality uncertainty, while $\theta > \sigma$ implies a preference for early resolution of consumption uncertainty. CR document evidence on a preference for late resolution of uncertainty regarding death, while the finance literature provides evidence for early resolution of consumption uncertainty. Utility (10) also includes a number of special cases: $\theta = \sigma$ is a standard EU model for consumption risk, but

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\(^7\)Age-specific discount factors are required in the calibration to match the risk-free rate.

\(^8\)Normalization $D_{i,a,t} = 0$ allows us to identify $\gamma$ from estimates of the VSL at age 40. Absent this normalization, it is not possible to calibrate the distribution of $D_{i,a,t}$ from the data, unless one imposes additional assumptions. From equation (8) we have that $D_{i,a,t} = \xi_{i,a,t}$, where $\xi_{i,a,t}$ is the imputed consumption upon death. If we assumed $\xi_{i,a,t} = \xi_{a,t}$, then there would be income effects as in the case of the EU model. Assuming $\xi_{i,a,t} = \kappa c_{i,a,t}$, where $\kappa$ is some constant, would eliminate income effects, but it would imply that $D_{i,a,t} = \kappa c_{i,a,t}$, so that inequality persists after death, since richer individuals are also richer in the after-life. Such ability of transferring resources between the alive and the dead seems to us somewhat unrealistic. Our assumption that $D_{i,a,t} = 0$ renders the model homothetic, a key property of the EZW model.
not for mortality risk. Case $\sigma = \gamma$ corresponds to EU for mortality risk, but not for consumption risk, while case $\theta = \sigma = \gamma$ is the full EU model for both consumption and mortality risks.

### 3.3 Closed-form solution of the value function

Closed-form solutions are possible given the set up of the problem, as shown in the following proposition.

**Proposition 1.** The utility of individual $i \in I(a, t)$ is given by

$$V_{i,a,t} = \pi_{a,t}^{-\gamma} V_{i,a,t} = \pi_{a,t}^{-\gamma} A_{i,a,t} e_{i,a,t} \text{ for all } i, t \text{ and all } a,$$

where $A_{i,a,t}$ is the saddle path solution of the following first order difference equation,

$$A_{i,a,t} = \begin{cases} 
1 - \beta_a + \beta_a \left[ \frac{1}{\pi_{a+1,t+1} A_{i,a+1,t+1}} \frac{\lambda_{i,a+1,t+1}}{\lambda_{i,a,t}} e^{\mu_{a+1}-\theta \eta_{a+1}/2} \right]^{1-\sigma} & \text{for } \sigma \neq 1, \\
\frac{1}{\pi_{a+1,t+1} A_{i,a+1,t+1}} \frac{\lambda_{i,a+1,t+1}}{\lambda_{i,a,t}} e^{\mu_{a+1}-\theta \eta_{a+1}/2} \beta_a & \text{for } \sigma = 1.
\end{cases}$$

In the baseline, $\lambda_{i,a,t} = 1$ and $A_{i,a,t} = A_{a,t}$, so that $A_{i,a,t}$ is independent of $i$. The linearity of the utility in consumption results from the fact that the EZW aggregator is homogeneous of degree one. An implication is that the utilitarian planner is not averse to inequality, as we discuss below.

Sequence $A_{i,a,t}$ is a central object because it contains all the welfare information (rate of time preference, mortality and consumption risk aversion, and intertemporal substitution) as well as survival rates. It can be shown that $A_{i,a,t}$ decreases with age, so older individuals derive less utility from consumption. Consider the case in which all parameters $(\mu_a, \eta_a, \pi_a, \beta_a)$ become constant after a certain age $a \geq a^*$. In particular, survival is still positive but becomes constant, arbitrarily small and equal to $\pi$. Sequence $A_{i,a,t}$ then converges to a terminal value $\tilde{A}$ which can be written as

$$\tilde{A} = \begin{cases} 
(1 - \beta_a)^{1/\sigma} \left[ 1 - \beta_a \left( \pi^{1/(1-\gamma)} e^{\mu - \theta \eta^2/2} \right)^{1-\sigma} \right]^{1/\sigma} & \text{for } \sigma \neq 1, \\
\frac{1}{\pi} \left[ \pi^{1/\sigma} e^{\mu - \theta \eta^2/2} \right] \beta_a & \text{for } \sigma = 1.
\end{cases}$$

It is easy to show that starting from $\tilde{A}$ and iterating backwards results in a sequence $A_{i,a,t}$ that decreases with age, which mirrors the life-cycle profile of survival probabilities.

### 3.4 Value of statistical life

Although the elderly derive lower continuation utility from consumption than the young, the WTP to reduce the risk of dying is not necessarily lower for the elderly. To see this we now compute
the value of statistical life (VSL) in the model, which captures the trade-offs between survival and consumption. The VSL corresponds to the marginal rate of substitution between survival and consumption defined as

$$VSL_{i,a,t} = \frac{\partial V_{i,a,t}}{\partial \pi_{a,t}} \frac{1}{1 - \gamma} \frac{1}{1 - \beta_a} \frac{1}{\pi_{i,a,t}}$$

for any $\sigma$. (14)

Using (11), this expression simplifies to

$$VSL_{i,a,t} = \frac{1}{1 - \gamma} \frac{1}{1 - \beta_a} A_{a,t}^{1-\sigma} c_{i,a,t}$$

for any $\sigma$. (15)

This equation shows the special role of $\gamma$, the coefficient of relative mortality aversion, in determining the VSL. The VSL increases with $\gamma$ and $\lim_{\gamma \rightarrow 1} VSL_{i,a,t} = \infty$. This expression will be used to calibrate $\gamma$. The time preference parameter $\beta_a$ plays an analogous role to that of $\gamma$ but the mechanism is different. While a larger $\gamma$ represents a larger aversion to mortality risk, a larger $\beta_a$ represents an increase in the weight that individuals place on the future, which increases the value of surviving. The effect of $A_{a,t}$ on the VSL depends on the value of $\sigma$. Since $A_{a,t}$ decreases with age, then the VSL would tend to increase with age if $\sigma > 1$, but it would decrease it if $\sigma < 1$. The intuition for this result is that when $\sigma > 1$ there is complementary in consumption over time, so everything else equal, those with fewer periods left to live will be willing to pay more to extend life. In addition, lower survival increases the VSL through term $1/\pi_{a,t}$, which reflects an increase in the expected marginal utility of consumption, as current consumption only takes places if the individual survives. Finally, the VSL is linear in consumption.

### 3.5 Annuities and asset pricing

Given an exogenous process for consumption, an approach in the finance literature is to use the Euler equations to price assets. We now use this approach to price a one-period riskless bond. This price will be used to calibrate preference parameters. In the absence of mortality risk, a riskless bond pays one unit of consumption in any state of the world. In the presence of mortality risk, an annuitized riskless bond pays one unit of consumption only if the individual survives, while a non-annuitized riskless bond pays even if the individual dies. Let $p^A_{a,t} = \pi_{a+1,t+1}/(1 + r)$ and $p^{NA} = 1/(1 + r)$ be the prices of these two bonds respectively and let $p^\delta_{a,t} = \delta p^A_{a,t} + (1 - \delta) p^{NA}$ the price of a bond that is partly annuitized to an extent determined by $\delta \in [0, 1]$. If consumers’ only savings vehicle is the semi-annuitized bond, then $p^\delta_{a,t}$ is the relevant marginal rate of transformation. Focusing on the case $\sigma \neq 1$, it can be shown that the Euler equation corresponding to this asset market structure and (10) is given by

$$p^\delta_{a,t} = \frac{1 - \beta_a}{1 - \beta_a} \beta_a \pi_{a+1} \frac{1}{\pi_a} e^{-\sigma \mu_{a+1} + \theta (1 + \sigma) \mu_{a+1}/2},$$

(16)
where the effective discount factor is \( \bar{\beta}_a = \beta_a^{(1-\sigma)/(1-\gamma)} \). In the quantitative exercise we consider the cases of perfect annuity markets, \( \delta = 1 \), and no annuity markets, \( \delta = 0 \).

### 3.6 Social welfare

Suppose the planner only cares about the welfare of the population alive at time 0.\(^9\) Define social welfare as 

\[
W_0 = \left[ \sum_a \int_{i \in I(a,0)} M_{i,a,0} \hat{V}_{i,a,0}^{1-\psi} di \right] \frac{1}{1-\psi}.
\]

where curvature parameter \( \psi \) describes planner’s aversion to inequality. A classical utilitarian planner is represented by the case \( \psi = 0 \). Since EZW utility is linear in consumption, this utilitarian planner would have no aversion to inequality. Although it might be natural to assume \( \psi = \sigma \), since \( \sigma \) reflects the curvature of the utility flow, this is not obvious in the case of EZW utility which features two additional curvature parameters: \( \gamma \) and \( \Theta \). We leave \( \psi \) as a separate curvature parameter for the planner and report results for several values of \( \psi \) in the quantitative section.

The expression above can be simplified when \( \lambda_{i,a,0} = \lambda_{a,0} \), which holds in our model. In this case \( A_{i,a,0} = A_{a,0} \) as well, and the social welfare can be written as

\[
W_0 = \left[ \sum_a M_{a,0} \left( \pi_{a,0}^{1-\gamma} A_{a,0} \lambda_{a,0} e^{v_a - g_a - \psi \Theta_a^2 / 2} \right)^{1-\psi} \right]^{\frac{1}{1-\psi}},
\]

where \( \Theta_a^2 = \Omega_a^2 + \Phi_a^2 \). Social welfare can be seen as a weighted composite of cohort-specific average utilities defined as \( \hat{V}_a = \pi_{a,0}^{1/(1-\gamma)} A_{a,0} \lambda_{a,0} e^{v_a - g_a - \psi \Theta_a^2 / 2} \). \( \hat{V}_a \) tends to increase with age due to life-cycle consumption growth (\( v_a \)), but tends to decrease with age due to lower survival, lower \( A_{a,0} \), lower initial consumption (if \( g > 0 \)), and larger inequality. If both \( M_{a,0} \) and \( \hat{V}_a \) decrease with age, then social welfare would reflect to a larger extent the welfare of the young rather than the old. A larger \( \psi \), however, could counteract a declining role of the old on social welfare unless life-cycle inequality is relatively large.

### 3.7 Willingness to pay to avoid COVID-19

In this section we derive consumption equivalent variations or the WTP to avoid COVID-19. We calculate measures for individuals and for the social planner.

