# OPTIMAL POPULATION ON A FINITE PLANET

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**Abstract**

The question of optimal population size on a finite planet is at the forefront of current policy issues. The purpose of this paper is to use economic theory to understand the dynamics of population in the context of fixed resources. We merge the Barro and Becker (1989) dynastic model of endogenous fertility choice with a neoclassical production structure augmented to include fixed resources, land in particular. Fertility decisions are driven by the trade-off between dynastic altruism on children and the cost of having children. The presence of a fixed resource allows the model to have a well-defined steady state population level. We use the model to explore the conditions and understand the mechanisms under which population implodes, explodes or stabilizes.

*Key words:* fixed resources, land, CES production, fertility choice, dynastic altruism, technological progress, Malthusian equilibrium

## 1 Introduction

Two conflicting concerns regarding population coexist. The first concern is that by 2050 the population of virtually all countries in Europe, as well as in Japan, will have declined (2001 *Replacement Migration,* UNPD). Given the below-replacement fertility levels, baseline population projections for Europe predict a 14% decline (101 million less people) between 2000 and 2050. For Japan alone, this decline is estimated to be 17% (22 million less people). Even in the United States, fertility rates post-Great Recession have not recovered, reaching a historic low of 1.76 children per woman (CDC, 2018). Many European countries and Japan have implemented pro-natal policies to address the consequences of these trends.

The second concern about population evokes the Malthusian principles. On a finite planet with limited resources, sustaining the world's population numbers does not seem possible without an environmental collapse. Although fertility rates have declined everywhere in the developing world, the world's population is estimated to continue growing through the rest of this century, reaching 11.2 billion by 2100. Median projections show a subsequent population decrease followed

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by a stabilized population at around 9 billion in the 22nd century (2015 UN Projections). Those concerned about environmental degradation advocate for child taxation, a policy that contrasts with the pro-natal policies in Europe (Bohn and Stuart, 2015). Although different on their motivation, these policies to limit population growth echo those implemented in developing countries the 1970s, when it was believed that overpopulation was hindering economic growth and perpetuating poverty.

These two conflicting concerns on population size bring the question of optimal population on a finite planet to the forefront of current issues. The purpose of this paper is to use economic theory to understand the dynamics of population in the context of fixed and exhaustible resources. We revisit the neoclassical growth model with endogenous population to study the macroeconomic implications of population dynamics. In particular, we merge the Barro and Becker (1989) dynastic model of endogenous fertility choice with a neoclassical production structure augmented to include fixed resources, land in particular. Fertility decisions are driven by the trade-off between dynastic altruism on children and the cost of having children. The presence of a fixed resource allows the model to have a well-defined steady state population level. We use the model to explore the conditions and understand the mechanisms under which population implodes, explodes or stabilizes.

In addition to the current relevance of population size issues, our analysis is also motivated by two cross-country facts in the data. As illustrated in Figure 1, there is a cross-country positive correlation between population and (usable) land area. On the other hand, as shown in Figure 2, there is no correlation between land area and GDP per capita. We show that our model with fixed resources and endogenous fertility is consistent with both of these facts. Our model features what we call a steady-state "dichotomy": any changes in land-related resources only affect the level of population, but not per capita income, consumption, or physical capital per person. In contrast, other non-land related shocks to productivity do affect these variables. In particular, land-augmenting technological progress, which is the only way to increase effective usable land in the model, results in higher population but not in higher steady-state per capita income. This sort of "resource curse" is consistent with the patterns portrayed in Figures 1 and 2.

The model is also able to generate scenarios with population implosion or explosion. The steady-state dichotomy in the model implies that if labor or capital-augmenting technological progress is faster than land-augmenting progress, then steady-state per capita income and consumption increase while population implodes, resembling the recent dynamics in Europe and Japan. Finally, our model solves for the efficient population dynamics on a finite planet. From this perspective, it suggest that the increase in population in response to more fixed resources is efficient.

[To be completed]

# 2 Theory

## 2.1 Social planner's problem

Our theory extends the Barro and Becker (1988, 1989) framework of endogenous fertility to include three factors of production: capital, labor and land. Consider an infinite horizon deterministic economy endowed with a fixed amount of land, $Z$, an initial population mass, $N_0$, and an initial amount of physical capital, $K_0$. Time is discrete. Individuals live for two periods, one as a child and one as an adult. Children do not consume. During adulthood individuals work, consume, and become parents. Each adult can give birth to a maximum of $\overline{n}$ children. There are two costs of raising a child: a time cost, $\lambda$, and a goods cost, $\pi$.

### 2.1.1 Aggregate constraints

Population evolves according to

$$N_{t+1} = N_t n_t \tag{1}$$

where $n_t$ is the realized fertility.

Production requires capital $K$, labor $L$, and land $Z$. Inputs are transformed into output according to a constant returns to scale production function $F(K, L, Z)$. Production is utilized for consumption, investment, and raising children,

$$F\left(K_t, L_t, Z\right) \geq c_t N_t + I_t + \pi_t N_{t+1},$$

where $c_t$ is individual consumption, and $I_t$ is investment in physical capital. Investment is used to accumulate capital according to the law of motion

$$K_{t+1} = (1 - \delta) K_t + I_t.$$

The initial amount of capital, $K_0$, is exogenously given. The last two equations can be combined into the single resource constraint

$$F\left(K_t, L_t, Z\right) \geq K_{t+1} - (1 - \delta) K_t + \left(c_t + \pi_t n_t\right) N_t \tag{2}$$

Each individual is endowed with one unit of labor. Aggregate labor input thus equals total population minus the amount of time spent raising children,

$$L_t = N_t - \lambda N_{t+1}. \tag{3}$$

### 2.1.2 Individual welfare

The lifetime utility of an individual born at time $t \geq 0$ is of the altruistic form,

$$U_t = u\left(c_t\right) + \beta \Phi\left(n_t\right) U_{t+1} + \beta\left(\Phi\left(\overline{n}\right) - \Phi\left(n_t\right)\right) \underline{U}, \tag{4}$$

where $u(.)$ is the utility flow from consumption, $\beta$ is a time discount factor, $\Phi(n)$ is the weight that a parent attaches to the welfare of her $n$ born children, $\Phi(\overline{n}) - \Phi(n)$ is the weight attached to the unborn children, $U_{t+1}$ is the utility of a born child and $\underline{U}$ is the utility of an unborn child.[1] These preferences are discussed in Cordoba and Ripoll (2011) who show that (4) satisfies the fundamental axiom of altruism. Specifically, parental utility increases with the number of born children if and only if children are better off born than unborn, that is $U_{t+1} > \underline{U}$.

#### 2.1.3 Social welfare

We consider a social planner who cares about the welfare of all *potential* individuals. The potential population at time $t$ is $\overline{N}_t = \overline{n}^t \overline{N}_0$. In particular, social welfare takes the utilitarian form:

$$SW_1 = \begin{cases} \sum_{t=0}^{\infty} \phi^t \left(N_t U_t + \left(\overline{N}_t - N_t\right) \underline{U}\right) & \text{if } \phi > 0 \\ N_t U_t + \left(\overline{N}_t - N_t\right) \underline{U} & \text{if } \phi = 0 \end{cases} \tag{5}$$

where $\phi^t$ is the weight that the planner gives to generation $t$. The case $\phi = 0$ refers to a planner who cares only about the initial generation as well as future generations but only to the extent that the initial generation does. In this case social discounting equals private discounting and the problem becomes one of dynastic maximization. $\phi > 0$ refers to a planner who is more patient than individuals, as in Farhi and Werning (2007).

We assume throughout that parameter values are such that social welfare is bounded. Most of the paper will focus on what we call the Barro-Becker case which requires the following assumptions.

**Assumption 1.** $\underline{U} = 0$, $\phi = 0$, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $\Phi(n) = n^{1-\varepsilon}$, and $1 > \sigma > \varepsilon > 0$.

The formulation normalizes the utility of the unborn to zero, and assumes isoelastic utility and altruistic functions. The last part of the assumption guarantees strict concavity of preferences for a well-behaved problem. We relax some of the assumptions later in the analysis.

Under Assumption 1, the social welfare function is reduces to:

$$N_0^{\varepsilon} \sum_{t=0}^{\infty} \beta^t N_t^{1-\varepsilon} u(c_t) \tag{6}$$

An advantage of focusing on the Barro-Becker case is that it is well-understood. For example, Golosov *et al.* (2007) have shown that the equilibrium allocation arising from the decentralized version of the Barro-Becker model is $\mathcal{A}$-efficient and $\mathcal{P}$-efficient.

An allocation $\{c_t, n_t, K_{t+1}, N_{t+1}\}_{t=0}^{\infty}$ is feasible if it satisfies (2), (1), and (3). Given initial population, $N_0$, and capital stock, $K_0$, the planner's problem is to choose a feasible allocation that maximizes social welfare (6). An efficient, or optimal allocation, is a feasible allocation that maximizes social welfare.

[1] The population ethics literature refers to $\underline{U}$ as the "neutral" utility level, a level above which a life is worth living (Blackorby *et al.* 2005, page 25).

