Design of a Wooden Pile-and-Plank Retaining Wall

ENGR0145, Statics & Mechanics of Materials II

Dr. William Slaughter

July 3, 2017

Seth Strayer

Liam Stubanas

Brian Walsh

Abstract

This report details our calculations, data, and analysis used to design a wooden pile-and-plank retaining wall that is 80 feet long and will be used to create a 5-foot-high terrace. The wall is to be made of standard structural timber and is subjected to a load in the form of a pressure distribution.

We initially construct free-body and shear force diagrams to determine the maximum bending moment experienced by both the planks and the piles. We then relate maximum bending moment to several different variables, including plank thickness, plank length, and minimum required section modulus. Once we find a relationship between the minimum section modulus and the length of the planks, we can start analyzing costs. Using a Microsoft Excel spreadsheet, we calculate the minimum overall cost based off the most optimum plank and pile combination.

Introduction

A plank and pile retaining wall is used to alter topography and mediate erosion due to storm water runoff. An example of such a retaining wall is shown in *Figure A*. Generally, these retaining walls can be constructed from wood, stone, or concrete. In the case of this design however, we will consider a wall made of only standard structural timber. The wall must be able to support a load specified by what is being held up, which is most frequently going to be soil. For this specific design, the load is non-uniformly distributed and ranges from $500\,\frac{lb}{ft^2}$ to $100\,\frac{lb}{ft^2}$ over a height of 5 feet as shown in *Figure B*. Using this distributed load, the supplied allowable flexural stress of structural timber, $1200\ psi$, and the dimensions of the planks and piles, such as length and cross section, we can design the retaining wall so as to minimize the overall cost. Some constraints are given:

- The wall must be 80 feet long and at least 5 feet high
- Piles should all be of the same size, be evenly spaced, be square in cross section, extend 5 feet below grade, and there should be a pile at either end of the retaining wall
- All planks should be the same size, and the length of the planks should be the same as the spacing between piles, so that each plank is supported by piles at either end

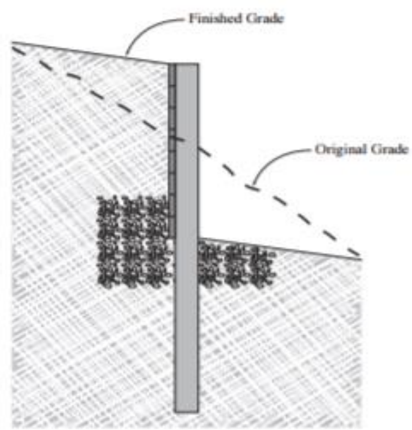


Figure A

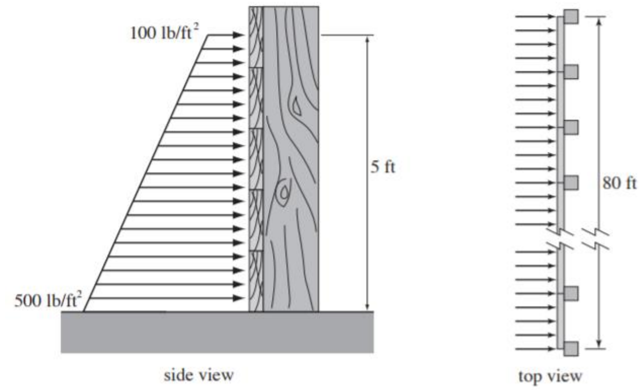


Figure B

Standard structural timbers are available in 8, 10, and 12 feet lengths and cost $14 per cubic foot (dressed size). There is also an additional cost of $40 per pile for the concrete footing needed to support each pile.

The overall goal is to design a wall the minimizes the cost of the wall that can also safely support the load. Since the length of boards is a function of the number of piles, and the cost is a function of the total volume of material and the number of piles, we expect the optimum cost to fall where total volume and the number of piles is near a minimum value.