\(^9\) A more general social welfare function will include the welfare of all potential individuals, present and future. Implicitly, we are assuming a social discount factor of zero.
3.7.1 Individual WTP

Let \( \pi_{a,0} = (\pi_{a+1,s})_{s \geq 0} \) denote a sequence of survival probabilities and \( \lambda_{i,a,0} = (\lambda_{i,a+s,s})_{s \geq 0} \) a vector of consumption shifter parameters. While \( \lambda_{i,a,0} = 1 \) in the baseline economy, in this section we use \( \lambda_{i,a,0} \) to calculate the welfare costs of the pandemic. Using this notation, express (11) as

\[
\nabla_{i,a,0} \equiv \nabla (\pi_{0,0}, \lambda_{i,a,0}, c_{i,a,0}) = \pi_{i,a,0}^{1-\gamma} \cdot A_{a} \cdot \lambda_{i,a,0} \cdot c_{i,a,0}; \quad (19)
\]

where \( A_{a+1,1}, \lambda_{i,a,0} \) is defined by the recursion in (12). Baseline time-0 utility in the absence of the pandemic is given by \( \nabla_{i,a,0} = \nabla (\pi_{a,0}^{*}, 1, c_{i,a,0}) \).

We model the COVID-19 pandemic as a drop in survival probabilities for only one year, \( t = 0 \). Let \( \Delta \pi_{a,0} = \pi_{a,0} - \pi_{a,0}^{*} \leq 0 \) for all \( a \) while \( \Delta \pi_{a,t} = 0 \) for all \( a \geq 0 \) and all \( t > 0 \). Initial utility, \( V_{i,a,0} = \pi_{a,0}^{1/(1-\gamma)} \) \( V_{i,a,0} \), is thus affected by the pandemic while the utility of surviving individuals, \( V_{i,a,0} \), is not affected because it depends only on future survival rates. To calculate equivalent variation measures, we compare two economies: one in which individuals face the pandemic but keep the baseline consumption levels, and one in which individuals enjoy baseline survival probabilities but lower consumption. The welfare cost calculation computes the consumption shift, \( \lambda_{i,a,0} \leq 1 \), that would make individuals indifferent between these two economies. The WTP can then be defined as \( WTP_{i,a,0} = 1 - \lambda_{i,a,0} \), which corresponds to the notion of WTP relative to consumption, or the welfare cost of the pandemic discussed in Section 2. Therefore \( \lambda_{i,a,0} \) can be implicitly defined by the equation

\[
\nabla (\pi_{a,0}^{*} + \Delta \pi_{a,0}, 1, c_{i,a,0}) = \nabla (\pi_{a,0}^{*}, [\lambda_{i,a,0}, 1], c_{i,a,0}). \quad (20)
\]

The left-hand side is the expected utility of individual \( i \) under the pandemic. The right-hand side is the utility without the pandemic but with a level of effective initial consumption just low enough to deliver the same utility as the one obtained under the pandemic. Using equations (11) and (12), this expression simplifies to \( (\pi_{a,0}^{*} + \Delta \pi_{a,0})^{1/(1-\gamma)} A_{a,0}^{*} = (\pi_{a,0}^{*})^{1/(1-\gamma)} A_{a,0} \lambda_{i,a,0} \), where

\[
A_{a,0} \lambda_{i,a,0} = \begin{cases} 
\left[ (\pi_{a,0}^{*})^{-\beta_{a}} + (1 - \beta_{a}) \left( \lambda_{i,a,0}^{1-\sigma} - 1 \right) \right]^{1/\sigma} & \text{for } \sigma \geq 0, \sigma \neq 1, \\
\lambda_{a,0}^{1-\beta_{a}} A_{a,0}^{*} & \text{for } \sigma = 1,
\end{cases}
\]

and \( A_{a,0}^{*} = A_{a} \) \( \pi_{a,0}^{*} + \Delta \pi_{a,0} \) is the baseline level of \( A_{a,0} \). A closed-form solution for \( \lambda_{i,a,0} \) is then given by\(^{10}\)

\[
\lambda_{i,a,0} = \lambda_{a,0} = \begin{cases} 
1 + \left[ \frac{(\pi_{a,0}^{*} + \Delta \pi_{a,0})^{1/\gamma} - 1}{(\pi_{a,0}^{*})^{1/(1-\gamma)} - \lambda_{a,0}^{1-\beta_{a}}} \right]^{1/\sigma} & \text{for } \sigma \neq 1, \\
\frac{1}{(\pi_{a,0}^{*} + \Delta \pi_{a,0})^{1/(1-\gamma)} - \lambda_{a,0}^{1-\beta_{a}}} & \text{for } \sigma = 1.
\end{cases}
\quad (21)
\]

Notice that the WTP is independent of \( i \) but depends on age: individuals of the same age are

\(^{10}\)Notice that the WTP \( \alpha_{i,a} \) in equation (5) was derived for the case of an economy with deterministic consumption inequality (Section 2). \( \lambda_{i,a} \) in equation (21) does not exactly correspond to \( 1 - \alpha_{i,a} \) because the model in this section considers stochastic inequality.
willing to pay the same fraction of their initial consumption to avoid the pandemic. According to (21), for the case \( \sigma = 1 \), \( \lambda_{a,0} \) is mostly determined by the extent to which survival drops due to the pandemic \( (\Delta \pi_{a,0} < 0) \), a magnitude affected by exponent \( 1/(1 - \gamma) \times 1/(1 - \beta_a) \). In the case of COVID-19, survival decreases by much more for older individuals and therefore they are more willing to pay. Notice also the link between the WTP in (21) and the VSL-to-consumption ratio in (15). For the case \( \sigma = 1 \), exponent \( 1/(1 - \gamma) \times 1/(1 - \beta_a) \) equals \( VSL_{a,0}/c_{a,0} \times \pi_{a,0} \). Therefore for \( \sigma = 1 \)

\[
\lambda_{a,0} = \left( \frac{\pi_{a,0} + \Delta \pi_{a,0}}{\pi_{a,0}} \right)^{\frac{VSL_{a,0}}{c_{a,0} \pi_{a,0}}},
\]

which provides a link between the WTP and the VSL-to-consumption ratio. As we explain below, we calibrate \( \gamma \) to match \( VSL_{a,0}/c_{a,0} \) at age \( a = 40 \). The larger this ratio, the larger the calibrated \( \gamma \) and the WTP.

When \( \sigma \neq 1 \) there is an additional effect that comes from \( A_{a,0}^* \), and a first-order effect from \( \beta_a \). Focusing on the case \( \sigma > 1 \), which is more relevant in quantitative macro, recall that since \( A_{a,0}^* \) decreases with age, this effect tends to make the WTP higher for older individuals. The intuition here is again related to the complementarity in consumption across ages for the case \( \sigma > 1 \), which makes life relatively more valuable for those who have less periods left. An additional effect occurs if \( \beta_a \) decreases with age, as it would be the case in the calibrated model with \( \sigma > 1 \). This effect tends to decrease the WTP for older individuals. The average WTP is calculated as the population-weighted average \( \overline{WTP}_0 = 1 - \sum_a M_{a,0} \lambda_{a,0} \).

### 3.7.2 Social WTP

Consider now the social WTP measured by single proportional shift, \( \lambda_{i,a,0} = \lambda_{a,0} = \lambda_0 \), for everyone in the economy. Let \( W_{0}^{\text{pandemic}} \) denote social welfare under the COVID-19 pandemic at time \( t = 0 \), and \( W_0^{\text{baseline}} \) the social welfare in the economy with the baseline survival but lower consumption. The one equation in one unknown defining \( \lambda_0 \) is \( W_{0}^{\text{pandemic}} = W_0^{\text{baseline}} \). Using (18), this expression becomes

\[
\sum_a M_{a,0} \left[ \left( \pi_{a,0} + \Delta \pi_{a,0} \right)^{1/(1-\gamma)} A_{a,0}^* e^{\nu_a - ga - \psi \pi_a^2/2} \right]^{1-\psi} = \sum_a M_{a,0} \left[ \left( \pi_{a,0} \right)^{1/(1-\gamma)} A_{a,0} \lambda_0 e^{\nu_a - ga - \psi \pi_a^2/2} \right]^{1-\psi}
\]

where

\[
A_{a,0} \lambda_0 = \begin{cases} 
\left( A_{a,0}^* \right)^{1-\sigma} + (1 - \beta_a) (\lambda_0^{1-\sigma} - 1) \left( 1 - \lambda_0^{1-\sigma} \right)^{1-\sigma} & \text{for } \sigma \neq 1 \\
\lambda_0^{1-\beta_a} A_{a,0}^* & \text{for } \sigma = 1 
\end{cases}
\]

(22)

and \( A_{a,0}^* \) is the baseline computed from the recursion in (12) with \( \lambda_{a,t} = 1 \) for all \( t \). In order to better understand the role of the inequality aversion parameter \( \psi \) on the social WTP, and to also see the relationship between the social and age-specific WTP, notice that for the case \( \sigma = 1 \) and
$\beta_a = \beta$ for all $a$, there is a closed-form solution for $\lambda_0$ given by

$$\lambda_0 = \left[ \frac{\sum_a M_{a,0} \left( \pi_{a,0} + \Delta \pi_{a,0} \right)^{1-\psi} \left( A_{a,0}^* e^{v_a - ga - \psi \Phi_a^2/2} \right)^{1-\psi}}{\sum_a M_{a,0} \left( \pi_{a,0}^* \right)^{1-\gamma} \left( A_{a,0}^* e^{v_a - ga - \psi \Phi_a^2/2} \right)^{1-\psi}} \right]^{(1-\beta)(1-\psi)}. \quad (23)$$

The following proposition derives some results for this special case.

**Proposition 2.** For the case $\sigma = 1$ and $\beta_a = \beta$ for all $a$, the $\lambda_0$ for a planner with aversion to inequality parameter $\psi$ can be written as

$$\lambda_0 = \left[ \sum_a f_{a,0}(\psi) (\lambda_{a,0})^{(1-\beta)(1-\psi)} \right]^{(1-\beta)(1-\psi)} \quad (24)$$

where $\lambda_{a,0}$ is the age-specific WTP as defined in equation (21), and $f_{a,0}(\psi)$ is a density function given by

$$f_{a,0}(\psi) = \frac{M_{a,0} \left( \pi_{a,0}^* \right)^{1-\gamma} A_{a,0}^* e^{v_a - ga - \psi \Phi_a^2/2}^{1-\psi}}{\sum_a M_{a,0} \left( \pi_{a,0}^* \right)^{1-\gamma} A_{a,0}^* e^{v_a - ga - \psi \Phi_a^2/2}^{1-\psi}}. \quad (25)$$

Proposition 2 provides two main insights. First, equation (24) suggests that the social costs of the pandemic $\lambda_0$ can be expressed as a CES function of the age-specific social costs ($\lambda_{a,0}$). In fact, since $\beta$ is close to 1, equation (24) is close to a Cobb-Douglas of the form $\lambda_0 \approx \prod_a \lambda_{a,0} f_{a,0}(\psi)$, so social welfare is approximately a geometric average of age-specific welfare, where the weights are given by densities $f_{a,0}(\psi)$. Consider the cases with $\psi = 1$ (log planner) and $\psi = 0$ (linear planner, no aversion to inequality), with

$$f_{a,0}(1) = \frac{M_{a,0}}{\sum_a M_{a,0}} \quad \text{and} \quad f_{a,0}(0) = \frac{M_{a,0} \left( \pi_{a,0}^* \right)^{1-\gamma} A_{a,0}^* e^{v_a - ga}}{\sum_a M_{a,0} \left( \pi_{a,0}^* \right)^{1-\gamma} A_{a,0}^* e^{v_a - ga}}. \quad (26)$$