### 2.1.4 Optimal allocation

We now characterize the optimal allocation. Proofs are provided in the Appendix. The Lagrangian characterizing the planner's problem can be written as:

$$\begin{aligned}\mathcal{L} &= \textstyle\sum_{t=0}^{\infty} \beta^t N_t^{1-\varepsilon} u(c_t) \\ &\quad +\mu_t \left[F(K_t, N_t - \lambda N_{t+1}, Z) - c_t N_t - \pi_t N_{t+1} + (1-\delta)K_t - K_{t+1}\right].\end{aligned}$$

The first order conditions with respect to $c_t$, $N_{t+1}$ and $K_{t+1}$ for $t \geq 0$ are, respectively,

$$\beta^t N_t^{-\varepsilon} u'(c_t) = \mu_t,$$

$$\begin{aligned}&\beta^{t+1}(1-\varepsilon) N_{t+1}^{-\varepsilon} u(c_{t+1}) \\ =\ &\mu_t \left[\lambda F_L(K_t, L_t, Z) + \pi_t\right] - \mu_{t+1}\left[F_L(K_{t+1}, L_{t+1}, Z) - c_{t+1}\right],\end{aligned}$$

and

$$\mu_t = \mu_{t+1}\left[F_K(K_{t+1}, L_{t+1}, Z) + 1 - \delta\right].$$

The first equation states that the marginal benefit of one unit of consumption is equal to the shadow price of consumption. The second equation equates marginal benefits to marginal costs of more population. The marginal benefit includes the welfare the individual brings to the social planner plus the extra labor supply increased by the individual in the next period. The marginal cost includes the cost of providing consumption and the cost of raising the individual. The third equation equates the marginal cost of physical capital in terms of consumption to its marginal benefit, which is the marginal product of an extra unit of capital discounted to the present. Using the first order conditions for consumption and capital, one can derive the corresponding Euler equation as

$$u'(c_t) = \beta \left(N_{t+1}/N_t\right)^{-\varepsilon} u'(c_{t+1})\left[F_K(K_{t+1}, L_{t+1}, Z) + 1 - \delta\right]. \tag{7}$$

**Lemma 1** *Optimal consumption satisfies*

$$c_{t+1} = \frac{1-\sigma}{\sigma-\varepsilon}\left[\frac{N_t^{-\varepsilon} u'(c_t)}{\beta N_{t+1}^{-\varepsilon} u'(c_{t+1})}\left(F_L(K_t, L_t, Z)\lambda + \pi_t\right) - F_L(K_{t+1}, L_{t+1}, Z)\right]. \tag{8}$$

**Proposition 2** *An optimal allocation* $(c_t, N_{t+1}, K_{t+1}, L_t)$ *is characterized by (2), (3), (7), (8), and the transversality conditions for physical capital and population,*

$$\lim_{T\to\infty} \beta^T N_T^{-\varepsilon} u'(c_T) K_{T+1} = 0,$$

*and*

$$\lim_{T\to\infty} \beta^T N_T^{1-\varepsilon} u'(c_T) \left[F_2\left(K_T, N_T - \lambda N_{T+1}, Z\right)\lambda + \pi_T\right] = 0.$$

### 2.1.5 Steady state

We now focus on steady state situations with constant population, capital, consumption and production. For capital to be used in the steady state, the production function needs to be further restricted to avoid situations in which land fully substitutes capital, or in which the marginal product of capital remains high enough so that the Euler equation cannot hold with equality for constant consumption and constant population. Restrictions are also needed on the cost of raising children to be large enough, so that consumption is positive.

Let lowercase variables denote variables in per-capital terms. At steady state the allocations $(y, k, z, c)$ satisfy,

$$y = F(k, 1-\lambda, z),$$

$$\delta k = y - c,$$

$$1 = \beta\left(F_K(k, 1-\lambda, z) + 1 - \delta\right),$$

$$c = \frac{1-\sigma}{\sigma-\varepsilon} F_L(k, 1-\lambda, z)\left(\lambda\beta^{-1} - 1\right) + \frac{1-\sigma}{\sigma-\varepsilon}\frac{\pi}{\beta}$$

The next proposition is the main result of the paper. It states that the steady-state allocation $(y, k, c)$ is independent of the amount land, $Z$, and of land-augmenting productivity, $A_Z$. As we show, these two variables only affect the steady-state population level, $N$. Let labor-augmenting productivity be $A_L$, and capital-augmenting productivity be $A_Z$.

**Proposition 3** *Assume that the production function takes the following general form* $Y = F(A_K K, A_L L, A_Z Z)$, *where* $F(.)$ *is constant returns to scale in efficiency units of capital, labor and land and let* $\delta > 0$. *Then steady state allocation satisfies* $\frac{\partial c}{\partial Z} = \frac{\partial y}{\partial Z} = \frac{\partial k}{\partial Z} = \frac{\partial z}{\partial Z} = 0$, $\frac{\partial c}{\partial A_Z} = \frac{\partial y}{\partial A_Z} = \frac{\partial k}{\partial A_Z} = 0$, *and* $\frac{\partial N}{\partial A_Z} > 0$.

We refer to Proposition 3 as the steady-state dichotomy. This dichotomy, which separates the effects of land-related variables from non-land related factors, is key to reconcile the patterns of the data displayed in Figures 1 and 2. The steady-state dichotomy is a general result and holds for any constant returns to scale production function, accommodating any degree of substitution among factors of production.

The generality of the dichotomy rests on two pillars. One is the use of more reproducible factors (capital and labor) than fixed factors (land) in production. More importantly, in the model capital is fundamentally different than land because it depreciates. The second pillar is the endogenous fertility choice. Labor input in the model evolves as a result of a trade-off: dynastic altruistic parents derive utility from creating children, who will be part of the labor force when adults. At the same time, parents incur a time cost of raising children, which reduces the current labor supply. The dynamics of wages becomes key in resolving this trade-off.

In our model land is in fixed supply and it will be used in production as long as its marginal product is positive. However, capital might not necessarily be used in production. We now establish the conditions under which physical capital is used. The following equation characterizes the condition for the usage of capital when the production is a CES function.

**Proposition 4** *Assume the production function takes the form*

$$Y_t = \left[\alpha \left(A_{Lt} L_t\right)^{-\theta} + (1-\alpha)\left(X_t\right)^{-\theta}\right]^{\frac{-1}{\theta}}, \tag{9}$$

*with*

$$X_t = \left[\gamma \left(A_{Zt} Z\right)^{-\rho} + (1-\gamma)\left(A_{Kt} K_t\right)^{-\rho}\right]^{\frac{-1}{\rho}}. \tag{10}$$

*(i) Suppose $-1 < (\rho, \theta) \leq 0$. Then a steady state exists if and only if*

$$1/\beta + \delta - 1 > (1-\alpha)^{-\frac{1}{\theta}} \left[1-\gamma\right]^{-\frac{1}{\rho}} A_K.$$

*(ii) Suppose $\rho = -1$ and $-1 < \theta \leq 0$. Then a steady state exists if and only if*

$$\begin{aligned} &(1-\alpha)(1-\gamma) A_K \left\{\alpha A_L^{-\theta} + (1-\alpha)\left[\gamma \left(A_Z z\right)\right]^{-\theta}\right\}^{-\frac{1+\theta}{\theta}} \left[\gamma A_Z z\right]^{-(\theta+1)} \\ > \;& 1/\beta + \delta - 1 > (1-\alpha)^{-\frac{1}{\theta}} \left[1-\gamma\right] A_K \end{aligned}$$

## 2.2 Decentralization

In this section we show that the social planner's allocation above can be decentralized by a competitive market economy. We add a superscript $c$ to allocations to represent competitive allocations. The utility of a parent in period $t$ is given by

$$U_t = u\left(c_t\right) + \beta n_t^{1-\varepsilon} U_{t+1}. \tag{11}$$

An individual is endowed, by their parents, with initial land holdings of $z_t$ and capital holding $k_t$. Individuals in period $t$ face the budget constraint

$$c_t + n_t\left(\pi_t + q_t z_{t+1} + k_{t+1}\right) \leq w_t\left(1 - \lambda n_t\right) + \left(q_t + r_{Zt}\right) z_t + (1 + r_{Kt} - \delta) k_t, \tag{12a}$$

where $q_t$ is the price of land in terms of goods. The parent transfers $z_{t+1}$ units of land and $k_{t+1}$ units of capital to each of the children. The level of population evolves according to

$$N_{t+1} = N_t n_t = N_0 \prod_{j=0}^{t-1} n_j = \prod_{j=0}^{t-1} n_j$$

where $N_0 = 1$. Forward iteration of equation (11) results in the following sequential representation of parental utility

$$U(k_t, z_t) = \max_{\{c_t, N_{t+1}, k_{t+1}, z_{t+1}\}} \sum_{t=0}^{\infty} \beta^t N_t^{1-\varepsilon} u(c_t).$$

The parental budget constraint in (12a) can also be written in terms of population levels as

$$c_t + \frac{N_{t+1}}{N_t}(\pi_t + q_t z_{t+1} + k_{t+1}) \leq w_t \left(1 - \lambda \frac{N_{t+1}}{N_t}\right) + (q_t + r_{Zt}) z_t + (1 + r_{Kt} - \delta) k_t. \tag{13}$$

**Proposition 5** *Given initial level of capital $k_0$, land $z_0$ and population $N_0$, the competitive equilibrium allocation $\left(c_t^c, n_t^c, k_{t+1}^c, z_{t+1}^c\right)$, and prices $(w_t, r_{Kt}, r_{Zt}, q_t)$, satisfy the budget constraint (13) and the following equations,*

$$c_{t+1}^c = \frac{1-\sigma}{\sigma - \varepsilon}\left[(1 + r_{Kt+1} - \delta)(w_t \lambda + \pi_t) - w_{t+1}\right],$$

$$\beta (n_t^c)^{-\varepsilon} = \frac{p_t}{r_{t+1} + p_{t+1}} \frac{u'(c_t^c)}{u'(c_{t+1}^c)}, \tag{14}$$

$$Z = N_t^c z_t^c,$$

$$1 + r_{Kt} - \delta = \frac{q_t + r_{Zt}}{q_{t-1}},$$

$$r_{Kt} = F_K\left(k_t^c, 1 - \lambda n_t^c, z_t^c\right),$$

$$r_{Zt} = F_Z\left(k_t^c, 1 - \lambda n_t^c, z_t^c\right),$$

$$w_t = F_L\left(k_t^c, 1 - \lambda n_t^c, z_t^c\right).$$

*The transversality conditions for physical capital and fertility are*

$$\lim_{T \to \infty} \beta^T (N_T^c)^{-\varepsilon} u'(c_T^c) K_{T+1}^c = 0,$$

*and*

$$\lim_{T\to\infty} \beta^T \left(N_T^c\right)^{1-\varepsilon} u'\left(c_T^c\right)\left(\lambda w_T + \pi_T\right) = 0.$$

The following proposition shows that the competitive allocation is identical to the planner's allocation.

**Proposition 6** *Given initial land and population $K_0$ and $N_0$, there exists a competitive equilibrium with initial land $k_0 = \frac{K_0}{N_0}$ and equilibrium prices satisfying $\frac{r_{t+1}+p_{t+1}}{p_t} = F_k\left(k_t, 1-\lambda n_t, z_t\right) + 1 - \delta$ that decentralizes the social planner's problem.*

# 3 Data and calibration

This section brings the model to the data. Key elements of our theory include the existence of a fixed factor (land), and the differential roles of labor, capital, and land-augmenting productivity, or $A_{L_t}$, $A_{K_t}$ and $A_{Z_t}$. We start by using panel data for a sample of countries to back out measures of $A_{L_t}$, $A_{K_t}$ and $A_{Z_t}$ across countries and over time. This exercise generalizes Caselli (2005) for the case in which the production includes capital, labor and land. We then test the model's predictions in both the cross-section and the time series for a sample of countries.