Before starting the analysis, we note some key variables that will be used throughout the entirety of the report:

- $\sigma_{max} - maximum\ flexural\ stress$
- $M_{max} - maximum\ bending\ moment$
- $S_{min} - minimum\ required\ section\ modulus$
- $h - depth$
- $t - thickness$
- $L - length$
- $n - number\ of\ piles$

Analysis & Design

We approach this problem by analyzing the planks and the piles separately. We will first consider the planks, treating them as simply-supported beams. Since the overall force acting on the planks decreases as we move above the ground, we can simply analyze the bottom-most plank. To be conservative, we assume that the pressure on this plank is uniform and equal to the maximum pressure of $(500\frac{lb}{ft^2})$. Since the soil load on the plank wall is in terms of a pressure distribution, we can multiply by the depth (call this h) in order to find the force per until length acting on the planks. We obtain the following free-body diagram for the bottom plank:

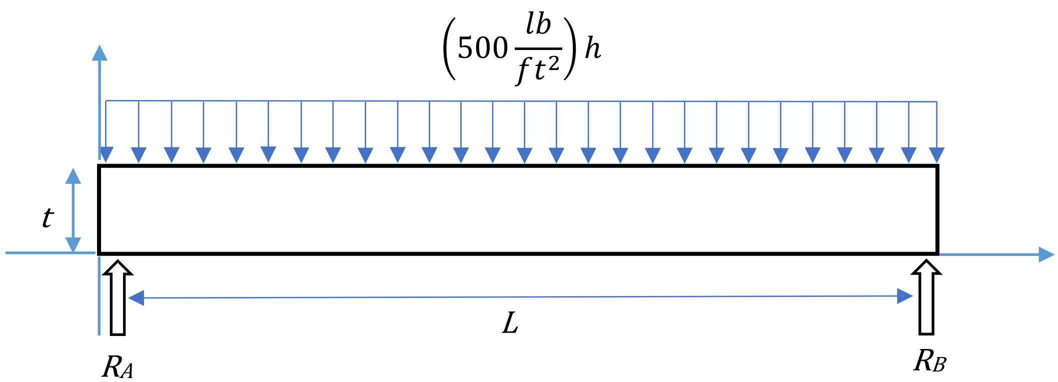


Figure 1

Since the plank is of rectangular geometry, we can replace the distributed load with its statically equivalent concentrated load of $(500\frac{lb}{ft^2})hL$ acting at $\frac{L}{2}$. It is important that we accurately keep track of our units in these early steps. We now have:

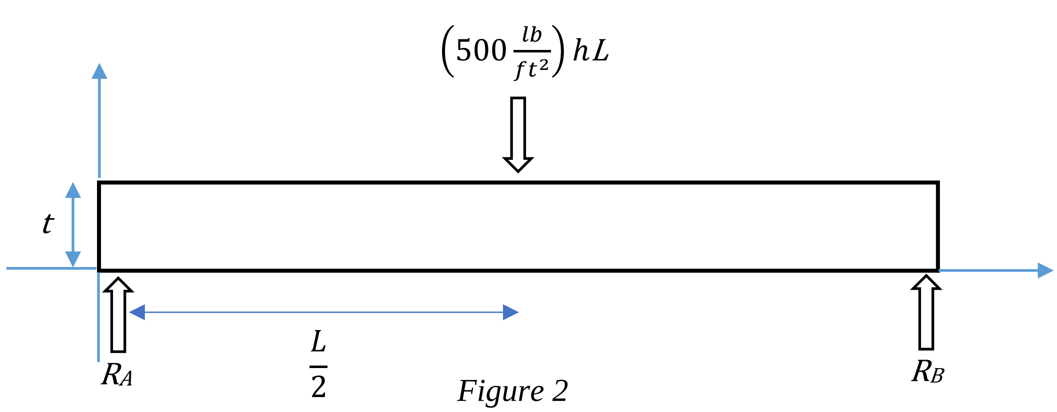


Figure 2

Summing forces in the y-direction, we have

$$\sum F_y = R_A + R_B - \left(500\,\frac{lb}{ft^2}\right) hL = 0$$

From symmetry,

$$R_A = R_B = \left(250\,\frac{lb}{ft^2}\right) hL$$

We can now construct our shear force diagram and calculate our corresponding maximum bending moment, M_{max}.