When $\psi = 1$, the planner weights the age-specific welfare costs just using population weights. When $\psi = 0$, weights depend not only on population, but also on survival probabilities ($\pi_{a,0}^*$) and utilities ($A_{a,0}^*$). Since both survival and continuation utility are larger for the young, then the young weight relatively more when $\psi = 0$ than when $\psi = 1$. Since younger individuals are relatively less affected by the pandemic, their WTP $(1 - \lambda_{a,0})$ is lower, resulting in the planner exhibiting a lower WTP $(1 - \lambda_0)$ when $\psi = 0$ than when $\psi = 1$. For the intermediate case in which $\psi = \gamma$, weights $f_{a,0}(\psi)$ closely resemble an utilitarian planner in the expected utility case: utilities are weighted by population and the expression is linear in survival probabilities.
Second, Proposition 2 also shows the role of cross-sectional inequality on the value of $\lambda_0$. Notice first from (25) that weights $f_{a,0}(\psi)$ are affected by life-cycle inequality, $\Phi_a^2$. Since $\Phi_a^2$ is increasing in age, the effective weight of each term in the summation gets smaller for higher $a$. Therefore for given $0 < \psi < 1$, the weight on the elderly gets downplayed in the computation of social welfare. In other words, the fact that inequality increases during the life-cycle tends to reduce the aggregate WTP to avoid the pandemic.

4 Calibration

We calibrate the model to US data. Regarding exogenous parameters, we first compute the conditional survival probabilities $\pi_a$ using the CDC 2017 life tables for the US (National Vital Statistics, 2019). Second, the distribution of population by age is obtained from the US Census for 2017. Last, we use PSID data to estimate the means and variances of the individual age-specific consumption shocks ($\mu_a - \eta_a^2/2$ and $\eta_a^2$), as well as the mean and variance of the lognormal distribution for age-0 consumption ($\ln c_{0,t} - \Omega_t^2/2$ and $\Omega_t^2$), as detailed below. We also set $g = 0$ to consider a stationary state.

The rest of the calibration strategy proceeds as follows. In the benchmark calibration we set $\sigma = \theta = 1$ so that there is no separate role for aversion to consumption risk. We also report a calibration with $\sigma = 2$, a commonly used value. The online appendix reports alternative calibrations of $\theta$ using values from the finance literature. We set $r = 2\%$ and back out the age-specific discount factors ($\beta_a$) so that the Euler equation holds for every age, as we explain below. Since $\beta_a$ depends on all the exogenous parameters and $\sigma$, we recalibrate this age-specific discount factors for every $\sigma$ we report. Last, we calibrate the mortality risk aversion parameter $\gamma$ to match the ratio of the value of statistical life to consumption at age 40.

4.1 Consumption process

We use 1999-2017 PSID biennial non-durable consumption data in order to compute cross-sectional means and variances by age.\footnote{Non-durable consumption is measured as the sum of the following categories: food (home, away and delivery), utilities, transportation (gasoline, auto insurance, vehicle repair, parking, bus, cab and other, and excluding vehicle loan and down payment), housing expenditures (rent and house insurance, excluding mortgages and property taxes), childcare, education and health. Homeowners rent is imputed as 6\% of the home value reported by households.} We estimate the values for $\mu_a$ and $\eta_a$ using cross-sectional consumption. For this purpose, we first remove the effects of differences in family size across households. We then construct 5-year overlapping age groups and non-overlapping 2-year cohort groups, and compute the mean and the variance for each age-group and cohort-group cell.\footnote{The data includes age groups 25 to 75 and cohorts 1934 to 1990 (even years).} Next, we separately regress the mean and the variance of each cell on a full set of age-group dummies and control for cohort effects. Finally, we use a Hodrick-Prescott filter to smooth out the estimates of the mean and variance of consumption by age. For ages older than 75 we set $\mu_a = \mu_{75}$ and $\eta_a = \eta_{75}$, and we
extrapolate \( \mu_a \) and \( \eta_a \) for ages between 18 and 24.\(^{13}\) As shown in the top-left panel of Figure 3, mean consumption increases up to age 50, flattens out until about age 70 and then decreases. As implied by the random walk assumption, the variance of consumption increases with age (top-center panel of Figure 3).\(^{14}\)

4.2 Time horizon

The model so far has been specified with age-specific survival rates in infinite horizon. As discussed in CR, if the horizon is finite so that dying becomes a sure state in finite time, and the utility upon dying is zero, then the utility of the alive is also zero whenever \( \sigma \geq 1 \). This result arises due to the implied strong complementarity of consumption over time. A tractable way to close the model for any \( \sigma \), while still maintaining homotheticity, is to assume that after age \( a^* \) the survival probability is arbitrarily small but positive, a sort of "perpetual old" assumption. We use this approach in our benchmark calibration by assuming that after certain age \( a^{*} \) all parameters are age invariant.\(^{15}\)

Suppose that for \( a \geq a^{*} = 99 \), \( \mu_a = \mu \), \( \eta_a^2 = \eta^2 \), and \( \pi_{a,t} = \pi \). In this case \( A_{a,t} \) in equation (12) is constant and given by the terminal value of \( \bar{A} \) specified in (13). Given \( \bar{A} \) we compute the sequence of \( A_{a,t} \) going backwards from age \( a^{*} \) and using equation (12).

4.3 Age-specific discount factors

We follow the standard asset-pricing literature practice of calibrating the discount factor to match the risk free interest rate. In our life-cycle model Euler equations (16) are age specific due to the survival rates and the consumption growth process. As a result, age-specific discount factors are needed to match the risk free interest target for every age group.\(^{16}\) We set \( r = 2\% \) and use equation (16) to calibrate \( \beta_a \) given an assumption on annuity markets being perfect (\( \delta = 1 \)) or absent (\( \delta = 0 \)). To back out \( \beta_a \), solve for \( \beta_a \) as a function of \( \beta_{a+1} \),

\[
\frac{1 - \beta_a}{\beta_a} = \frac{1 + r}{\delta \pi_a + 1 - \delta} \left( \frac{1 - \beta_{a+1}}{\pi_{a+1}} \right)^{1 + \sigma} e^{-\sigma \pi_{a+1} + (1 + \sigma)(\eta_a^2 + 1)/2}.
\]

This equation suggests that \( \beta_a \) typically depends on age. A constant \( \beta \) would result only in specific cases such as \( \delta = 0 \), \( \sigma = 1 \), \( \mu_{a+1} = \mu \) and \( \eta_{a+1} = \eta \). But in general, an age-dependent \( \beta_a \) has to be backed-out for the equation above to hold for every age. Like before, assume that for \( a \geq a^{*} = 99 \) parameters are constant so that \( \mu_a = \mu \), \( \eta_a^2 = \eta^2 \), and \( \pi_{a,t} = \pi \). In this case a terminal \( \bar{\beta} \) is given.

\(^{13}\) We assume that \( \mu_a = \mu_{26} \) and \( \eta_a = \eta_{26} \) for ages 19 through 25, and then use the estimated \( \mu_{25} \) and \( \eta_{25} \) to construct \( \mu_{18} \) and \( \eta_{18} \) so as to preserve the mean and variance of the lognormal distribution for age-25 consumption. We check that our results are not sensitive to this extrapolation.

\(^{14}\) The same patterns have also been documented by others in the literature (Aguiar and Hurst, 2013).

\(^{15}\) We check the robustness of our results to an alternative formulation in which the utility upon death is positive once death becomes a sure state: \( D_{i,a,t} > 0 \) for ages above \( a^* \). Under this alternative, the horizon is strictly finite and non-homothetic. As we show in the online appendix, our exercise is robust to this alternative case.

\(^{16}\) Murphy and Topel (2006) and Cordoba and Ripoll (2017) use age-specific health indexes to match the target.
by
\[ \tilde{\beta} = \frac{1}{1 + \frac{r_{\pi+1}}{\pi(1-\gamma) e^{-\sigma \mu + \theta (1+\sigma) \eta^2 / 2}}}. \] 
(27)

The sequence \( \beta_a \) is computed backwards from age \( a^* \) using equation (26). Since the estimated \( \mu_a \) and \( \eta_a \) are small in magnitude, then if \( \delta = 0 \) equation (26) implies that \( \beta_a \approx \pi_a^{(\sigma-1)/(1-\gamma)} \). For the most relevant case with \( \sigma > 1 \), since \( \gamma < 1 \) then we have that \( \beta_a \) is decreasing in age. Although \( \beta_a \) varies with age in our model, notice that the effective discount factor is not \( \beta_a \) but \( \tilde{\beta}_a = \beta_a \pi_{a+1,t+1}^{(1-\sigma)/(1-\gamma)} \).

4.4 Aversion to mortality risk

We follow CR and calibrate the mortality risk aversion parameter \( \gamma \) to match the average ratio of the value of statistical life (VSL) to consumption at age 40, which is given by equation (15).\(^{17}\) We calibrate \( \gamma \) so that the average ratio \( VSL_i,a,t/c_{i,a,t} \) at age 40 is 150, a number similar to the one in CR and Murphy and Topel (2006). Given our calibration strategy for \( \beta_a \), it is instructive to discuss how it interacts with other determinants of the VSL in the equation above. Consider first the case in which \( \sigma = 1 \), there are no annuities (\( \delta = 0 \)), and the parameters of the consumption shock process are constant across ages (\( \mu_{a+1} = \mu \) and \( \eta_{a+1} = \eta \)). In this case, \( \beta_a \) is constant across ages so that the VSL is only affected by \( \pi_{a,t} \) and \( c_{i,a,t} \) as seen in equation (15). Therefore unless there is a strong drop in consumption at older ages, the VSL tends to increase in age. For the more common case \( \sigma > 1 \), \( \beta_a \) decreases with age regardless of the annuity market structure, which tends to make the VSL decrease with age. However, the effects from \( A_{a,t}^{1-\sigma} \) and \( 1/\pi_{a,t} \) work in the opposite direction than the effect from \( \beta_a \), making the age profile of the VSL a quantitative issue.