## 3.1 Backing out non-neutral productivities

Caselli (2005) uses a cross-section of countries in 1996 to construct measures of non-neutral productivities in physical capital and labor (human capital). In this section we follow a similar approach, but extend Caselli's analysis in two ways: first, we include land in the production function, and second, we construct both cross-sectional and time series measures of non-neutral technological progress.

This approach requires observing in the data $Y_t$, $L_t$, $K_t$, $Z_t$, as well as the income shares of each of the factors: $S_{Lt}$, $S_{Kt}$ and $S_{Zt}$, all of which are available from Penn World Tables (PWT), the World Development Indicators (WDI), and from computations following Monge-Naranjo *et al.* (2019), which we discuss next subsection.

To back out $A_{L_t}$, $A_{K_t}$ and $A_{Z_t}$, we start from the expressions for the shares of factors for the general CES production function in (9) and (10), which are given by,

$$S_{Lt} = \frac{w_t L_t}{Y_t} = \frac{\alpha Y_t^{\theta}}{\left(A_{Lt} L_t\right)^{\theta}}, \tag{15}$$

$$S_{Kt} = \frac{r_{Kt} K_t}{Y_t} = \frac{(1-\alpha)(1-\gamma) Y_t^{\theta} X_t^{\rho-\theta}}{\left(A_{Kt} K_t\right)^{\rho}}, \tag{16}$$

and

$$S_{Zt} = \frac{r_{Zt} Z_t}{Y_t} = \frac{(1-\alpha)\gamma Y_t^{\theta} X_t^{\rho-\theta}}{(A_{Zt} Z_t)^{\rho}}. \tag{17}$$

Given the observed factor shares in the data together with data on $Y_t$, $L_t$, $K_t$, $Z_t$, we can back out $A_{Lt}$, $A_{Kt}$ and $A_{Zt}$ from

$$A_{Lt} = \left[\frac{\alpha}{S_{Lt}}\right]^{\frac{1}{\theta}} \frac{Y_t}{L_t}, \tag{18}$$

$$A_{Kt} = \left[\frac{(1-\alpha)(1-\gamma)}{S_{Kt}}\right]^{\frac{1}{\rho}} \left[\frac{X_t}{Y_t}\right]^{\frac{\rho-\theta}{\rho}} \frac{Y_t}{K_t}, \tag{19}$$

and

$$A_{Zt} = \left[\frac{(1-\alpha)\gamma}{S_{Zt}}\right]^{\frac{1}{\rho}} \left[\frac{X_t}{Y_t}\right]^{\frac{\rho-\theta}{\rho}} \frac{Y_t}{Z_t}. \tag{20}$$

Notice from equation (18) that we can directly obtain $A_{Lt}$ from the data and from a value of $\theta$. With $A_{Lt}$ at hand we can obtain $X_t$ directly from the production function from

$$X_t = \left[\frac{Y_t^{-\theta} - \alpha\left(A_{Lt} L_t\right)^{-\theta}}{(1-\alpha)}\right]^{\frac{-1}{\theta}},$$

so that $X_t$ is measured only from data and an assumed value for $\theta$, but does not depend on $\rho$. Finally, with $X_t$ at hand, we can compute $A_{Kt}$ from (19) and $A_{Zt}$ from (20).

Two comments are in order. First, identification of $A_{L_t}$, $A_{K_t}$ and $A_{Z_t}$ from the data using the equations above requires that $\theta \neq 0$ and $\rho \neq 0$. In what follows we set $\theta = -0.2$ following the estimates of Karabarbounis and Neiman (2014). This implies that labor and the composite of physical capital and land ($X_t$) are more substitutes than Cobb-Douglas. Since less information is known about the elasticity of substitution between capital and land from aggregate data, we calibrate this parameter within the model.

Second, equations (18), (19) and (20) imply that if factor shares, $S_{Lt}$, $S_{Kt}$ and $S_{Zt}$, are uncorrelated with output per worker, then parameters $\theta$ and $\rho$ do not matter much for how $A_{Lt}$, $A_{Kt}$ and $A_{Zt}$ correlate across countries. In this case, $A_{Lt}$ will be an increasing function of $Y_t/L_t$. If in addition $X_t/Y_t$ is uncorrelated with output per worker, then $A_{Kt}$ will be an increasing function of $Y_t/K_t$, and $A_{Zt}$ will be an increasing function of $Y_t/Z_t$. However, this is not the case in the data. As we show below, in the data $S_{Lt}$ is mostly uncorrelated with output per worker, but $S_{Kt}$ increases with output per worker, and $S_{Zt}$ strongly decreases with output per worker. In this case, the cross-sectional correlation between $A_{Lt}$, $A_{Kt}$ and $A_{Zt}$ depends on a number of data patterns, as well as the values of $\theta$ and $\rho$. In the case of labor-augmenting technological progress, equation (18) implies that if with $S_{Lt}$ is mostly uncorrelated with output per worker, then $A_{Lt}$ increases with output per worker. The patterns regarding $A_{Kt}$ and $A_{Zt}$ depend on more factors, as well as on the relative sizes of $\rho$ and $\theta$. However, combining equations (20) and (19) we have that

$$\frac{A_{Zt}}{A_{Kt}} = \left[\frac{\gamma S_{Kt}}{(1-\gamma)S_{Zt}}\right]^{\frac{1}{\rho}} \frac{Y_t/Z_t}{Y_t/K_t}, \tag{21}$$

so that $A_{Zt}/A_{Kt}$ only depends on observables in the data, namely $S_{Zt}$, $S_{Kt}$, $Y_t/K_t$ and $Y_t/Z_t$, and parameter $\rho$. Ratio $A_{Zt}/A_{Kt}$ is independent of $\theta$. With these insights in mind, we now describe the data sources to back out $A_{Lt}$, $A_{Kt}$ and $A_{Zt}$, as well as the patterns we obtain from this exercise.

## 3.2 Data

We use three main data sources: PWT, WDI and the estimates of land shares from Monge-Naranjo *et al.* (2019).

### 3.2.1 Factor shares

We define our fixed factor $Z$ as agricultural land. In particular, agricultural land refers to the land area that is arable (land under temporary crops, temporary meadows for pasture, and market or kitchen gardens), under permanent crops (excluding trees grown for wood or timber), and under permanent pastures. We compute the share of agricultural land from Monge-Naranjo *et al.* (2019). Specifically, we consider two of the share components they estimate: croplands and pasturelands, which correspond to our definition of $Z$. They compute these shares directly from rent flow estimates for a sample of countries from 1970 to 2005. Given the share of agricultural land, we use the labor share from PWT and compute the capital share as a residual.

Figure 3 portrays the labor share against GDP per worker for a cross-section of countries in 2005. As can be seen on the figure, the labor share is mostly uncorrelated with GDP per worker, which is verified in Table 1, panel 1.1. Figure 4 shows the capital share against GDP per worker, which have a correlation of 0.25 (Table 1). Last, as shown in Figure 5, the agricultural land share is strongly decreasing in GDP per worker with a correlation of -0.74 (Table 1).

### 3.2.2 Output and factor inputs

Data on output, and factor inputs (labor, capital and land) is taken from the PWT and from the World Development Indicators. For cross-sectional comparisons, we measure GDP and capital in current PPPs (variables *cgdpo* and *cn* from PWT, in millions of 2011 US \$). For time-series computations we use GDP and capital in constant 2011 national prices (variables *rgdpna* and *rnna* from PWT). We measure labor input as the product of employment (number of persons engaged) and the human capital index based on years of schooling and returns to education from PWT (variable *hc*). Last, we measure agricultural land from the WDI.

Before backing out $A_{Lt}$, $A_{Kt}$ and $A_{Zt}$, it is instructive to characterize how the output-capital ratio ($Y_t/K_t$), the output-land ratio ($Y_t/Z_t$), and the output-composite ratio ($Y_t/X_t$) correlate with output per worker in the cross section. Together with the factor shares, these ratios are key components in equations (18), (19) and (20). As shown in Table 1, panel 1.2, $Y_t/K_t$ is negatively correlated with output per worker, with a correlation of -0.23. This cross-sectional fact implies that countries high higher average product of labor ($Y_t/L_t$) tend to have a lower average product of capital ($Y_t/K_t$). This fact is purely data based, independent of any parameter values. Recall that labor here really means human capital, as labor input adjusts workers by a measure of human capital that reflects schooling and returns to experience. Therefore, richer countries tend to exhibit relatively higher average product of human capital and lower average product of physical capital.

In contrast, as shown in Table 1 (panel 1.2), there is a strong positive correlation of 0.66 between the average product of land ($Y_t/Z_t$) and output per worker ($Y_t/L_t$). In other words, richer countries have higher average product of land and labor (human capital). Last, when computing the capital-land composite $X_t$, which depends both on data and on parameter $\theta = -0.2$, we find that the output-composite ratio ($Y_t/X_t$) and output per worker have a slight negative correlation of -0.09. Notice how this correlation is independent on the elasticity of substitution between capital and land ($\rho$).

The cross-sectional correlations in Table 1 provide interesting insights on how the average products of different factors correlate with output per worker. Given the data, we now use equations (18), (19) and (20) to construct $A_{Lt}$, $A_{Kt}$ and $A_{Zt}$. We present our results first for a cross section of countries in 2005, the last date for which we have estimates of the agricultural land share, and then for the time series from 1970 to 2005.

### 3.3 Cross-sectional analysis

In order to back out $A_L$, $A_K$ and $A_Z$ from the data and from the assumption that $\theta = -0.2$, we also need to set parameters $\alpha$, $\gamma$ and $\rho$. We set $\alpha = 0.6$ and $\gamma = 0.04$ which respectively correspond to the mean labor share and the mean land share in the sample for 2005. As we explain below in more detail, we calibrate $\rho$ so that the implied $A_L$, $A_K$ and $A_Z$ for the United States are consistent with the asymptotic balanced-growth predictions of the model. The calibration results in $\rho = -1$, so that in the aggregate, capital and land are perfect substitutes in production. As shown above, under certain conditions both capital and land are used in production even if they are perfect substitutes. In addition, in the limit, land will be used in production, but its share declines to a small positive number.

Figure 6 displays measured capital and labor-augmenting productivity, $A_K$ and $A_L$, for the cross-section of countries in 2005. The figure suggests that countries that are more efficient using labor (human capital), are less efficient using capital. In fact, as reported in Table 2 (panel 2.1), the correlation between $A_L$ and $A_K$ in the cross-section is -0.73. As shown in Figure 7, measured land and labor-augmenting productivity, $A_Z$ and $A_L$, also have a negative relationship across countries, although the correlation is smaller at -0.22 (Table 2, panel 2.1). Last, Figure 8 shows a positive relationship between measured land and capital-augmenting productivity, $A_Z$ and $A_K$, with a correlation of 0.42 (Table 2). In sum, we find that countries that are more efficient at using land are also more efficient at using capital, but are less efficient at using human capital (labor).