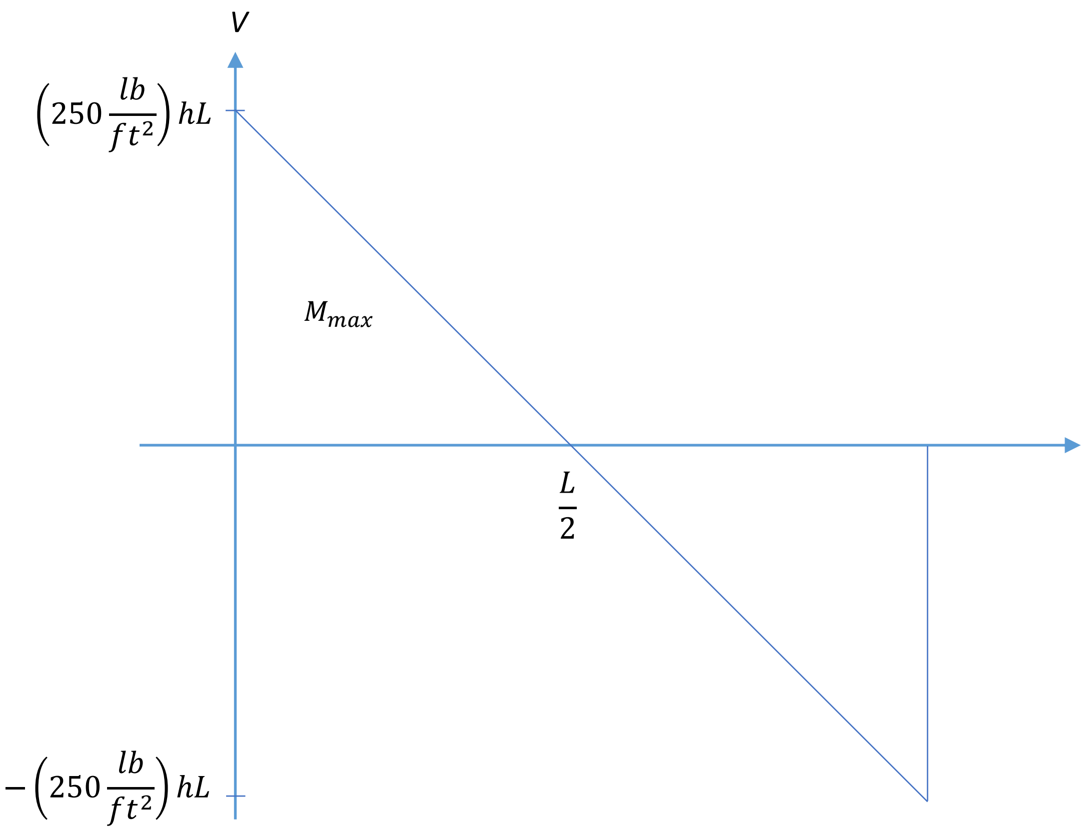


Figure 3

By definition, our maximum bending moment for the plank is

$$M_{max} = \frac{1}{2}\left(\frac{L}{2}\right)\left(\left(250\,\frac{lb}{ft^2}\right) hL\right) = \left(62.5\,\frac{lb}{ft^2}\right) hL^2$$

Converting this result to units of $\frac{lb}{in^2}$,

$$M_{max} = \left(.43403\,\frac{lb}{in^2}\right) hL^2$$

Given that $\sigma_{max} = \frac{M_{max}}{S_{min}}$, we have

$$\sigma_{max} = 1200\,\frac{lb}{in^2} = \frac{\left(.43403\,\frac{lb}{in^2}\right) hL^2}{S_{min}} \tag{1}$$

We obtain S_{min} by examining the plank's cross section:

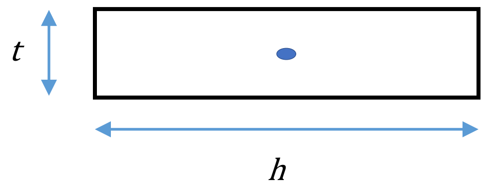


Figure 4

For this rectangular cross section,

$$S_{min} = \frac{I}{c} = \frac{\frac{1}{12}(h)(t)^3}{\frac{1}{2}t}$$

$$S_{min} = \frac{1}{6}ht^2$$

We insert this result into equation (1) to obtain:

$$\sigma_{max} = 1200\,\frac{lb}{in^2} = \frac{\left(\frac{.43403\ lb}{in^2}\right) hL^2}{\frac{1}{6}ht^2}$$

$$\sigma_{max} = 1200\,\frac{lb}{in^2} = \frac{\left(2.6042\,\frac{lb}{in^2}\right) L^2}{t^2}$$