4.5 Benchmark calibration

The benchmark with \( \sigma = \theta = 1 \) and \( \delta = 0 \) features two additional characteristics, both of which can be seen in equation (26). First, in the benchmark parameter \( \gamma \) does not affect the Euler equation and therefore does not affect the calibrated \( \beta_a \). This implies that \( \gamma \) is completely identified from the targeted \( VSL/c \) ratio in equation (15). Second, with \( \sigma = 1 \) and \( \delta = 0 \), discount factor \( \beta_a \) only reflects the variations in \( \mu_a \) and \( \eta_a \). Since these variations are quantitatively small, \( \beta_a \) would be almost constant across ages. These multiple desirable features make the benchmark our preferred calibration. For this calibration we obtain a mortality risk aversion parameter of \( \gamma = 0.675 \). Since \( \sigma > \gamma \), the benchmark calibration implies a preference for late resolution of uncertainty regarding mortality. Figure 3 displays additional features of the benchmark calibration. The implied cross-sectional Gini coefficients of consumption start around 0.26 at age 18 and increase to 0.33 by age 60, suggesting our estimates from the PSID are a good representation of US consumption inequality.\(^{18}\)

\(^{17}\)We calibrate the model to match the VSL-to-consumption ratio rather than the VSL level because the PSID data we use in our calibration includes about 70% of the consumption categories of the CEX.

\(^{18}\)The Gini coefficient for the lognormal distribution is given by
\[ Gini_{a,t} = 2\Theta \left( \bar{\Phi} \left( \frac{\Omega_c^2 + \Phi_c^2}{2} \right) \right) - 1, \] where \( \Theta(.) \) is the cumulative density of a standard normal distribution, and \( \Omega_c^2 + \Phi_c^2 \) is the variance of the lognormal cross-sectional
Finally, the top-right panel displays the calibrated age-varying $\beta_a$. As expected for the benchmark calibration with $\sigma = 1$ and $\delta = 0$, $\beta_a$ varies very little across ages, between 0.984 and 0.974.

5 Results

5.1 Distributional welfare costs of COVID-19

In order to simulate the effect of COVID-19, we take the age-specific pre-lockdown fatality rates from Ferguson et al. (2020), but adjust them to obtain the aggregate fatality rate of 0.58% from Menachemi et al. (2020). Table 2 summarizes the welfare costs of COVID-19 for different assumptions regarding annuity markets and $\sigma$. The table reports the following three statistics: the average WTP ($\overline{WTP}_0$), which corresponds to the population-weighted average of age-specific WTP $(1 - \lambda_{a,0})$; the WTP of the 46-year old median voter (median age of population above 18); and the standard deviation of the age-specific WTP. For our benchmark calibration with no annuity markets, $\sigma = 1$ and $VSL/c = 150$, we find that the costs of COVID-19 are high: the average WTP to avoid the pandemic is 41.2% of one-year consumption. More interestingly, we find that the median voter is willing to pay much less, 22.7%. In addition, we find substantial disagreement across individuals: the standard deviation of the distribution of age-specific WTP is 41.2% of one-year consumption.

The bottom-left panel of Figure 3 displays the distribution of age-specific WTP $(1 - \lambda_{a,0})$, which sharply increases with age and is close to 100% after age 80. Recall from (21) that when $\sigma = 1$ and $\delta = 0$, since $\beta_a$ is almost constant across ages the WTP is mostly driven by the age-specific change in survival $\Delta \pi_{a,0} < 0$. As the change in the survival probability is substantially different across ages, the dispersion of the WTP across ages is also substantial. The bottom-center panel shows the contribution of the age-specific WTP to the average WTP ($\overline{WTP}_0$), suggesting that those ages 55-70 contribute the most. Last, the bottom-right panel shows the distribution of WTP at age 40. It is similar to that in Figure 2, although the dispersion is now larger since in the presence of consumption risk, the variance of consumption increases with age.

As shown in Table 2, when $\sigma = 2$ and there are no annuities, the statistics are lower: the average WTP is 34.3% of one-year consumption, the median voter’s is 20.5%, and the standard deviation is 35.2%. The intuition for this result is that due to the CES form of our EZW utility in (8), higher $\sigma$ results in more complementarity between current and future utility. Since in the data consumption mostly increases with age, the higher survival probability in the economy with no pandemic relative to the pandemic economy results in higher future utility, which requires a higher complementary current consumption, a larger $\lambda_{a,0}$, and a lower WTP.

Table 2 also reports the WTP for the case of perfect annuity markets, case in which all the statistics are also lower than in the benchmark calibration. In this case the reason can be traced distribution of consumption at age $a$.

---

19 We use (log) interpolation to compute the decrease of survival probabilities for each age using the following fatality rates by age bracket from: 0.001% for ages 0-9; 0.003% for ages 10-19; 0.015% for ages 20-29; 0.04% for ages 30-39; 0.08% for 40-49; 0.3% for 50-59; 1.2% for 60-69; 2.7% for 70-79; and 4.9% for ages 80+.
to the calibration of \( \beta_a \): for equation (16) to hold at every age when annuity markets are perfect, \( \beta_a \) has to be lower, or individuals have to become less patient at every age. Equation (15) implies that less patient individuals discount the future more, have lower VSL, and have lower WTP out of present consumption.

5.2 Social welfare costs

Table 3 presents the social costs of COVID-19 from the perspective of the planner for our benchmark calibration. Our preferred scenario is that with no annuities, since annuities are rarely used in practice. In this case, a social planner who does not care about inequality (\( \psi = 0 \)) would be willing to pay only 4.1\% to avoid COVID-19. However, this number changes rapidly as inequality aversion increases: when \( \psi = \gamma = 0.675 \) the WTP more than doubles to 10.8\%. When \( \psi = 0.9 \) the planner’s WTP is 41.2\%, the same as the average across individuals. As we continue to increase \( \psi \) up to the log-planner with \( \psi = \sigma = \theta = 1 \), the aggregate WTP becomes 72.7\%. In sum, in the case of no annuities the social WTP is sensitive to the planner’s inequality aversion, spanning a wide range of values.

As seen in Table 3 the planner’s WTP is higher when annuities are perfect. This result emerges because the calibrated profiles of \( A_a \) are different in these two cases. In the absence of annuities \( A_a \) rapidly declines with age, while the decline is much slower when annuities are perfect.

5.3 Full recessions

Our computations so far have used pre-lockdown fatality rates, which are appropriate to compute the WTP to fully avoid the pandemic. In practice, lockdown policies have been implemented to avoid the loss of lives, which have resulted in contraction of economic activity. In this section we use our model to calculate full recessions, which correspond to a combined measure of consumption and lives lost. Our notion of full recession is analogous to that of full income, which has been used in cross-country literature to compute a measure of welfare that augments income with life expectancy (see CR). We compute full recessions using the following version of individual utility in equation (20),

\[
\nabla \left( \pi_{a,0}^* + \Delta \pi_{a,0}, [\lambda_{i,a,0}^r, 1], c_{i,a,0} \right) = \nabla \left( \pi_{a,0}^*, [\lambda_{i,a,0}^r, 1], c_{i,a,0} \right),
\]

where \( \lambda_{i,a,0}^r \) corresponds to the time-0 change in consumption due to the associated lockdown recession. The left-hand side is the expected utility of individual \( i \) under the one-year lockdown recession and the change in survival probabilities from the pandemic. The right-hand side is the utility without the pandemic but with a level of effective consumption adjusted by \( \lambda_{i,a,0}^r \) and that delivers the same utility as the left-hand side. We assume \( \lambda_{i,a,0}^r = \lambda_0^r \), so that the lockdown recession is the same for all individuals and all ages. Similar to what we found before, \( \lambda_{i,a,0}^r \) only depends on age. We refer to \( \alpha_{a,0}^r = (1 - \lambda_{a,0}^r) \) as the full recession in the sense that it is the drop in consumption
that would reflect both the change in survival due to the pandemic and the economic contraction from the lockdown.

Table 4 presents the value of the full recessions in terms of the percentage of consumption lost for a 3.5% and a 4.5% drop of economic activity (consumption) during the year of the pandemic, and for 500 and 600 thousand lives lost. As shown, for a 3.5% contraction in consumption during the year of the pandemic and 500 thousand deaths, the full recession is 24.2% for the average individual, 13.4% for the median voter, 7.0% for the planner with moderate inequality aversion, and 15.8% for the planner with higher aversion. In order to interpret these findings, first notice that for the former planner, a 3.5% recession becomes a 7.0% full recession, with the additional 3.5% corresponding to the planner’s valuation the lives lost. Using a back-of-the-envelope computation where GDP is $22 trillion, this roughly corresponds to a planner’s valuation of about $1.5 million per life lost. But for the planner with higher aversion to inequality, this valuation becomes $5.4 million per life lost. On the other hand, the 46-year old median voter values each life at roughly $4.1 million, while on average each life lost is valued at $9 million. The reason why the average individual values life much more than either planner is that those above age 75 would be willing to pay a sizeable amount to save their lives, since the pandemic reduces their already shorter life spans. The planner with lower inequality aversion places an effectively lower weight on older individuals, whose continuation utility is lower.

5.4 Consumption-lives frontier

In this section we use our model to compute the trade-off between consumption and lives along a frontier. We compute this frontier by using the following version of equation (20)

$$V_{a;0} = V_{a;0} + \varphi \Delta \pi_{a;0} \left[ \lambda_{i,a;0}, 1 \right], c_{i,a;0}.$$ (28)

The left-hand side is the expected utility of individual $i$ under the COVID-19 death rates at pre-lockdown levels. The right-hand side is the utility under a milder pandemic, with $\varphi \in [0, 1]$, but with a level of initial consumption adjusted by $\lambda_{i,a;0}$ that delivers the same utility as the left-hand side. At one extreme, when $\varphi = 0$, we obtain the WTP $(1 - \lambda_{i,a;0})$ to avoid all COVID deaths, which are the same ones reported in Table 2. At the other extreme, with $\varphi = 1$, by definition we obtain $\lambda_{i,a;0} = 1$ or a WTP of zero: if no deaths can be avoided, then no consumption would be traded off. Table 5 presents our computation of the consumption-lives frontier for the benchmark calibration. This is an isouitility frontier, independent of any underlying technologies that trades off consumption for lives saved. The frontier for the average individual is relatively more concave than that of the median voter. Starting at 1.9 million deaths, the average individual is initially willing to reduce consumption sharply to save lives: the first 1.1 million lives are traded-off by a 32.9% consumption cut in the year of the pandemic. But the trade-off is smaller for the next 800 thousand lives, adding a extra 8% consumption cut. The concavity of the average frontier reflects the large variation in WTP across individuals of different ages. For the median voter, the frontier
is less concave, sacrificing 13.9% of consumption to save the first 1.1 million lives, an additional 8.8% for the next 800 thousand people.

5.5 Discussion and relationship to the literature

Our paper is most related to Hall et al. (2020). They use the EU framework, consider only mortality risk, and assume perfect annuity markets. While the aggregate welfare costs we compute are within the range of theirs, as shown in Figures 1 and 2, and discussed at length in Section 2, the key differences between our model and Hall et al. are on the distribution of the welfare costs of COVID-19. As shown in Section 2, when both models are compared under the same assumptions and calibration, our model implies higher welfare costs. In addition, we model the planner’s social welfare function to include aversion to inequality, which allows us to rationalize a wide range of social WTP to avoid the pandemic. Like Hall et al. (2020), other recent papers in the COVID-19 literature use the EU framework. Some of them specifically incorporate the spread of the pandemic using a SIR model, age-specific mortality rates, and consumption/income heterogeneity (Hur, 2020; Glover et al., 2020; and Eichenbaum et al., 2020, among others). Although their focus is mostly on policy implications, these papers also match the VSL to data and imply a distribution of VSL by age and consumption. Since they all introduce the non-homothetic term $c$ as in Hall et al., they also have the same type of income effects we discussed in Section 2 regarding income-poor and rich individuals.