We now provide some insights on the origins of these findings. First, regarding labor-augmenting productivity, Table 2 reports that the correlation between $A_L$ and the labor share is 0.82, and that between $A_L$ and output per worker is 0.59. With $\theta < 0$, the strong positive correlation between $A_L$ and the labor share stems from the fact that as shown in Figure 3, the labor share is uncorrelated with output per worker. As seen in equation (15), the labor share can be uncorrelated with output per worker only if countries in which GDP per worker is higher, $A_L$ is also proportionally higher. In other words, countries the use labor more efficiently exhibit higher wages and higher GDP per worker.

Second, equation (21) helps explain Figure 8. According to the equation, $A_Z/A_K$ is proportional to the ratio of $Y/Z$ to $Y/K$. Recall from Table 1 that $Y/Z$ and $Y/K$ are negatively correlated (-0.21), so countries with high average product of land tend to have lower average product of capital, which would imply a higher relative efficiency in the use of land, or a higher $A_Z/A_K$. This is in fact what Figure 8 portrays, where the correlation between $A_Z$ and $A_K$ is 0.42 (Table 2, panel 2.1). In addition, equation (21) also indicates that with $\rho < 0$, ratio $A_Z/A_K$ is positively related to ratio $S_Z/S_K$. Recall from Table 1 that $S_Z$ and $S_K$ are strongly negatively correlated (-0.93), so countries with high land share tend to have low capital share, which implies a higher relative efficiency in the use of land, or higher $A_Z/A_K$. In sum, with $\rho < 0$, countries with higher average product of land, and higher land share also exhibit higher efficiency in the use of land.

Third, panel 2.2 in Table 2 links our measures of non-neutral technological change with GDP per worker. We find that $A_L$, $A_K$ and $A_Z$ are all positively correlated with GDP per worker, with respective correlations of 0.59, 0.02 and 0.16. However, as shown in panel 2.3, ratios $A_K/A_L$ and $A_Z/A_L$ are negatively correlated with GDP per worker, with respective correlations of -0.34 and -0.25. Regarding the correlation between $A_K/A_L$ and GDP per capita, Caselli (2005) also documented that richer countries are relatively less efficient at using capital than labor (human capital). By extending the production function to include land, our analysis adds the insight that richer countries are also relatively less efficient at using land than labor (human capital). Last, our analysis also suggests that richer countries are relatively more efficient at using land than capital: the correlation between $A_Z/A_K$ and GDP per worker is 0.15 (Table 2, panel 2.3).

Last, panel 2.2 in Table 2 links our measures of non-neutral technological change with population. The strongest correlation we measure is between population and land-augmenting productivity $A_Z$, of 0.26. This is consistent with the prediction of our model that increases in $A_Z$ result in increases in population. Figure 9 portrays both $A_Z$ and population for the cross-section of countries in 2005. The correlation between $A_K$ and population is also positive but weaker, at 0.11. Last the cross-sectional correlation between $A_L$ and population is small and negative, in the order of -0.09.

### 3.4 Time-series analysis

In this section we construct time-series measures of $A_{Lt}$, $A_{Kt}$ and $A_{Zt}$ for each country in our sample between 1970 and 2005. We use the same parameters as for the cross-sectional analysis.

Figures 10 and 11 show the time series evolution of $A_{Lt}$, $A_{Kt}$ and $A_{Zt}$ for the US. We extract the trends of the time series of $A_{Lt}$, $A_{Kt}$, $A_{Zt}$ and GDP per worker using a Hodrick-Prescott filter. Figure 10 shows population and the trend of $A_{Zt}$, where both variables have been normalized to one in 1970. The figure shows how between 1970 and 2005, population in the United States multiplied by a factor of about 1.45 (left axis), while $A_{Z_t}$ did by a factor of 1.55 (right axis). Population and $A_{Z_t}$ show parallel trends in the US. Figure 11 displays the filtered $A_{Lt}$, $A_{Kt}$, $A_{Zt}$, and $Y_t/L_t$ for the US, again with each series normalized to one in 1970. As shown in the figure, GDP per worker grew relatively faster until about 2000 (right axis), showing a slowdown afterwards. Labor-augmenting productivity declined between 1970 and 1990 (right axis), then grew between 1990 and 2000, and has decelerated since then. In contrast, capital-augmenting productivity grew substantially until the 1990s (right axis), then decelerated, but had almost fully recovered by 2005.

As mentioned, we use the case of the United States to calibrate $\rho$, which is the elasticity of substitution between capital and land. We find that with $\rho = -1$, the measured and filtered series for $A_{Lt}$, $A_{Kt}$, $A_{Zt}$, and $Y_t/L_t$ converge very closely to the conditions that allow for an asymptotic balanced growth path to exist for the United States. As we discuss in detail below, these conditions include that the growth rate of $A_{Kt}$ is zero, and that the growth rate of $A_{Zt}$ equals the growth rate of $A_{Lt}$ plus the growth rate of $L_t$. A calibration with $\rho = -1$ gets us the closest to these conditions for the United States. In particular, if we look at the average growth rate of $A_{Kt}$ after 1995, it is almost zero (-0.08%). The average growth rate of $A_{Zt}$ after 1995 is 1.85%, that of $A_{Lt}$ is 1.32%, and that of $L_t$ is 0.9%, which brings us as very close to the required conditions.

As seen in Figure 11, both $A_{Kt}$ and $A_{Zt}$ exhibit substantial growth over time. To interpret these trends, recall that capital is growing over time, while land is a fixed factor. With $\rho = -1$, or perfect substitution between capital and land, for both of these factors to be used in production, $A_{Zt}$ would have to grow overtime.

Our time series analysis allows us to estimate the correlation between $A_{Lt}$, $A_{Kt}$, $A_{Zt}$, and both GDP per worker and population. Table 3 present these results. Panel 3.1 reports the results of country-fixed effect panel regressions (with time dummies) in which the dependent variable is the trend of GDP per worker and the independent variables are the trends of $A_{Lt}$, $A_{Kt}$, $A_{Zt}$. Each of these regressors enter one by one in columns (1) through (3), and then together in column (4). Column (1) includes more observations than the rest of the columns because we are able to back out $A_{Lt}$ from 1970-2017, while our measures of $A_{Kt}$ and $A_{Zt}$ require the land share, which is available until 2005. Column (1) in panel 3.1 reveals a positive and significant association between $A_{Lt}$ and GDP per worker. This relationship is robust to the including also $A_{Kt}$ and $A_{Zt}$ in column (4). While in this column we don't find an effect of $A_{Zt}$ on GDP per worker in the time series, the effect of $A_{Kt}$ is positive and statistically significant. Therefore it is the changes in $A_{Lt}$ and $A_{Kt}$ what explain the changes in GDP per worker over time.

Panel 3.2 in Table 3 reports the country-fixed effects panel regression with population as dependent variable. As shown in column (1), the effect of $A_{Lt}$ in population is negative and statistically significant. In terms of our model, this can be thought of as rising wages having negative effects on fertility and population over time. Most interestingly, column (3) shows how $A_{Zt}$ has a positive and statistically significant effect on population. This stands in sharp contrast with the lack of a statistically significant effect of $A_{Zt}$ on GDP per worker in panel 3.1, column (3). Last, column (4) in panel 3.2 shows that while increases over time in $A_{Lt}$ and $A_{Kt}$ result in population decreases, growth in $A_{Zt}$ results in population growth, exactly as our model predicts.

As a final test of our model, we use our time series analysis to compute the average annual growth rate of $A_{Lt}$, $A_{Kt}$, $A_{Zt}$, GDP per worker and population during 1970-2005, as well as the correlations among these average growth rates. Results are reported in Table 4, which confirm the model's predictions, as well as the predictions in Table 3. Notably, growth in land-augmenting productivity translates into population growth, but not into growth in GDP per worker. In contrast, growth in labor or capital-augmenting productivity result in lower population growth, but higher growth in GDP per worker. Figure 12 displays the positive correlation between the average growth of $A_{Zt}$ and the average growth of population in our sample of countries. Last, Table 4 also shows that while there is a negative correlation between the average growth rate of $A_{Lt}$ and $A_{Zt}$, and also between the growth rate of $A_{Lt}$ and $A_{Kt}$, the correlation between the growth rate of $A_{Kt}$ and $A_{Zt}$ is positive during 1970-2005.

# 4 Extensions

## 4.1 Transitional dynamics

[To be completed]

## 4.2 Balanced growth

[To be completed]

# 5 Concluding comments

[To be completed]

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## APPENDIX

**Proof of Lemma 1.** In the CRRA case the Euler equation is given by

$$C_t^{-\sigma} = \beta \left(N_{t+1}/N_t\right)^{\sigma-\varepsilon} C_{t+1}^{-\sigma} \left[F_K\left(K_{t+1}, L_{t+1}, Z\right) + (1-\delta)\right].$$

Next, using the $c_t$ condition into the $N_{t+1}$ we have

$$\begin{aligned} & \beta(1-\varepsilon) N_{t+1}^{-\varepsilon} u\left(c_{t+1}\right) \\ = \; & N_t^{-\varepsilon} u'\left(c_t\right)\left[F_L\left(K_t, L_t, Z\right)\lambda + \pi_t\right] - \beta N_{t+1}^{-\varepsilon} u'\left(c_{t+1}\right)\left[F_L\left(K_{t+1}, L_{t+1}, Z\right) - c_{t+1}\right]. \end{aligned}$$

For the case of CRRA utility we have

$$\begin{aligned} \beta(1-\varepsilon) N_{t+1}^{-\varepsilon} \frac{c_{t+1}^{1-\sigma}}{1-\sigma} &= N_t^{-\varepsilon} c_t^{-\sigma}\left[F_L\left(K_t, L_t, Z\right)\lambda + \pi_t\right] \\ & \quad - \beta N_{t+1}^{-\varepsilon} c_{t+1}^{-\sigma}\left[F_L\left(K_{t+1}, L_{t+1}, Z\right) - c_{t+1}\right] \end{aligned}$$

We can simplify consumption as the formula in the Proposition. ■

**Proof of Proposition 2.** The proof of this proposition uses Lemma 1 and the constraints given above. To derive the transversality conditions we assume the last period is $T$. The derivative of the objective with respect to fertility in the last period is

$$-\mu_T\left[\lambda F_2\left(K_T, N_T - \lambda N_{T+1}, Z\right) + \pi_T\right]$$