We wish to solve for t. Namely, this value will be the minimum thickness required for the planks, t_{min}^{plank}:

$$\left(1200\frac{lb}{in^2}\right)t^2 = \left(2.6042\frac{lb}{in^2}\right)L^2$$

$$t_{min}^2 = 0.0021702L^2$$

$$\boxed{t_{min}^{plank} = 0.046585L} \qquad (2)$$

This result yields the minimum required thickness for the planks as a function of the length of the planks. We can now begin to analyze the piles. We consider the piles to act as cantilevered beams subjected to a trapezoidal distribution of load. Consider the given side view of the retaining wall and it's respective 90° rotation:

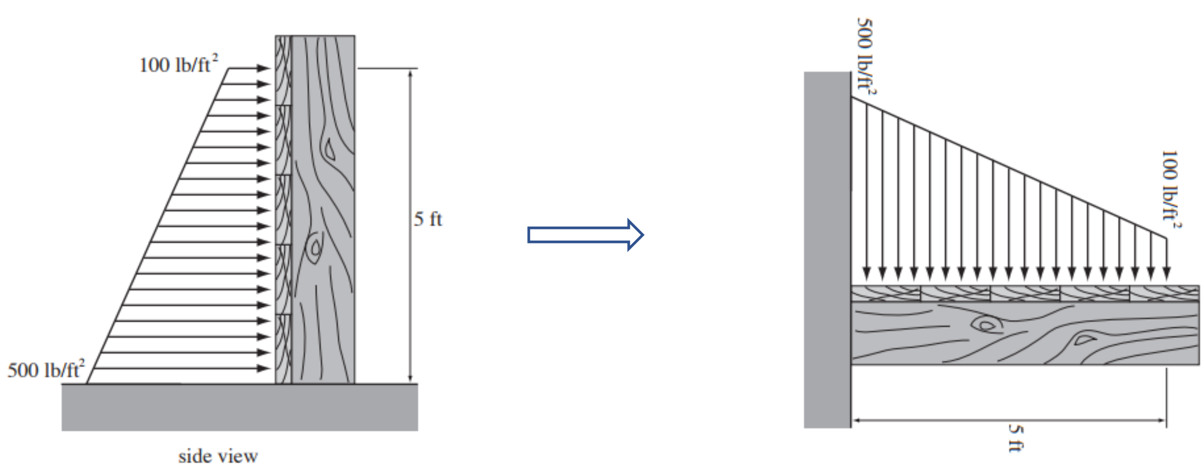


Figure 5

Indeed, the pile acts as a cantilevered beam. We can divide the trapezoidal distribution of load into a rectangular distributed load and a triangular distributed load. Let y_1 be the centroid of the rectangular distributed load and A_1 be the area of the rectangular distributed load. Let y_2 be the centroid of the triangular distributed load and A_2 be the area of the triangular distributed load.

We have the following formulas for the centroid y_C and area A of rectangular and triangular areas, where b is the base of the area and h is the height of the area:

$$y_{rectangle} = \frac{b}{2} \qquad A_{rectangle} = bh$$

$$y_{triangle} = \frac{b}{3} \qquad A_{triangle} = \frac{1}{2}bh$$

We use these formulas to find the values of y_1, A_1, y_2, and A_2, given the values for b and h from *Figure 5*. We disregard units for now to find:

$$y_1 = \frac{5}{2} = 2.5 \qquad A_1 = (5)(100) = 500$$

$$y_2 = \frac{5}{3} = 1.6667 \qquad A_2 = \frac{1}{2}(5)(400) = 1000$$

We let y_C be the centroid of the trapezoidal distributed load consisting of the rectangular and triangular distributed load. The formula for the centroid of the area is:

$$y_C = \frac{1}{A}\sum_{i=1}^{n} y_{Ci}A_i = \frac{y_1A_1 + y_2A_2}{A_1 + A_2}$$