6 Concluding comments

The COVID-19 pandemic has brought back to light the difficult trade-offs between consumption and survival, as well as the inherent conflicts among individuals who bear very different costs from a rare but potentially fatal shock. Our analysis provides a framework to study these trade-offs as well as the distributional welfare effects of pandemics. We find that the welfare costs of COVID-19 are large; that the differences in the WTP to avoid the pandemic across individuals of different ages is substantial; and that the WTP of the median voter is less than the average. Our framework can also rationalize planners with very different perspectives about the pandemic.

We hope our framework is useful to analyze the trade-offs between consumption and survival from other mortality shocks. We think it provides a meaningful and rigorous integration of traditional models of economic shocks, with models of mortality shocks. In future work we plan to extend our model to include other aspects that may affect the value of life, such as leisure. We also plan to further explore the value of life from the perspective of the social planner.

References


TABLE 1
Welfare costs of COVID-19 in the expected utility and our Epstein-Zin-Weil models
Willingness to pay to avoid a one-year pandemic (% of consumption)

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Median voter</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected utility model (EU)</td>
<td>31.3%</td>
<td>19.2%</td>
<td>32.5%</td>
</tr>
<tr>
<td>Our Epstein-Zin-Weil model (EZW)</td>
<td>34.1%</td>
<td>20.4%</td>
<td>35.2%</td>
</tr>
</tbody>
</table>

Notes: Both models are calibrated assuming no annuity markets, \( \sigma = 2 \), and age-varying discount factors. Minimum consumption (\( \bar{c} \)) in the EU model is calibrated so that the average \( VSL/c = 150 \) at age 40. This same target is used to calibrate mortality risk aversion (\( \gamma \)) in the EZW model. COVID-19 age-specific deaths are fitted from Ferguson et al. (2020) in combination with Menachemi et al. (2020). Average WTP corresponds to the average of age-specific WTP using population weights. Median voter WTP is that of individuals age 46. Standard deviation is computed over the age-specific WTP.
<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Median voter</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No annuity markets</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>41.2%</td>
<td>22.7%</td>
<td>41.2%</td>
</tr>
<tr>
<td>$\sigma = 2$</td>
<td>34.3%</td>
<td>20.5%</td>
<td>35.2%</td>
</tr>
<tr>
<td><strong>Perfect annuity markets</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>35.9%</td>
<td>21.1%</td>
<td>32.9%</td>
</tr>
<tr>
<td>$\sigma = 2$</td>
<td>29.7%</td>
<td>19.2%</td>
<td>25.9%</td>
</tr>
</tbody>
</table>

Notes: Table displays the willingness to pay (WTP) for different values of $\sigma$ and for the cases of no annuities and perfect annuity markets. Calibration assumes average $VSL/c = 150$ at age 40, $\theta = 1$ and age-varying discount factors. COVID-19 age-specific death rates are as in Table 1.
<table>
<thead>
<tr>
<th>Alternative social welfare functions</th>
<th>Aggregate WTP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No annuity markets</strong></td>
<td></td>
</tr>
<tr>
<td>$\psi = 0$</td>
<td>4.1%</td>
</tr>
<tr>
<td>$\psi = \gamma = 0.675$</td>
<td>10.8%</td>
</tr>
<tr>
<td>$\psi = 0.90$</td>
<td>41.0%</td>
</tr>
<tr>
<td>$\psi = \sigma = \theta = 1$</td>
<td>72.7%</td>
</tr>
<tr>
<td><strong>Perfect annuity markets</strong></td>
<td></td>
</tr>
<tr>
<td>$\psi = 0$</td>
<td>40.1%</td>
</tr>
<tr>
<td>$\psi = \gamma = 0.81$</td>
<td>58.2%</td>
</tr>
<tr>
<td>$\psi = \sigma = \theta = 1$</td>
<td>63.1%</td>
</tr>
</tbody>
</table>

*Notes: All exercises on the table use the benchmark calibration of our EZW model with $\sigma = 1$, average $\text{VSL}/c = 150$ at age 40, $\theta = 1$, and age-varying discount factors. $\psi = 0$ represents the case in which the planner has linear preferences or is indifferent to inequality across ages. The rest of the scenarios introduce some curvature in the planner’s utility ($\psi > 0$).*
### TABLE 4

**Full recessions: COVID-19 with concurrent economic contraction**  
(\% of consumption)

<table>
<thead>
<tr>
<th>Total number of one-year COVID deaths</th>
<th>Death rate</th>
<th>3.5% recession</th>
<th>4.5% recession</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500,000</td>
<td>0.15%</td>
<td>24.2%</td>
<td>25.0%</td>
</tr>
<tr>
<td>600,000</td>
<td>0.18%</td>
<td>25.7%</td>
<td>26.4%</td>
</tr>
<tr>
<td><strong>Median voter</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500,000</td>
<td>0.15%</td>
<td>13.4%</td>
<td>14.3%</td>
</tr>
<tr>
<td>600,000</td>
<td>0.18%</td>
<td>14.5%</td>
<td>15.4%</td>
</tr>
<tr>
<td><strong>Planner with ( \psi = \gamma = 0.675 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500,000</td>
<td>0.15%</td>
<td>7.0%</td>
<td>7.9%</td>
</tr>
<tr>
<td>600,000</td>
<td>0.18%</td>
<td>7.6%</td>
<td>8.6%</td>
</tr>
<tr>
<td><strong>Planner with ( \psi = 0.90 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500,000</td>
<td>0.15%</td>
<td>15.8%</td>
<td>16.7%</td>
</tr>
<tr>
<td>600,000</td>
<td>0.18%</td>
<td>18.0%</td>
<td>18.9%</td>
</tr>
</tbody>
</table>

*Notes: All exercises on the table use the benchmark calibration of our EZW model with no annuity markets, \( \sigma = 1 \), average \( VSL/c = 150 \) at age 40, \( \theta = 1 \), and age-varying discount factors.*
### TABLE 5

*The consumption - lives frontier (% of consumption)*

<table>
<thead>
<tr>
<th>Total number of COVID deaths</th>
<th>Death rate</th>
<th>Average age-specific</th>
<th>Median voter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0%</td>
<td>41.2%</td>
<td>22.7%</td>
</tr>
<tr>
<td>200,000</td>
<td>0.06%</td>
<td>39.5%</td>
<td>20.6%</td>
</tr>
<tr>
<td>400,000</td>
<td>0.12%</td>
<td>37.6%</td>
<td>18.4%</td>
</tr>
<tr>
<td>600,000</td>
<td>0.18%</td>
<td>35.5%</td>
<td>16.2%</td>
</tr>
<tr>
<td>800,000</td>
<td>0.24%</td>
<td>32.9%</td>
<td>13.9%</td>
</tr>
<tr>
<td>1,000,000</td>
<td>0.30%</td>
<td>30.0%</td>
<td>11.5%</td>
</tr>
<tr>
<td>1,200,000</td>
<td>0.36%</td>
<td>26.3%</td>
<td>9.1%</td>
</tr>
<tr>
<td>1,400,000</td>
<td>0.42%</td>
<td>21.6%</td>
<td>6.6%</td>
</tr>
<tr>
<td>1,600,000</td>
<td>0.48%</td>
<td>15.3%</td>
<td>4.0%</td>
</tr>
<tr>
<td>1,800,000</td>
<td>0.54%</td>
<td>6.3%</td>
<td>1.4%</td>
</tr>
<tr>
<td>1,900,000</td>
<td>0.58%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

*Notes: Same as Table 4. The largest number of deaths, 1.9 million people, corresponds to the fatality rate estimated in Menachemi et al. (2020).*
Notes: EU stands for expected utility model and EZW is the Epstein-Zin-Weil model. Both models feature age-specific survival rates and deterministic consumption inequality. There is no social mobility. Consumption is distributed log normal with a mean that increases with age and constant variance. Both models are calibrated assuming no annuity markets, $\sigma = 2$, and age-varying discount factors. Minimum consumption in the EU model is calibrated so that the average $VSL/c = 150$ at age 40. This same target is used to calibrate mortality risk aversion in the EZW model.
Notes: Same as in Figure 1. WTP is computed as a one-year consumption payment to avoid death from a one-year COVID-19 pandemic. COVID-19 age-specific deaths are fitted from Ferguson et al. (2020) in combination with Menachemi et al. (2020).
Notes: EZW model refers to generalized Epstein-Zin-Weil model with mortality and consumption risks. Consumption inequality is stochastic and there is social mobility. Mean and variance of consumption are estimated from PSID data. Computations use the benchmark calibration with no annuity markets, $\sigma = 1$, average $VSL/c = 150$ at age 40, $\theta = 1$, and age-varying discount factors. COVID-19 age-specific deaths are fitted from Ferguson et al. (2020) in combination with Menachemi et al. (2020).
APPENDIX

Supplementary materials for online publication

The Full Recession: Private versus Social Costs of COVID-19

Juan Carlos Córdoba, Marla Ripoll and Siqiang Yang

July, 2021

1 Derivation of formulas in Section 2

Section 2 in the paper considers the case of an economy in which individuals face age-dependent mortality risk and there is deterministic consumption inequality. There is no social mobility and the distribution of consumption at any age is log normal. Individual consumption changes exogenously with age to reflect the life cycle profile from the data. We also assume there are no annuity markets.

This section of the appendix derives the formulas in Section 2 of the paper. Specifically, expressions for the value of statistical life (VSL) and the willingness to pay (WTP) to avoid the one-year COVID-19 pandemic are derived. The latter corresponds to the welfare costs of the pandemic (equivalent variation). Formulas are derived for the expected utility (EU) model and for our Epstein-Zin-Weil (EZW) model.

1.1 Formulas for EU model

The remaining lifetime utility in the EU model for an individual \(i\) of age \(a\) is given by

\[
V^{EU}_{i,a} = \pi_a V^{EU}_{i,a} \quad \text{with} \quad V^{EU}_{i,a} = c_{1,a}^{1-\sigma} - \xi^{1-\sigma} \quad \frac{1}{1-\sigma} + \beta_a V^{EU}_{i,a+1},
\]

where \(\pi_a\) is the probability of survival between \(a-1\) and \(a\), \(c_{1,a}\) is consumption, \(1/\sigma\) is the elasticity of intertemporal substitution, \(\xi\) is the level of minimum consumption, and \(\beta_a\) is an age-dependent
discount factor. Forward iteration on $V_{i,a}^{EU}$ yields

$$V_{i,a}^{EU} = \sum_{s=a}^{\infty} S_{a,s}^{EU} \frac{c_{i,s}^{1-\sigma} - c_{i,s}^{1-\sigma}}{1-\sigma},$$

where $S_{a,a}^{EU} = 1$ and $S_{a,s}^{EU} = \prod_{j=a}^{s-1} \beta_j \pi_{j+1}$ for $s > a$. The present-value budget constraint is given by

$$b_{i,a} = \sum_{s=a}^{\infty} (1 + r)^{a-s} [c_{i,s} - y_{i,s}],$$

where $y_{i,a}$ corresponds to income, $b_{i,a}$ are non-annuitized riskless bonds, and $r$ is the interest rate.