Multiply this term by $N_T$ and take limit we obtain the transversality condition for population. The derivative of the objective with respect to physical capital in the last period is $-\mu_T$. Multiply this value by $K_{T+1}$ and take limit, we obtain the transversality condition for physical capita. ■

**Proof of Proposition 3.** First, let $m_K \equiv \left(A_K K\right)/\left(A_Z Z\right)$ and $m_L \equiv \left(A_L L\right)/\left(A_Z Z\right)$. Using the constant returns to scale property we can write the production function as

$$Y = F\left(1, \frac{m_L}{m_K}, \frac{1}{m_K}\right) A_K K = F\left(\frac{m_K}{m_L}, 1, \frac{1}{m_L}\right) A_L L = F\left(m_K, m_L, 1\right) A_Z Z,$$

we can write per capita income as

$$y = \frac{Y}{N} = (1-\lambda) F\left(\frac{m_K}{m_L}, 1, \frac{1}{m_L}\right) A_L, \tag{22}$$

and we can write per capita capital as

$$k=\frac{K}{N}=\frac{K}{Y}y=\frac{(1-\lambda)\,F\left(\frac{m_K}{m_L},1,\frac{1}{m_L}\right)A_L}{F\left(1,\frac{m_L}{m_K},\frac{1}{m_K}\right)A_K}. \tag{23}$$

Next, in a steady state with $n=1$ the intergenerational Euler equation reduces to

$$1=\beta\left(1+F_K\left(m_K,m_L,1\right)A_K-\delta\right). \tag{24}$$

The optimality condition for fertility determines per capita consumption as a function of wages as follows

$$c=\frac{1-\sigma}{\sigma-\varepsilon}\left[\frac{\lambda-\beta}{\beta}F_L\left(m_K,m_L,1\right)A_L+\frac{1}{\beta}\pi\right], \tag{25}$$

and the aggregate resource constraint reads

$$\delta k=y-c-\pi,$$

which using the expressions for $k$ and $y$ can be written as

$$\delta\frac{(1-\lambda)\,F\left(\frac{m_K}{m_L},1,\frac{1}{m_L}\right)A_L}{F\left(1,\frac{m_L}{m_K},\frac{1}{m_K}\right)A_K}=(1-\lambda)\,F\left(\frac{m_K}{m_L},1,\frac{1}{m_L}\right)A_L-c-\pi. \tag{26}$$

Therefore, equations (24), (25) and (26) constitute a system of three equations in three unknowns: $c$, $m_K$ and $m_L$. These variables directly depend on $A_L$ and $A_K$, but do not depend on $A_Z$ or $Z$. Similarly, $y$ and $k$, as given by equations (22) and (23), do not depend on $A_Z$ or $Z$. Finally, once $m_K$ and $m_L$ are solved for, an increase in either $A_Z$ or $Z$ results in higher level of population $N$, proving the steady-state dichotomy result. ■

**Proof of Proposition 4.** Rewrite the CES production function at steady state as

$$\begin{aligned}&F(K,L,Z)\\=\;&\left\{\alpha\left(A_LL\right)^{-\theta}+(1-\alpha)\left[\gamma\left(A_ZZ\right)^{-\rho}+(1-\gamma)\left(A_KK\right)^{-\rho}\right]^{\theta/\rho}\right\}^{-\frac{1}{\theta}}\end{aligned}$$

Then

$$
\begin{aligned}
& F_K(\frac{K}{N}, 1-\lambda, \frac{Z}{N}) \\
= \; & (1-\alpha)(1-\gamma)A_K^{-\rho}\left\{\alpha A_L^{-\theta}k^{\theta}(1-\lambda)^{-\theta} + (1-\alpha)\left[\gamma(A_Z z)^{-\rho}k^{\rho} + (1-\gamma)A_K^{-\rho}\right]^{\theta/\rho}\right\}^{-\frac{1+\theta}{\theta}} \\
& * \left[\gamma(A_Z z)^{-\rho}k^{\rho} + (1-\gamma)A_K^{-\rho}\right]^{(\theta-\rho)/\rho}
\end{aligned}
$$

Note that

$$
\begin{aligned}
& \lim_{k\to 0}\left\{\alpha A_L^{-\theta}k^{\theta}(1-\lambda)^{-\theta} + (1-\alpha)\left[\gamma(A_Z z)^{-\rho}k^{\rho} + (1-\gamma)A_K^{-\rho}\right]^{\theta/\rho}\right\}^{-\frac{1+\theta}{\theta}} \\
= \; & \left\{\alpha A_L^{-\theta}(1-\lambda)^{-\theta} + (1-\alpha)\left[\gamma(A_Z z)^{-\rho}\right]^{\theta/\rho}\right\}^{-\frac{1+\theta}{\theta}} \lim_{k\to 0} k^{-(1+\theta)} \\
= \; & \infty
\end{aligned}
$$

and

$$
\begin{aligned}
& \lim_{k\to 0}\left[\gamma(A_Z z)^{-\rho}k^{\rho} + (1-\gamma)A_K^{-\rho}\right]^{(\theta-\rho)/\rho} \\
= \; & \lim_{k\to 0}\left[\gamma(A_Z z)^{-\rho}k^{\rho}\right]^{(\theta-\rho)/\rho} = \left[\gamma(A_Z z)^{-\rho}\right]^{(\theta-\rho)/\rho}\lim_{k\to 0} k^{\theta-\rho}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
& \lim_{k\to 0} F_K(k, 1-\lambda, z) \\
= \; & (1-\alpha)(1-\gamma)A_{Kt}^{-\rho}\left\{\begin{array}{c} \alpha A_L^{-\theta}(1-\lambda)^{-\theta} \\ +(1-\alpha)\left[\gamma(A_Z z)^{-\rho}\right]^{\theta/\rho} \end{array}\right\}^{-\frac{1+\theta}{\theta}} \left[\gamma(A_Z z)^{-\rho}\right]^{(\theta-\rho)/\rho}\lim_{k\to 0} k^{-1-\rho}
\end{aligned}
$$

Hence $\lim_{k\to 0} f_k(k,z) = \infty$ if $\rho \in (-1, 0]$. If $\rho = -1$,

$$
\begin{aligned}
& \lim_{k\to 0} F_K(k, 1-\lambda, z) \\
= \; & (1-\alpha)(1-\gamma)A_{Kt}^{-\rho}\left\{\alpha A_L^{-\theta}(1-\lambda)^{-\theta} + (1-\alpha)\left[\gamma(A_Z z)^{-\rho}\right]^{\theta/\rho}\right\}^{-\frac{1+\theta}{\theta}}\left[\gamma(A_Z z)^{-\rho}\right]^{(\theta-\rho)/\rho}
\end{aligned}
$$

Next let us consider the case when $k$ converges to infinity.

$$
\lim_{k\to\infty} F_k(k,z) = (1-\alpha)^{-\frac{1}{\theta}}\left[1-\gamma\right]^{-\frac{1}{\rho}} A_K
$$

Therefore, when $-1 < (\rho, \theta) \leq 0$, a steady state exists if and only if

$$F_K(k, 1-\lambda, z) > (1-\alpha)^{-\frac{1}{\theta}} [1-\gamma]^{-\frac{1}{\rho}} A_K$$

And if $\rho = -1$ and $-1 < \theta \leq 0$, a steady state exists if and only if

$$\begin{aligned}
& (1-\alpha)(1-\gamma)A_{Kt}^{-\rho} \left\{ \alpha A_L^{-\theta} (1-\lambda)^{-\theta} + (1-\alpha) \left[ \gamma (A_Z z)^{-\rho} \right]^{\theta/\rho} \right\}^{-\frac{1+\theta}{\theta}} \left[ \gamma (A_Z z)^{-\rho} \right]^{(\theta-\rho)/\rho} \\
> \quad & r_K > (1-\alpha)^{-\frac{1}{\theta}} [1-\gamma]^{-\frac{1}{\rho}} A_K.
\end{aligned}$$

By the Euler equation (7), at steady state

$$F_K(k, 1-\lambda, z) = 1/\beta + \delta - 1$$

The condition follows. ■

**Proof of Proposition 5.** Let the Lagrange multiplier associated with the constraint be $\mu_t$. In this proof we simplify the notation by dropping the superscript $c$ for equilibrium variables. First order condition with respect to $c_t$, $k_{t+1}$, $z_{t+1}$ and $N_{t+1}$ are

$$\beta^t N_t^{1-\varepsilon} u'(c_t) = \mu_t$$

$$u'(c_t) = \beta n_t^{-\varepsilon} u'(c_{t+1}) (1 + r_{Kt+1} - \delta)$$

$$u'(c_t) q_t = \beta n_t^{-\varepsilon} u'(c_{t+1}) (q_{t+1} + z_{t+1})$$

$$\begin{aligned}
& \beta^{t+1} (1-\varepsilon) N_{t+1}^{-\varepsilon} u(c_{t+1}) - \mu_t (\lambda w_t + \pi_t + q_t z_{t+1} + k_{t+1}) \frac{1}{N_t} \\
& + \mu_{t+1} (w_{t+1}\lambda + \pi_{t+1} + q_{t+1} z_{t+2} + k_{t+2}) \frac{N_{t+2}}{N_{t+1}^2} \\
= \quad & 0
\end{aligned}$$

By the first order conditions with respect to $k_{t+1}$ and $z_{t+1}$, we have

$$1 + r_{Kt+1} - \delta = \frac{q_{t+1} + z_{t+1}}{q_t}.$$

Plug $\mu_t$ in the first order condition with respect to $N_{t+1}$ and substitute out $u'(c_t)$ using Euler equation, we have

$$\begin{aligned}
& (1-\varepsilon) N_{t+1}^{-\varepsilon} \frac{u(c_{t+1})}{u'(c_{t+1})} - N_{t+1}^{-\varepsilon} (1 + r_{Kt+1} - \delta) (\lambda w_t + \pi_t + q_t z_{t+1} + k_{t+1}) \\
& + N_{t+1}^{-1-\varepsilon} (w_{t+1}\lambda + \pi_{t+1} + q_{t+1} z_{t+2} + k_{t+2}) N_{t+2} \\
= \; & 0.
\end{aligned}$$

Using the budget constraint, the utility form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and simplify the condition, we obtain the expression for consumption as

$$c_{t+1} = \frac{1-\sigma}{\sigma-\varepsilon} \left[ (1 + r_{Kt+1} - \delta)(\lambda w_t + \pi_t) - w_{t+1} \right].$$