$$y_C = \frac{(2.5)(500) + (1.6667)(1000)}{500 + 1000} = 1.9444$$

For this argument, a distance y is the distance above the ground. Thus, our units are in ft, and

$$y_C = 1.9444\ ft$$

Since the distributed load for the piles is also in terms of pressure, we consider the argument that each pile will take on a depth of L for the distributed load (piles are placed a distance L apart). We replace the trapezoidal distributed load with its statically equivalent concentrated load of $\left(1500\,\frac{lb}{ft}\right)L$ acting at the centroid of the area, $1.9444\ ft$, to obtain the following free-body diagram for the piles:

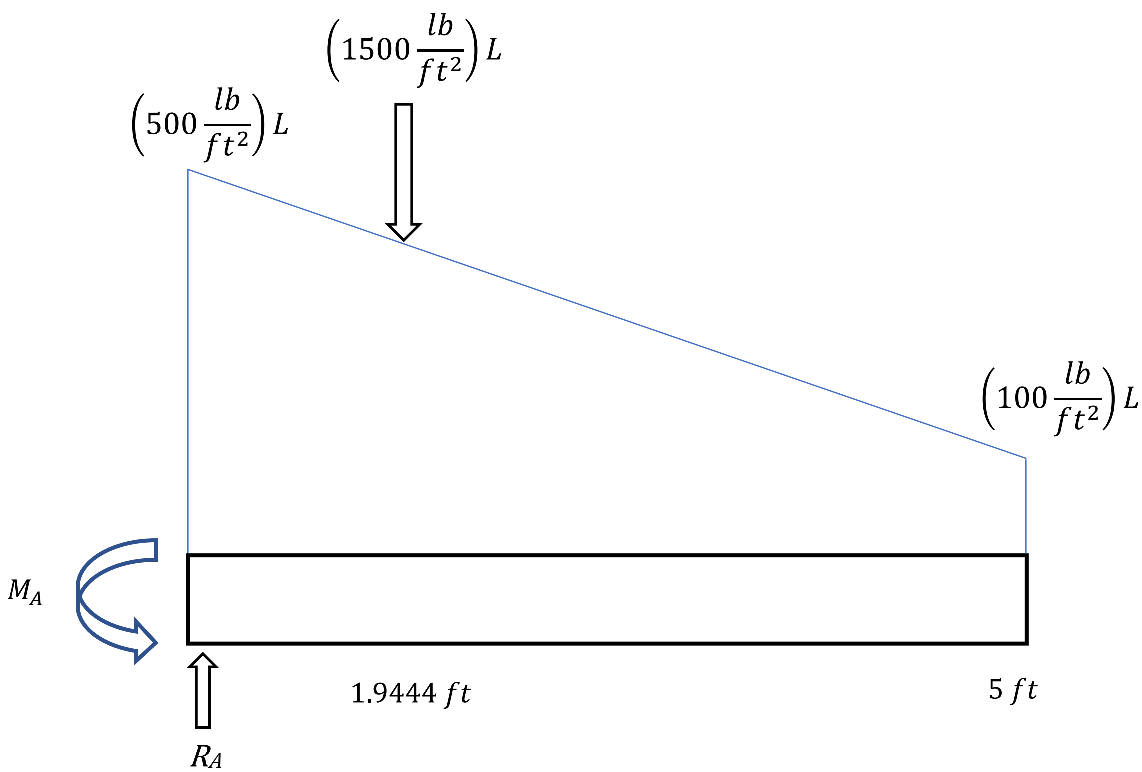


Figure 6

Summing forces and moments, we obtain:

$$\sum F_y = R_A - \left(1500\,\frac{lb}{ft}\right)L = 0$$

$$R_A = \left(1500\,\frac{lb}{ft}\right)L$$

$$\sum M_A = M_A - (1.9444\ ft)\left(1500\,\frac{lb}{ft}\right)L = 0$$

$$M_A = (2916.6\ lb)L$$

At this point we could construct shear force and bending moment diagrams for the piles, but we notice that at $y = 0$ (point A), the shear force and bending moment is a maximum since we only have one reaction force acting at A and only one distributed load acting on the pile. We have