The VSL corresponds to the marginal rate of substitution between survival and consumption defined as,

$$V_{i,a}^{EU}(\pi_{a} + \Delta \pi_{a}, 1) = V_{i,a}^{EU}(\pi_{a}, \lambda_{i,a}^{EU}),$$

which corresponds to

$$\left(1 + \frac{\Delta \pi_{a}}{\pi_{a}}\right) \frac{c_{i,a}^{1-\sigma} - c_{i,a}^{1-\sigma}}{1-\sigma} + \frac{\Delta \pi_{a}}{\pi_{a}} \beta_{a} V_{i,a+1}^{EU} = \frac{(\lambda_{i,a}^{EU} c_{i,a})^{1-\sigma} - c_{i,a}^{1-\sigma}}{1-\sigma}.$$

Using (2), the equation above can be written as

$$\frac{\lambda_{i,a}^{EU1-\sigma} - 1}{1-\sigma} = \frac{\Delta \pi_{a}}{\pi_{a}} \frac{V_{SL_{i,a}^{EU}}}{c_{i,a}},$$

as in the paper.

We now compute the welfare costs (equivalent variation) of the COVID-19 pandemic in the EU model. We model COVID-19 as a drop in survival probabilities for only one year $\Delta \pi_{a} < 0$, when the individual is age $a$. To calculate equivalent variation measures, we compare two economies: one in which individuals face the pandemic but keep the baseline consumption levels, and one in which individuals enjoy baseline survival probabilities but lower consumption. The welfare cost calculation computes the consumption shift, $\lambda_{i,a}^{EU} \leq 1$, that would make individuals indifferent between these two economies. The welfare cost $\lambda_{i,a}^{EU}$ should satisfy

$$\nabla_{i,a}^{EU} (\pi_{a} + \Delta \pi_{a}, 1) = \nabla_{i,a}^{EU} (\pi_{a}, \lambda_{i,a}^{EU}),$$

which corresponds to

$$\left(1 + \frac{\Delta \pi_{a}}{\pi_{a}}\right) \frac{c_{i,a}^{1-\sigma} - c_{i,a}^{1-\sigma}}{1-\sigma} + \frac{\Delta \pi_{a}}{\pi_{a}} \beta_{a} V_{i,a+1}^{EU} = \frac{(\lambda_{i,a}^{EU} c_{i,a})^{1-\sigma} - c_{i,a}^{1-\sigma}}{1-\sigma}. $$

Using (2), the equation above can be written as

$$\frac{\lambda_{i,a}^{EU1-\sigma} - 1}{1-\sigma} = \frac{\Delta \pi_{a}}{\pi_{a}} \frac{V_{SL_{i,a}^{EU}}}{c_{i,a}},$$

as in the paper.
and using first order Taylor expansion around $\lambda_{i,a}^{EU} = 1$ we have

$$\alpha_{i,a}^{EU} = - (\lambda_{i,a}^{EU} - 1) \approx - \Delta \pi_{a} \frac{VSL_{i,a}^{EU}}{c_{t}},$$

where $\alpha_{i,a}^{EU}$ is the welfare cost of a one-year pandemic, or the WTP as a fraction of one-year consumption to avoid death as in the paper.

### 1.2 Formulas for EZW model

This part of the appendix derives the formulas in Section 2 for the EZW model. Since there is no consumption risk, but only mortality risk, the EZW model disentangles intertemporal substitution $(1/\sigma)$ from mortality risk aversion $(\gamma)$. There is no separate role for a consumption risk aversion parameter as in the generalized model in Section 3 of the paper.

The remaining lifetime EZW utility for an individual $i$ of age $a$ in is given by,

$$V_{EZW}^{i,a} = \frac{1}{\pi_{a}^{1-\gamma}} V_{EZW}^{i,a} \quad \text{where} \quad V_{EZW}^{i,a} = \left[ (1 - \beta_{a}) c_{i,a}^{1-\sigma} + \beta_{a}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$  

Iterating forward on $V_{i,a}$, the following sequential form can be obtained,

$$V_{EZW}^{i,a} = \left[ \sum_{s=a}^{\infty} S_{EZW}^{a,s} (1 - \beta_{s}) c_{i,s}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad \text{(3)}$$

where $S_{a,a} = 1$ and $S_{EZW}^{a,s} = \prod_{j=a}^{s-1} \beta_{j}^{\frac{(1-\sigma)}{(1-\gamma)}}$ for $s > a$. Using the Euler equation from maximizing $V_{EZW}^{i,a}$ subject to lifetime budget constraint (1) results in

$$\left( V_{EZW}^{i,a} \right)^{1-\sigma} = (1 - \beta_{a}) c_{i,a}^{-\sigma} \sum_{s=a}^{\infty} (1 + r)^{a-s} c_{i,s}. \quad \text{(4)}$$

The VSL corresponds to the marginal rate of substitution between survival and consumption defined as:

$$VSL_{EZW}^{i,a} = \frac{\partial V_{EZW}^{i,a}}{\partial \pi_{a}} \frac{\partial \pi_{a}}{\partial c_{i,a}} = \frac{1}{1 - \gamma} \frac{1}{1 - \beta_{a} \pi_{a}} \left( V_{EZW}^{i,a} \right)^{1-\sigma} c_{i,a}^{-\sigma}; \quad \text{(5)}$$

which in the EZW model corresponds to

$$VSL_{EZW}^{i,a} = \frac{1}{\pi_{a}} \left[ \sum_{s=a}^{\infty} \frac{1}{(1 + r)^{s-a}} \frac{1}{1 - \gamma} c_{i,s} \right], \quad \text{(6)}$$

as in the paper.
Regarding the welfare cost of COVID-19 in the EZW model, we have that $\lambda_{i,a}^{EZW}$ should satisfy

$$\nabla_{i,a}^{EZW} (\pi_a + \Delta \pi_a, 1) = \nabla_{i,a}^{EZW} (\pi_a, \lambda_{i,a}^{EZW}),$$

which using (5) can be written as

$$\frac{(\lambda_{i,a}^{EZW})^{1-\sigma} - 1}{1 - \sigma} = \left(1 + \frac{\Delta \pi_a}{\pi_a}\right)^{\frac{1-\sigma}{1-\gamma}} - 1 \left[1 - \gamma \frac{\pi_a}{\sigma} \frac{VSL_{i,a}^{EZW}}{c_{i,a}}\right].$$

Last, using a first order Taylor expansion around $\lambda_{i,a}^{EZW} = 1$ we have

$$\alpha_{i,a}^{EZW} = -(\lambda_{i,a}^{EZW} - 1) \approx - \left(1 + \frac{\Delta \pi_a}{\pi_a}\right)^{\frac{1-\sigma}{1-\gamma}} - 1 \left[1 - \gamma \frac{\pi_a}{\sigma} \frac{VSL_{i,a}^{EZW}}{c_{i,a}}\right],$$

where $\alpha_{i,a}^{EZW}$ is the welfare cost of a one-year pandemic in the EZW model as in the paper.

2 Proofs of propositions

2.1 Proposition 1

Proposition 1 derives the closed-form solution for the Bellman equation. Preferences are described by

$$V_{i,a,t} = \frac{1}{\pi_{a,t}} V_{i,a,t}, \text{ with } \gamma \in (0, 1),$$

and

$$V_{i,a,t} = \begin{cases} \left(1 - \beta_a\right) \bar{c}_{i,a,t}^{1-\sigma} + \beta_a \pi_{a+1,t+1}^{1-\sigma} \left(E_{a} \left[V_{i,a+1,t+1}^{1-\theta} \right] \right)^{\frac{1}{1-\theta}} & \text{for } \sigma \neq 1, \\
\bar{c}_{i,a,t}^{1-\beta_a} \left(\pi_{a+1,t+1}^{1-\gamma} E_{a} \left[V_{i,a+1,t+1}^{1-\theta} \right] \right)^{\frac{1}{1-\theta}} & \text{for } \sigma = 1, \end{cases}$$

where $V_{i,a,t} \geq 0$ is the utility of being alive, $\pi_{a,t}$ is the survival probability between ages $a - 1$ and $a$, $\gamma$ is the coefficient of mortality aversion, $\beta_a$ is an age-specific discount factor, $1/\sigma$ is the standard intertemporal substitution, and $\theta$ captures overall aversion to risk. Effective consumption is defined as $\bar{c}_{i,a,t} = \lambda_{i,a,t} c_{i,a,t}$, where $c_{i,a,t}$ is consumption and $\lambda_{i,a,t}$ is a proportional shift parameter used to calculate welfare under various scenarios. In the baseline $\lambda_{i,a,t} = 1$ for all $i$, $a$ and $t$. Closed-form solutions for the Bellman equation in (8) are possible as shown in the following proposition.

**Proposition 1.** The utility of individual $i \in I(a,t)$ is given by

$$V_{i,a,t} = \pi_{a,t}^{1-\gamma} V_{i,a,t} = \pi_{a,t}^{1-\beta_a} A_{i,a,t} \bar{c}_{i,a,t} \text{ for all } i, t \text{ and all } a,$$
where \( A_{i,a,t} \) is the saddle path solution of the following first order difference equation,

\[
A_{i,a,t} = \begin{cases} 
1 - \beta_a + \beta_a \left[ \frac{1}{\pi_{a+1,t+1}} A_{i,a+1,t+1} \frac{\lambda_{i,a+1,t+1}}{\lambda_{i,a,t}} e^{\mu_{a+1} - \theta \eta_{a+1}/2} \right]^{1-\sigma} \frac{1}{1-\sigma} & \text{for } \sigma \neq 1, \\
\frac{1}{\pi_{a+1,t+1}} A_{i,a+1,t+1} \frac{\lambda_{i,a+1,t+1}}{\lambda_{i,a,t}} e^{\mu_{a+1} - \theta \eta_{a+1}/2} \beta_a & \text{for } \sigma = 1.
\end{cases}
\] (10)

**Proof.** Guess that value function of the alive has the form \( V_{i,a,t} = A_{i,a,t} \bar{c}_{i,a,t} \) where \( A_{i,a,t} \) is a coefficient to be determined. Under this guess, and using the fact that \( \bar{c}_{i,a,t} = \lambda_{i,a,t} c_{i,a,t} \), it follows that:

\[
E [ V_{i,a,t+1,1}^{1-\theta} | a_t ] = (A_{i,a+1,t+1} \lambda_{a+1,t+1})^{1-\theta} E \left[ c_{i,a+1,t+1}^{1-\theta} | a_t \right]
= \left[ A_{i,a+1,t+1} \lambda_{a+1,t+1} c_{i,a,t} e^{\mu_{a+1} - \theta \eta_{a+1}/2} \right]^{1-\theta}.
\]

Substituting this expression into (8) we can write

\[
V_{i,a,t} = \begin{cases} 
\lambda_{i,a,t} c_{i,a,t} \left[ 1 - \beta_a + \beta_a \pi_{a+1,t+1} \frac{1}{\lambda_{i,a,t}} (A_{i,a+1,t+1} \frac{\lambda_{i,a+1,t+1}}{\lambda_{i,a,t}} e^{\mu_{a+1} - \theta \eta_{a+1}/2})^{1-\sigma} \right]^{1-\sigma} \frac{1}{1-\sigma} & \text{for } \sigma \neq 1, \\
\lambda_{i,a,t} c_{i,a,t} \left[ \frac{1}{\pi_{a+1,t+1}} A_{i,a+1,t+1} \frac{\lambda_{i,a+1,t+1}}{\lambda_{i,a,t}} e^{\mu_{a+1} - \theta \eta_{a+1}/2} \right] \beta_a & \text{for } \sigma = 1,
\end{cases}
\] (11)

which satisfies the guess if \( A_{i,a,t} \) satisfies (10).