To derive the transversality condition, we assume the last period is $T$. Multiply the derivative of the objective function with respect to fertility by population and then take limit, we obtain the transversality condition for population as

$$\lim_{T\to\infty} \mu_T (\lambda w_T + \pi_T) = 0.$$

In the same way, the transversality condition for physical capital is

$$\lim_{T\to\infty} \mu_T \frac{N_{T+1}}{N_T} k_{T+1} = 0.$$

Substitute out $\mu_T$ we obtain the condition in the proposition. ■

**Proof of Proposition 6.** Denote the social planner's optimal allocation by $\left(c_t^*, N_{t+1}^*, K_{t+1}^*, L_t^*\right)$ to distinguish with the competitive equilibrium. Then

$$k_t^* = \frac{K_t^*}{N_t^*},\; n_t^* = \frac{N_{t+1}^*}{N_t^*}.$$

Under the condition $\frac{r_{t+1}+p_{t+1}}{p_t} = F_k(k_t, 1-\lambda n_t^*, z_t^*) + 1 - \delta$ and $k_0 = \frac{K_0}{N_0}$, the Euler equations (7) and (14) are equivalent. In the competitive equilibrium the return to capital and the return to land are the same, we have

$$F_K(K_{t+1}, L_{t+1}, Z) + 1 - \delta = 1 + r_{Kt+1} - \delta.$$

In the social planner's problem,

$$\frac{N_t^{-\varepsilon} c_t^{-\sigma}}{\beta N_{t+1}^{-\varepsilon} c_{t+1}^{-\sigma}} = F_K(K_{t+1}, L_{t+1}, Z) + 1 - \delta.$$

so the consumption formulas in the two problems are the same. Sum up the budget constraints for all periods to write the present value budget constraint and use the No-ponzi game condition we have the resource constraint (2). The transversality condition of the two problems are equivalent as well. ■

TABLE 1

*Cross-sectional correlations in the data – 2005*

*1.1. Factor shares*

| | Output per worker | Labor share | Capital share | Agricultural land share |
|---|---|---|---|---|
| Output per worker ($Y/L$) | 1.000 | | | |
| Labor share | 0.033 | 1.000 | | |
| Capital share | 0.249 | -0.925 | 1.000 | |
| Agricultural land share | -0.741 | -0.008 | -0.372 | 1.000 |

*1.2. Average products*

| | Output per worker | Output-capital ratio | Output-land ratio | Output-composite ratio |
|---|---|---|---|---|
| Output per worker ($Y/L$) | 1.000 | | | |
| Output-capital ratio($Y/K$) | -0.233 | 1.000 | | |
| Output-land ratio ($Y/Z$) | 0.664 | -0.206 | 1.000 | |
| Output-composite ratio ($Y/X$) | -0.091 | -0.045 | 0.051 | 1.000 |

*Notes:* Output per worker $(Y/L)$, labor share, and capital $(K)$ are from the Penn World Tables 9.1. Workers are computed as employment times the Barro and Lee's human capital measure from Penn World Tables 9.1. The share of agricultural land is computed from Monge-Naranjo *et al*. (2019) to include the shares of cropland and pastures only. Agricultural land $(Z)$ is from the World Development indicators. Composite $(X)$ refers to the CES composite of capital and land assumes that the elasticity of substitution between labor and the capital-land composite is -0.2. The capital-land composite backed out from the data is independent of the elasticity of substitution between capital and land. Logs are taken for output per worker and the average products when computing the correlations.

TABLE 2

*Measured labor, capital, and land-augmenting productivity*

*Cross-sectional correlations - 2005*

*2.1. With factor shares*

| | Labor share | Capital share | Agricultural land share | Labor-augmenting productivity ($A_L$) | Capital-augmenting productivity ($A_K$) | Land-augmenting productivity ($A_Z$) |
|---|---|---|---|---|---|---|
| Labor share | 1.000 | | | | | |
| Capital share | -0.925 | 1.000 | | | | |
| Agricultural land share | -0.024 | -0.365 | 1.000 | | | |
| Labor-augmenting productivity ($A_L$) | 0.815 | -0.594 | -0.427 | 1.000 | | |
| Capital-augmenting productivity ($A_K$) | -0.949 | 0.902 | -0.048 | -0.727 | 1.000 | |
| Land-augmenting productivity ($A_Z$) | -0.429 | 0.358 | 0.106 | -0.219 | 0.421 | 1.000 |

*2.2. With output per worker and population*

| | Output per worker ($Y/L$) | Population | Labor-augmenting productivity ($A_L$) | Capital-augmenting productivity ($A_K$) | Land-augmenting productivity ($A_Z$) |
|---|---|---|---|---|---|
| Output per worker ($Y/L$) | 1.000 | | | | |
| Population | -0.031 | 1.000 | | | |
| Labor-augmenting productivity ($A_L$) | 0.593 | -0.093 | 1.000 | | |
| Capital-augmenting productivity ($A_K$) | 0.018 | 0.109 | -0.727 | 1.000 | |
| Land-augmenting productivity ($A_Z$) | 0.156 | 0.255 | -0.220 | 0.421 | 1.000 |

TABLE 2 – CONT'D

*Measured labor, capital, and land-augmenting productivity*
*Cross-sectional correlations - 2005*

*2.3. Relative productivities with output per worker*

| | Output per worker ($Y/L$) | Capital-to-labor productivity ($A_K/A_L$) | Land-to-labor productivity ($A_Z/A_L$) | Land-to-capital productivity ($A_Z/A_K$) |
|---|---|---|---|---|
| Output per worker ($Y/L$) | 1.000 | | | |
| Capital-to-labor productivity ($A_K/A_L$) | -0.338 | 1.000 | | |
| Land-to-labor productivity ($A_Z/A_L$) | -0.254 | 0.796 | 1.000 | |
| Land-to-capital productivity ($A_Z/A_K$) | 0.147 | -0.366 | 0.272 | 1.000 |

*Notes:* Same as in Table 1. Labor, capital, and land-augmenting productivities are backed out from the data using an elasticity of substitution between capital and land of -1. Except for the factor shares, all other variables are logged when computing the correlations.

TABLE 3

*Time series correlations*

*Panel regressions for 1970 – 2005*

*3.1. Dependent variable: Output per worker*

| *Explanatory variables* | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Labor-augmenting productivity ($A_L$) | 0.288*** | | | 0.572*** |
| | (5.13) | | | (4.45) |
| Capital-augmenting productivity ($A_K$) | | 0.063 | | 0.302*** |
| | | (1.08) | | (6.07) |
| Land-augmenting productivity ($A_Z$) | | | -0.051 | 0.002 |
| | | | (-1.03) | (0.04) |
| *Observations* | 4,200 | 2,471 | 2,364 | 2,364 |

*3.2. Dependent variable: Population*

| *Explanatory variables* | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Labor-augmenting productivity ($A_L$) | -0.103** | | | -0.143* |
| | (-2.47) | | | (-2.17) |
| Capital-augmenting productivity ($A_K$) | | -0.019 | | -0.176*** |
| | | (-0.51) | | (-4.95) |
| Land-augmenting productivity ($A_Z$) | | | 0.119*** | 0.156*** |
| | | | (4.74) | (7.18) |
| *Observations* | 4,200 | 2,471 | 2,364 | 2,364 |

*Notes:* Labor, capital, and land-augmenting productivities are backed out from the data using an elasticity of substitution between capital and land of -1, and an elasticity of substitution between labor and the capital-land composite of -0.2. All regressions include country fixed effects and time dummies. Regressions in columns (1) include more observations because labor-augmenting productivity can be constructed for 1970-2017. Labor, capital, and land-augmenting productivity measures are normalized to one in each country in 1970. All variables are logged, so coefficients represent elasticities. Standard errors are clustered at the country level and reported in parenthesis. Start superscripts: * $p < 0.10$, ** $p < .05$, *** $p < 0.01$.

TABLE 4

*Average annual growth for measured labor, capital, and land-augmenting productivity – 1970-2005*
*Correlations with average growth of output per worker and population*

| *Average annual growth rates 1970-2005:* | Output per worker ($Y/L$) | Population | Labor-augmenting productivity ($A_L$) | Capital-augmenting productivity ($A_K$) | Land-augmenting productivity ($A_Z$) |
|---|---|---|---|---|---|
| Output per worker ($Y/L$) | 1.000 | | | | |
| Population | -0.460 | 1.000 | | | |
| Labor-augmenting productivity ($A_L$) | 0.668 | -0.319 | 1.000 | | |
| Capital-augmenting productivity ($A_K$) | 0.139 | -0.039 | -0.524 | 1.000 | |
| Land-augmenting productivity ($A_Z$) | 0.080 | 0.406 | -0.343 | 0.620 | 1.000 |

*Notes:* Country sample is an in Table 3. Labor, capital, and land-augmenting productivities are backed out from the data using an elasticity of substitution between capital and land of -1, and an elasticity of substitution between labor and the capital-land composite of -0.2. Labor, capital, and land-augmenting productivity measures are normalized to one in each country in 1970.