### 2.2 Proposition 2

Proposition 2 characterizes the social welfare costs for the special case in which \( \sigma = 1 \) and \( \beta_a = \beta \) for all \( a \). As shown in the paper, in this special case the social welfare costs (equivalent variation) of a one-year COVID-19 pandemic at time \( t = 0 \) are given by

\[
\lambda_0 = \frac{\sum_a M_{a,0} (\pi^*_a + \Delta \pi_{a,0})^{\frac{1-\psi}{1-\gamma}} \left( A^*_a e^{v_a - ga - \psi \Phi_a^2/2} \right) \frac{1}{1-\psi} \frac{1}{(1-\sigma)(1-\psi)}}{\sum_a M_{a,0} (\pi^*_a)^{\frac{1-\psi}{1-\gamma}} \left( A^*_a e^{v_a - ga - \psi \Phi_a^2/2} \right) \frac{1}{1-\psi}},
\] (12)

where \( M_{a,0} \) is the mass of people age \( a \) at time 0, \( \pi^*_a \) is the baseline survival probability, \( \Delta \pi_{a,0} < 0 \) is the drop in survival at time 0 due to the pandemic, \( A^*_a \) is the value of \( A_{a,0} \) in the economy with no pandemic, and \( \psi \) is the parameter that controls the planner’s aversion to inequality. In
addition, the private welfare costs of the one-year pandemic are given by

\[
\lambda_{i,a,0} = \lambda_{a,0} = \begin{cases} 
1 + \left[ \frac{\left( \pi_{a,0}^* + \Delta \pi_{a,0} \right)}{\pi_{a,0}^*} \right]^{\frac{1-\gamma}{1-\beta a}} - 1 \left( \frac{\pi_{a,0}^*}{1-\beta a} \right)^{\frac{1-\gamma}{1-\beta a}} & \text{for } \sigma \neq 1, \\
\left( \frac{\pi_{a,0}^* + \Delta \pi_{a,0}}{\pi_{a,0}^*} \right)^{\frac{1}{1-\beta a}} & \text{for } \sigma = 1.
\end{cases}
\]  

(13)

The following proposition derives the link between the social and private costs of the pandemic for the special case with \( \sigma = 1 \) and \( \beta_a = \beta \) for all \( a \).

**Proposition 2.** For the case \( \sigma = 1 \) and \( \beta_a = \beta \) for all \( a \), the \( \lambda_0 \) for a planner with aversion to inequality parameter \( \psi \) can be written as

\[
\lambda_0 = \left[ \sum_a f_{a,0}(\psi) (\lambda_{a,0})^{(1-\beta)(1-\psi)} \right]^{\frac{1}{(1-\beta)(1-\psi)}}
\]  

(14)

where \( \lambda_{a,0} \) is the age-specific WTP as defined in equation (13), and \( f_{a,0}(\psi) \) is a density function given by

\[
f_{a,0}(\psi) = \frac{M_{a,0} \left( \left( \pi_{a,0}^* \right)^{\frac{1}{1-\gamma}} A_{a,0}^* e^{v_{a}-ga-\psi g^2/2} \right)^{1-\psi}}{\sum_a M_{a,0} \left( \left( \pi_{a,0}^* \right)^{\frac{1}{1-\gamma}} A_{a,0}^* e^{v_{a}-ga-\psi g^2/2} \right)^{1-\psi}}.
\]  

(15)

In addition, the marginal rate of substitution between consumption and survival for the planner is given by

\[
MRS(\psi) = -\frac{1}{1-\gamma} \frac{1}{1-\beta} \sum_a f_{a,0}(\psi) \left( \frac{\pi_{a,0}^*}{\pi_{a,0}^*} \right)^{\frac{1}{1-\psi}}.
\]  

(16)

**Proof.** Density (15) and equation (14) follow from (12). In order to derive (16), notice that the WTP of the planner is given by

\[
\alpha(\Delta \pi) = 1 - \lambda_0 = 1 - \left[ \sum_a f_{a,0}(\psi) \left( \frac{\pi_{a,0}^* + \Delta \pi_{a,0}}{\pi_{a,0}^*} \right)^{\frac{1}{1-\gamma}} \right]^{\frac{1}{(1-\beta)(1-\psi)}},
\]

where we have used equation (13) for the case \( \sigma = 1 \) to substitute for \( \lambda_{a,0} \). The marginal rate of substitution consumption and survival can be calculated as \( MRS(\psi) = \lim_{\Delta \pi \to 0} \frac{\alpha(\Delta \pi)}{\Delta \pi} \),and
using L’Hopital’s rule

\[
MRS(\psi) = \lim_{\Delta \pi \to 0} \frac{1}{\Delta \pi} \left[ 1 - \sum_a f_{a,0}(\psi) \left( \frac{\pi_{a,0} + \Delta \pi}{\pi_{a,0}} \right)^{\frac{1-\psi}{1-1+\psi}} \right],
\]

which results in (16).

3 Robustness checks

3.1 Alternative parameter values

This section presents a robustness analysis of our results to alternative parameter values. Table A.1 summarizes the results when we change certain parameter values, one at a time. In particular, we report results for alternative values of \( \theta \), for a calibration of the model that assumes constant \( \mu_a = \mu \) and \( \eta_a = \eta \), and for a finite-horizon calibration that considers \( D_{i,a,t} \neq 0 \). Table A.1 includes the benchmark calibration with \( \sigma = \theta = 1 \).

Table A.1 shows that alternative values of \( \theta \) have little effects on the WTP results. For both \( \theta = 10 \) and \( \theta = 20 \) the average and the median voter WTP are higher than in the calibrated benchmark (\( \sigma = \theta = 1 \)) with or without annuity markets, but the differences are quantitatively small. Recall from equation (13) that when \( \sigma = 1 \) the age-specific WTP depends on \( \Delta \pi_{a,0} \), the calibrated \( \beta_a \), and the calibrated \( \gamma \). As seen in the following equation,

\[
\frac{1 - \beta_a}{\beta_a} = \frac{1 + r}{\delta \pi_a + 1 - \delta} (1 - \beta_{a+1}) \frac{1+\sigma}{\pi_{a+1}} e^{-\sigma \mu_{a+1} + \theta(1+\sigma)\eta_{a+1}^2/2},
\]

parameter \( \theta \) affects the calibrated \( \beta_a \) only through term \( e^{-\sigma \mu_{a+1} + \theta(1+\sigma)\eta_{a+1}^2/2} \); a larger \( \theta \) results in a lower \( \beta_a \). However, since \( \eta_a^2 \) is very small in magnitude, the overall quantitative effect of \( \theta \) on \( \beta_a \) is small. For the case of no annuity markets, with \( \theta = 10 \) we obtain a calibrated \( \gamma = 0.877 \), while for \( \theta = 20 \) we obtain \( \gamma = 0.894 \). In this respect, higher consumption risk aversion results in a higher mortality risk aversion in the calibration. The bottom-left panel of Figure A.1 displays the full distribution of age-specific WTP for \( \theta = 1 \) and \( \theta = 10 \). As shown, there is little difference, although those between ages 50 and 60 have slightly higher WTP when \( \theta = 10 \). This is the case because the standard deviation of the consumption shock (\( \eta_a \)) is higher at those ages.

Table A.1 also shows that for the case of constant \( \mu_a = \mu \) and \( \eta_a = \eta \) (but still maintaining \( \sigma = \theta = 1 \)), the WTP results only change slightly for both the no annuities and the perfect annuity cases. The bottom-right panel of Figure A.1 displays the full distribution of age-specific WTP for constant and age-varying \( \mu_a \) and \( \eta_a \) for the case of no annuities. This is not surprising since the age-varying estimated values of \( \mu_a \) and \( \eta_a \) only affect the results through term \( e^{-\sigma \mu_{a+1} + \theta(1+\sigma)\eta_{a+1}^2/2} \), which tends to have a quantitatively second-order effect. The bottom-right panel of Figure A.1
displays the full distribution of age-specific WTP for constant and age-varying $\mu_a$ and $\eta_a$. Having said this, it is important to notice that the decrease in survival rates from COVID-19 are so large, particularly for the elder, that the WTP results are mostly driven by $\Delta \pi_{a,0}$. This is specially true in the case of $\sigma = 1$. If we were to use our model to compute the WTP to avoid pandemics with smaller effects in survival probabilities, other features of our model like age-varying $\mu_a$ and $\eta_a$ would play a larger role.

In the benchmark calibration we closed the model by assuming that the horizon is infinite, but that after certain age $a \geq a^* = 99$, the survival probability is constant and very low, but positive ($\pi_{a,t} = \pi$). This "perpetual old" assumption avoids that lifetime utility becomes zero when $\sigma \geq 1$. Table A.1 reports the robustness of the WTP results to an alternative way of closing the model, namely one in which the horizon is finite. In this case, rather than assuming that the utility upon death $D_{i, a, t} = 0$, we impute a positive value of consumption in the dead state for ages after which the survival probability is zero. In particular, a non-homotheticity in utility ($D_{i, a, t} > 0$) is assumed for ages over $a^* = 99$.