Figure 1. Population versus agricultural land area - 2015

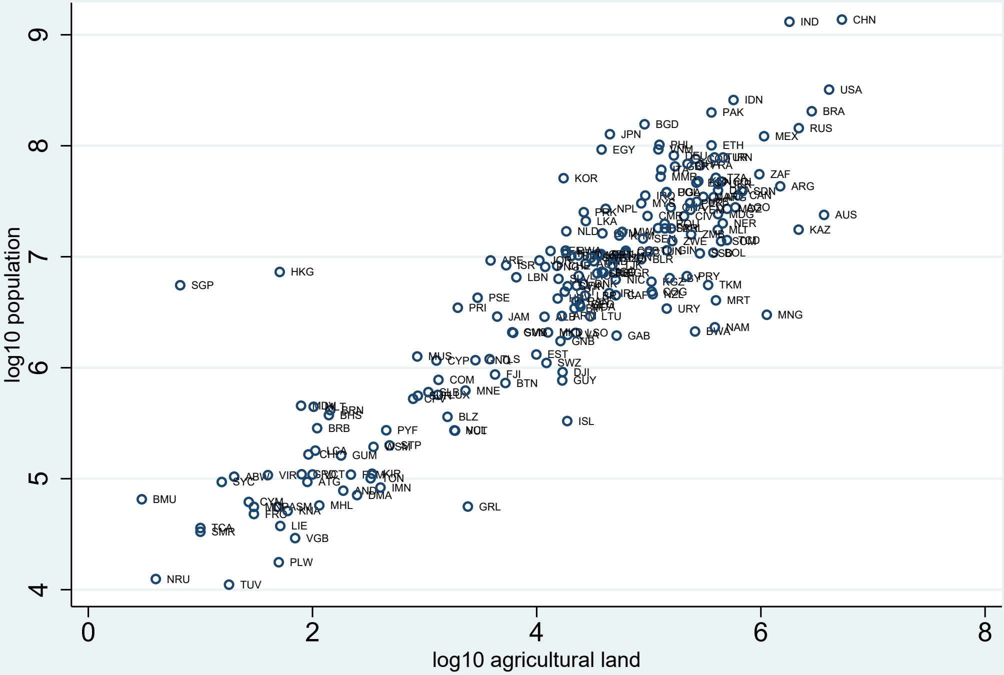


Figure 2. GDP per capita versus agricultural land area - 2015

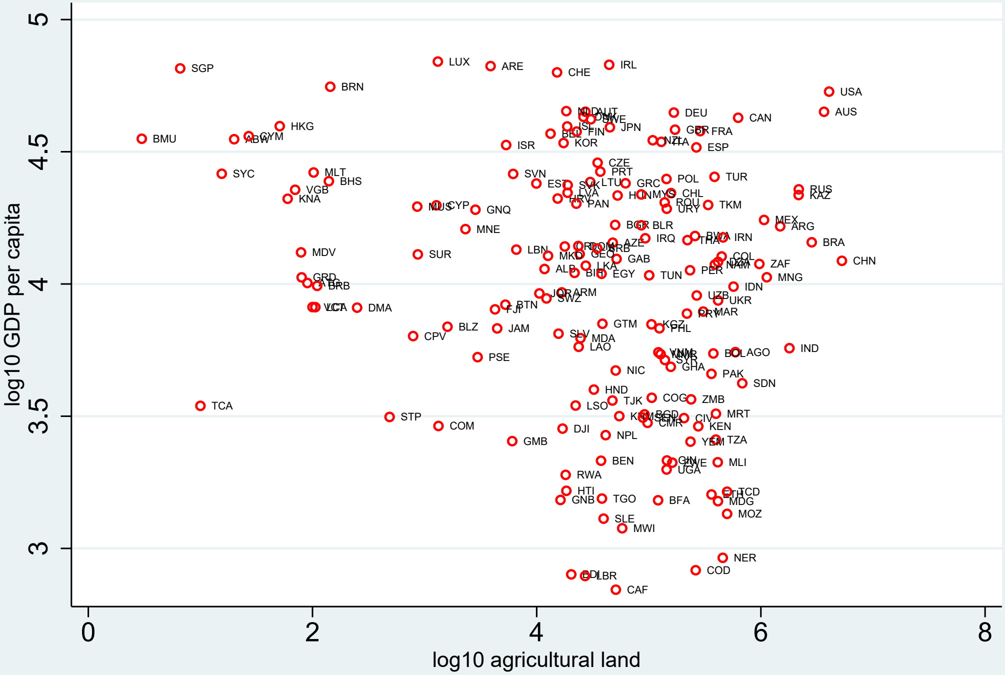


Figure 3. Labor share versus GDP per worker - 2005

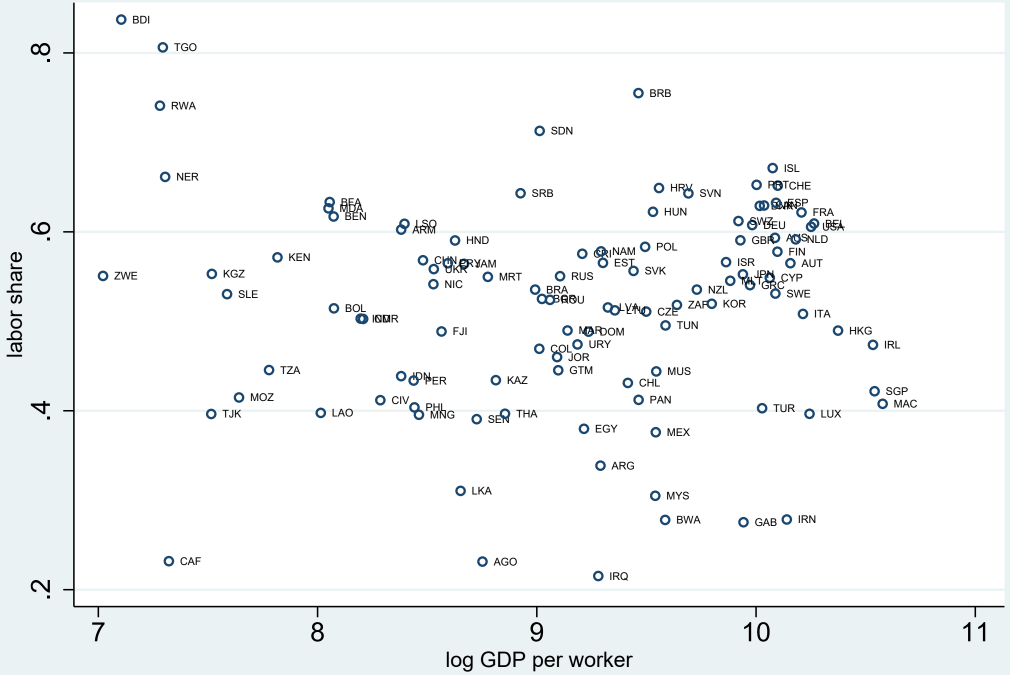


Figure 4. Capital share versus GDP per worker - 2005

# Figure 5. Agricultural land share versus GDP per worker - 2005

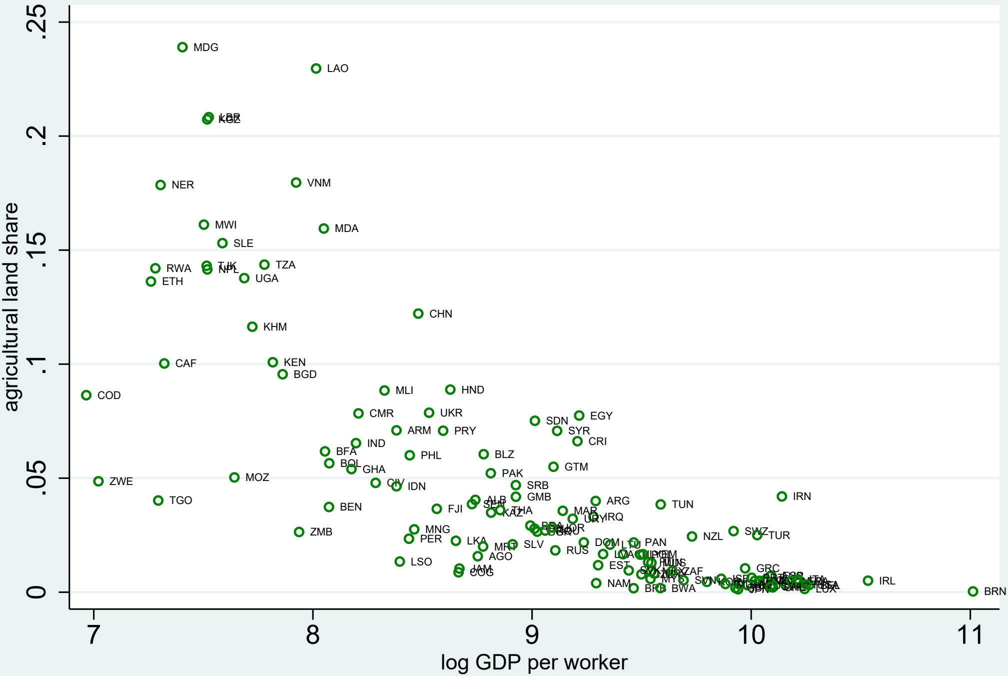


Figure 6. Capital-augmenting vs. labor-augmenting productivity

2005

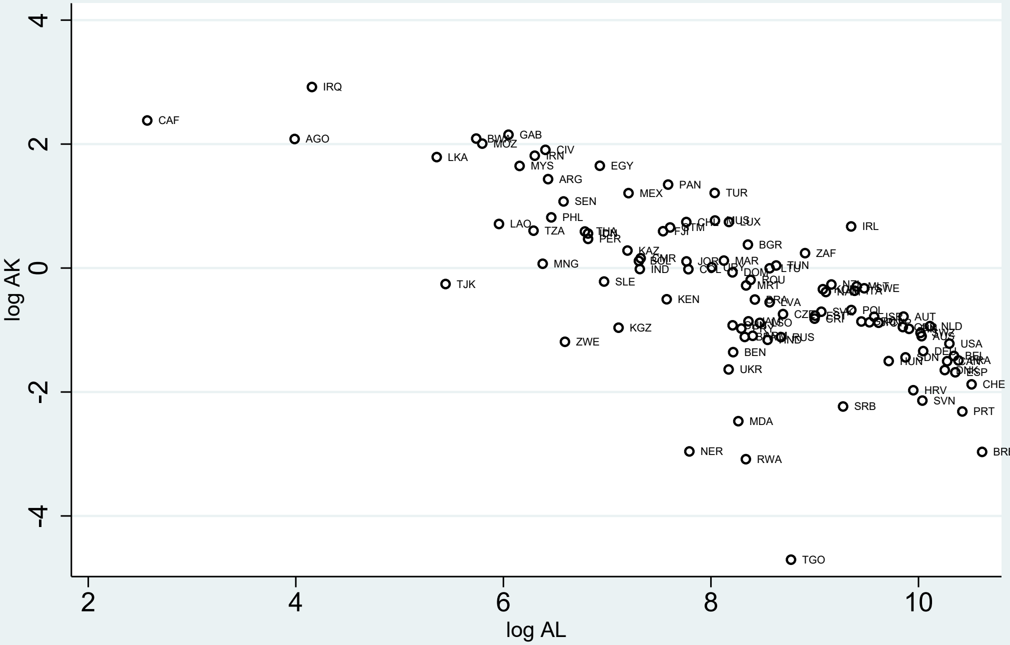


Figure 7. Land-augmenting vs. labor-augmenting productivity