The main difficulty of solving this case is that with the non-homotheticity after age 99, utility is no longer linear in consumption. In this finite-horizon model, utility in period $a^*$ is given by $V_{a^*} = D_{a^*}$ while utility in period $a^* - 1$ is given by

$$V_{a^*-1} = (c_{a^*-1})^{1-\beta_{a^*-1}} (D_{a^*})^{\beta_{a^*-1}}.$$ 

It can be shown that in this case utility is of the form

$$V_{i,a,t} = A_{a,t} (\lambda_{a,t} c_{i,a,t})^{\phi_a}$$

where $\phi_a = 1 - \beta_a + \beta_a \phi_{a+1}$ and

$$A_{a,t} = \left( \frac{1}{\pi_{a+1,t+1} A_{a+1,t+1}} \left( \frac{\lambda_{a+1,t+1}}{\lambda_{a,t}} e^{\phi_{a+1} (1 - \theta) - 1} \eta_{a+1}^2 / 2 \right)^{\phi_{a+1}} \right)^{\beta_a}.$$ 

The last two recursions can be solved using the following terminal conditions $\phi_{a^*-1} = 1 - \beta_{a^*-1}$, and $A_{a^*-1,t^*-1} = (D_{a^*})^{\beta_{a^*-1}}$ where $D_{a^*} > 0$.

As shown in Table A.1, our results are robust to this alternative way of closing the model. We set $D_{a^*} = 0.001$ and find that for the benchmark calibration with $\sigma = \theta = 1$, the average and the median voter’s WTP are virtually identical to the benchmark for both the cases with no annuities and annuities. These results are robust to other values of $D_{a^*}$, but we set $D_{a^*} = 0.001$ to guarantee a decreasing profile for $A_{a,t}$ with age.
3.2 Persistent consumption costs

In this section we extend the computations to consider the case in which the consumption costs are persistent. In this case the cost is paid gradually over time according to \( \lambda_{t,a,0} = \{ \lambda_{i,a,0}, \lambda_{i,a,0}, \lambda_{i,a,0}^2, \ldots \} \), where \( \rho \in [0,1) \) is a persistence parameter. The computations in the paper correspond to the special case in which \( \rho = 0 \). The case \( \rho > 0 \) seeks to describe a situation in which avoiding the pandemic could trigger a long-lasting recession. Sequence \( \lambda_{i,a,0} \) is fully characterized its first element \( \lambda_{i,a,0} \). In this case, although the pandemic decreases the survival probabilities only in year \( t = 0 \), the consumption costs persist up to time \( t = t^* \).

Specifically, assume that

\[
\ln \lambda_{a,t+1} = \rho \ln \lambda_{a,t}
\]

for \( t \geq 0 \), where \( \rho \in [0,1) \) characterizes the persistence of consumption costs. By construction, the WTP when \( \rho > 0 \) will be gradually declining over time, and the WTP at \( t = 0 \) will be smaller the larger the \( \rho \) is. In other words, the WTP at time \( t = 0 \) is smaller the larger \( t^* \) is.

The computation of the individual and the aggregate welfare costs from \( t = 0 \) to \( t = t^* \) is more complicated than what was shown above for the case \( \rho = 0 \). As shown in the paper, in the case \( \rho = 0 \), we have that

\[
A_{a,0} \lambda_{i,a,0} = \begin{cases} 
(A^*_a)^{1-\sigma} + (1 - \beta_a) (\lambda_{1-a,0} - 1) \right)^{\frac{1}{1-\sigma}} & \text{for } \sigma \neq 1, \\
\lambda_{1-a,0} A_{a,0}^* & \text{for } \sigma = 1,
\end{cases}
\]

so that \( A_{a,0} \lambda_{i,a,0} \) can be expressed as a function of the baseline \( A_{a,0}^* \). But when \( \rho > 0 \) this is not the case any longer because consumption goes back to the baseline in \( t = t^* \), not in \( t = 1 \). For the case \( \rho > 0 \) we have that

\[
A_{a,0} = \begin{cases} 
(1 - \beta_a) + \beta_a \left[ \frac{1}{\pi_{a+1,1}} A_{a+1,1} \lambda_{a,0}^{\rho-1} e^{\mu_{a+1} - \theta_{a+1}^2 / 2} \right]^{\frac{1}{1-\sigma}} & \text{for } \sigma \geq 0, \sigma \neq 1, \\
\frac{1}{\pi_{a+1,1}} A_{a+1,1} \lambda_{a,0}^{\rho-1} e^{\mu_{a+1} - \theta_{a+1}^2 / 2} & \text{for } \sigma = 1.
\end{cases}
\]

In this case one needs to compute separate sequences to obtain each \( A_{a,0} \), as older individuals will reach terminal age \( a^* \) before time \( t = t^* \), while younger individuals will reach this terminal age after period \( t = t^* \). These different \( A_{a,0} \) sequences reflect the fact that individuals experience the "transition" from \( t = 0 \) to \( t = t^* \) at different ages over their life cycle. Notice that by construction \( \lambda_{a,t^*} = \lambda_{t^*} = 1 \).

Using the equation above we compute the WTP each year from 0 to \( t^* \). Table A.2 presents the results for this extension, in particular for the cases \( \rho = 0.5 \) and \( \rho = 0.9 \). Value \( \rho = 0.5 \) implies \( t^* \approx 8 \), while \( \rho = 0.9 \) implies \( t^* \approx 30 \). The former illustrates lingering effects of a crisis similar to the 2007-2009 Great Recession, while the latter illustrates the limiting case as \( \rho \rightarrow 1 \).

Table A.2 reports the WTP statistics for \( t = 0 \), while Figure A.2 includes plots for the full
transition of WTP from \( t = 0 \) to \( t^* \). As seen in Table A.2, the WTP during period \( t = 0 \) falls as the persistence increases: individuals are willing to pay less to avoid the pandemic if they will be paying a percentage of consumption for a longer period of time. For instance, for the case with no annuities, the average WTP at \( t = 0 \) falls from 41.2\% when all costs are paid in one period \((\rho = 0)\), to 30.9\% when \( \rho = 0.5 \) and to 12.2\% when \( \rho = 0.9 \). As seen in the table, a similar pattern is observed for the WTP of the median voter. Finally, the disagreement among individuals of different ages still persists as seen on the standard deviation of the WTP across ages.

The overall message of Table A.2 is that even when the consumption costs are persistent, the average WTP to eliminate a one-year COVID-19 pandemic is still large. For the benchmark economy with \( \sigma = \theta = 1 \) and no annuity markets, simulating a \( \rho = 0.5 \) as a potentially realistic scenario for a lengthy recovery results in an average WTP of 30.9\% of time-0 consumption. In the presence of annuity markets this number falls to 23.4\%, still a large welfare cost.

The top-left panel in Figure A.2 shows the distribution of the WTP by age at time \( t = 0 \). As \( \rho \) increases, the WTP falls for all ages. In the limit, with \( \rho = 0.9 \), those above age 80 are willing to pay much less than when \( \rho = 0 \), between 55 and 65\% of time-0 consumption. The top-right panel in Figure A.2 displays the average WTP from period \( t = 0 \) to \( t = t^* \) for different values of \( \rho \). Notice that by construction the WTP falls over time. The bottom-left panel shows how the WTP for the median voter evolves over time, while the bottom-right displays the case of the average 65-year old individual.
TABLE A.1
Robustness checks to alternative parameters
Willingness to pay to avoid a one-year pandemic (% of consumption)

<table>
<thead>
<tr>
<th>Alternative parameters</th>
<th>Average age-specific</th>
<th>Median voter</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No annuity markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark calibration</td>
<td>41.2%</td>
<td>22.7%</td>
<td>41.2%</td>
</tr>
<tr>
<td>$\theta = 10$</td>
<td>42.9%</td>
<td>23.7%</td>
<td>41.4%</td>
</tr>
<tr>
<td>$\theta = 20$</td>
<td>43.9%</td>
<td>24.0%</td>
<td>41.5%</td>
</tr>
<tr>
<td>Constant $\mu$ and $\eta$</td>
<td>42.3%</td>
<td>23.9%</td>
<td>41.6%</td>
</tr>
<tr>
<td>Non-homothetic finite horizon</td>
<td>41.4%</td>
<td>22.8%</td>
<td>41.3%</td>
</tr>
<tr>
<td>Perfect annuity markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark calibration</td>
<td>35.9%</td>
<td>21.1%</td>
<td>32.9%</td>
</tr>
<tr>
<td>$\theta = 10$</td>
<td>40.2%</td>
<td>22.7%</td>
<td>38.1%</td>
</tr>
<tr>
<td>$\theta = 20$</td>
<td>42.5%</td>
<td>23.4%</td>
<td>40.2%</td>
</tr>
<tr>
<td>Constant $\mu$ and $\eta$</td>
<td>36.5%</td>
<td>21.7%</td>
<td>33.3%</td>
</tr>
<tr>
<td>Non-homothetic finite horizon</td>
<td>35.9%</td>
<td>21.1%</td>
<td>33.1%</td>
</tr>
</tbody>
</table>

Notes: Benchmark calibration refers to the case $\sigma = 1$, $\text{VSL}/c = 150$, $\theta = 1$ and age-varying discount factors. Each robustness check is introduced separately. Average WTP corresponds to the weighted average of age-specific WTP using population weights. Median voter WTP is that of individuals age 46, which corresponds to the median of the population above age 18. Standard deviation is computed over the age-specific WTP.
<table>
<thead>
<tr>
<th></th>
<th>Average age-specific</th>
<th>Median voter</th>
<th>Standard deviation</th>
</tr>
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<tbody>
<tr>
<td>No annuity markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark calibration ($\rho = 0$)</td>
<td>41.2%</td>
<td>22.7%</td>
<td>41.2%</td>
</tr>
<tr>
<td>$\rho = 0.5$</td>
<td>30.9%</td>
<td>12.1%</td>
<td>40.8%</td>
</tr>
<tr>
<td>$\rho = 0.9$</td>
<td>12.2%</td>
<td>2.9%</td>
<td>25.0%</td>
</tr>
<tr>
<td>Perfect annuity markets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark calibration ($\rho = 0$)</td>
<td>35.9%</td>
<td>21.1%</td>
<td>32.9%</td>
</tr>
<tr>
<td>$\rho = 0.5$</td>
<td>23.4%</td>
<td>11.1%</td>
<td>25.3%</td>
</tr>
<tr>
<td>$\rho = 0.9$</td>
<td>8.2%</td>
<td>2.6%</td>
<td>13.7%</td>
</tr>
</tbody>
</table>

**Notes:** Same as in Table A.1. In the benchmark calibration consumption costs are not persistent ($\rho = 0$) and are all paid in one period. For the cases with persistent consumption shocks ($\rho > 0$) the table displays the WTP on the first year (year of the COVID-19 pandemic).
Figure A.1. Robustness checks: Willingness to pay under alternative scenarios

**Intertemporal substitution**

- `sigma=1`
- `sigma=2`

**Annuity markets**

- `no annuities`
- `perfect annuities`

**Aversion to consumption risk**

- `theta = 1`
- `theta = 10`

**Consumption shock parameters**

- `age-varying`
- `constant`
Figure A.2. Willingness to pay under persistent consumption costs

WTP during year of pandemic

Average WTP over time

Median voter WTP over time

65-year old WTP over time