2005

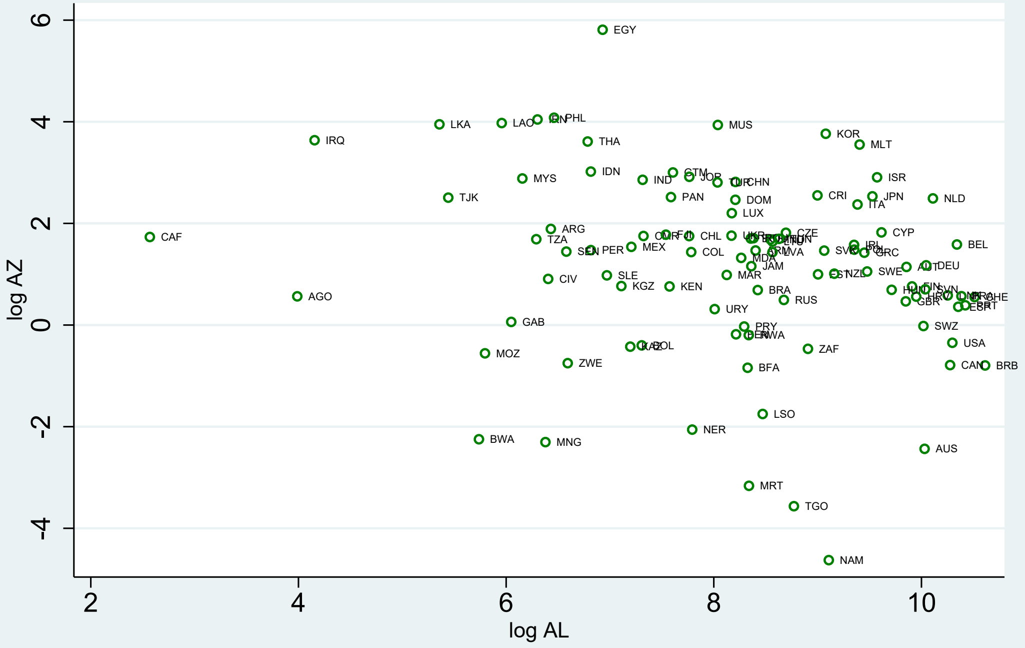
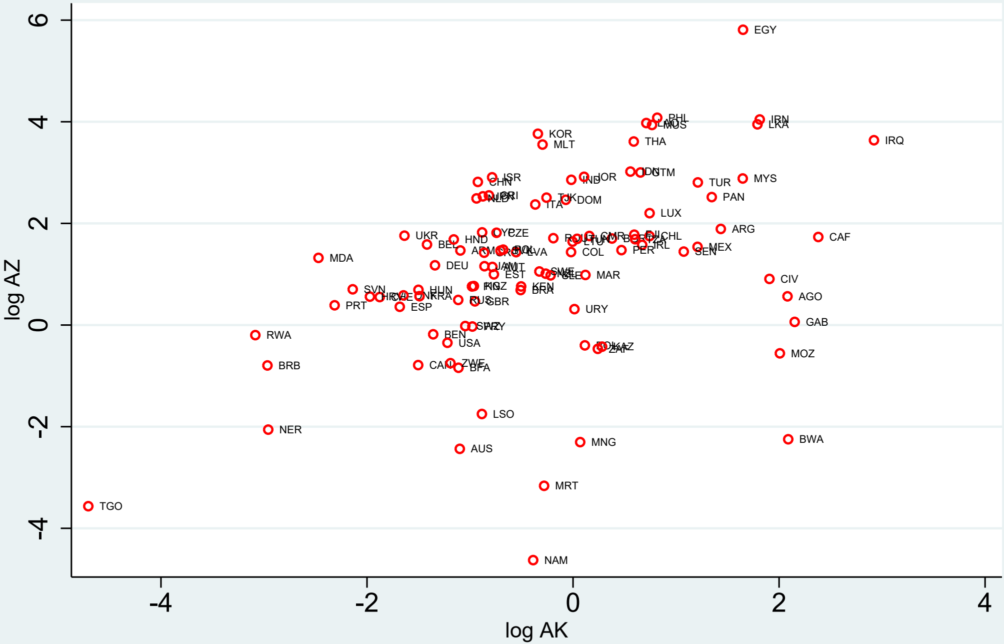


Figure 8. Land-augmenting vs. capital-augmenting productivity

2005



# Figure 9. Land-augmenting productivity versus population - 2005

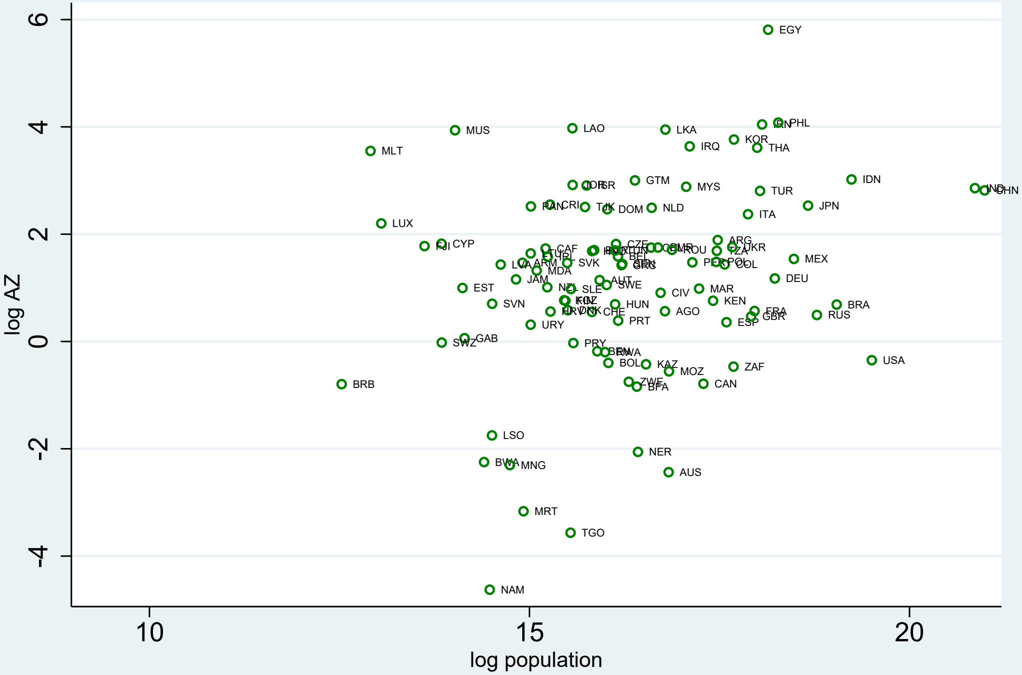
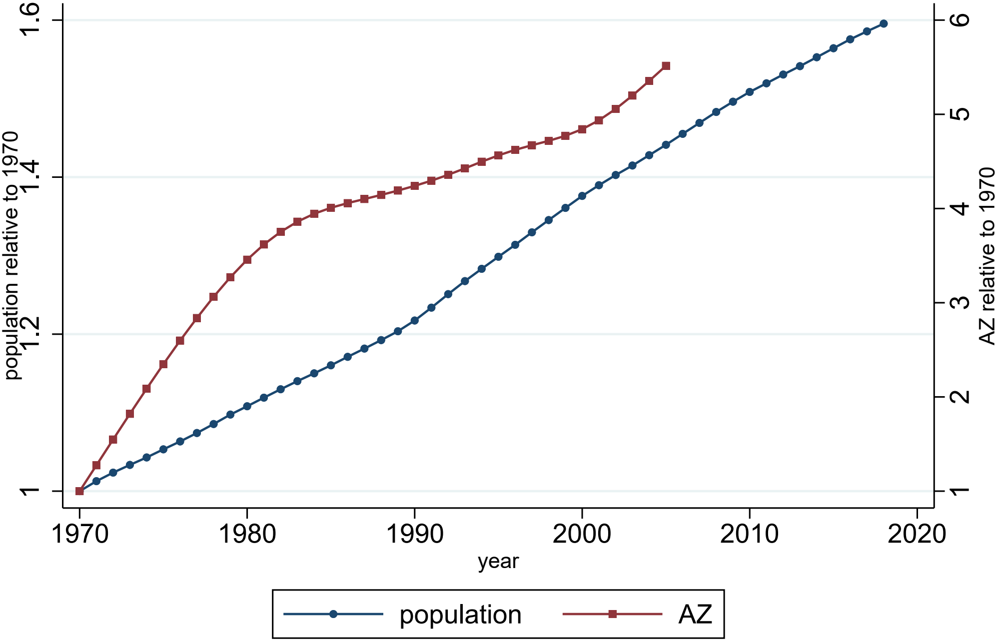


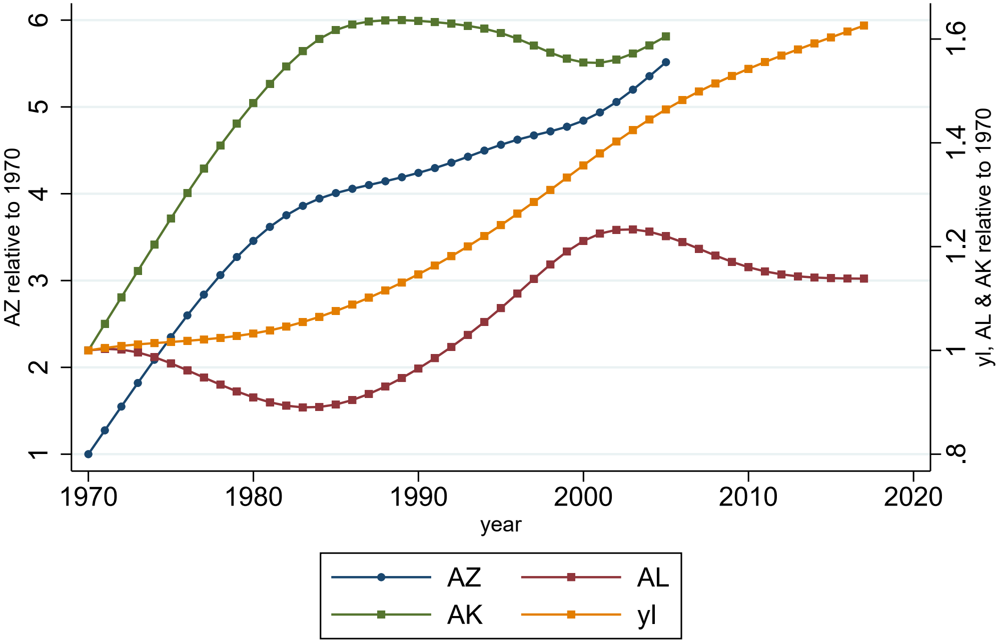
Figure 10. United States - Population and land-augmenting productivity

Variables measured relative to 1970



# Figure 11. United States - GDP per worker, AL, AK and AZ

Variables measured relative to 1970



# Figure 12. Population growth vs. land-augmenting productivity growth

1970-2005