$$M_{max} = (2916.6\ lb)L$$

Given that $\sigma_{max} = \frac{M_{max}}{S_{min}}$, we have

$$\sigma_{max} = 1200\,\frac{lb}{in^2} = \frac{(2916.6\ lb)L}{S_{min}}$$

$$\boxed{S_{min}^{pile} = (2.4305\ in^2)L} \qquad (3)$$

This result yields the minimum required section modulus for the piles as a function of length. We now have out two desired results; the minimum required thickness for the planks as a function of length, and the minimum section modulus required for the piles as a function of length. We can now finally start to analyze the costs for various sized retaining walls. We have the length of each plank as a function of the number of piles,

$$L = \frac{80}{n-1}$$

where 80 is the total length of the wall, in feet, and the quantity n being the number of piles. Given that the structural timbers are available only in 8, 10, and 12 foot lengths, we calculate our minimum number of piles to be 8. The maximum number of piles we are allotted is 41. Since the piles are to be square in cross section, we can pull the following minimum section moduli from Table A-15 of "Mechanics of Materials, 6th edition" by Riley:

Nominal Size	S_{all}(in^3)
4 × 4	7.94
6 × 6	27.7
8 × 8	70.3
10 × 10	143
12 × 12	253

Table 1

Given that $S_{min}^{pile} = (2.4305\ in^2)L$, we see the minimum section modulus for the piles varies linearly with the length of each plank. We can then test these values against the S_{all} values found in *Table 1*. Any timber piles with $S_{min}^{pile} < S_{all}$ will be denoted with a green stylized cell. These represent the possible cross-section sizes we *could* use for the piles.

Stress Analysis									
Number of Piles	Length of Planks	t.min	Dressed Thickness	S.min	Viability (S.all)				
n	L (ft)	t.min (in)	t (in)	S.min(in^3)	4 × 4	6 × 6	8 × 8	10 × 10	12 × 12
8	11.4286	6.3888	7.5	333.326	7.94	27.7	70.3	143	253
9	10.0000	5.5902	7.5	291.660	7.94	27.7	70.3	143	253
10	8.8889	4.9691	5.5	259.253	7.94	27.7	70.3	143	253
11	8.0000	4.4722	5.5	233.328	7.94	27.7	70.3	143	253
12	7.2727	4.0656	5.5	212.116	7.94	27.7	70.3	143	253
13	6.6667	3.7268	5.5	194.440	7.94	27.7	70.3	143	253
14	6.1538	3.4401	3.625	179.483	7.94	27.7	70.3	143	253
15	5.7143	3.1944	3.625	166.663	7.94	27.7	70.3	143	253
16	5.3333	2.9814	3.625	155.552	7.94	27.7	70.3	143	253
17	5.0000	2.7951	3.625	145.830	7.94	27.7	70.3	143	253
18	4.7059	2.6307	3.625	137.252	7.94	27.7	70.3	143	253
19	4.4444	2.4845	3.625	129.627	7.94	27.7	70.3	143	253
20	4.2105	2.3538	3.625	122.804	7.94	27.7	70.3	143	253
21	4.0000	2.2361	3.625	116.664	7.94	27.7	70.3	143	253
22	3.8095	2.1296	3.625	111.109	7.94	27.7	70.3	143	253
23	3.6364	2.0328	3.625	106.058	7.94	27.7	70.3	143	253
24	3.4783	1.9444	3.625	101.447	7.94	27.7	70.3	143	253
25	3.3333	1.8634	3.625	97.220	7.94	27.7	70.3	143	253
26	3.2000	1.7889	3.625	93.331	7.94	27.7	70.3	143	253
27	3.0769	1.7201	3.625	89.742	7.94	27.7	70.3	143	253
28	2.9630	1.6564	3.625	86.418	7.94	27.7	70.3	143	253
29	2.8571	1.5972	1.625	83.331	7.94	27.7	70.3	143	253
30	2.7586	1.5421	1.625	80.458	7.94	27.7	70.3	143	253
31	2.6667	1.4907	1.625	77.776	7.94	27.7	70.3	143	253
32	2.5806	1.4426	1.625	75.267	7.94	27.7	70.3	143	253
33	2.5000	1.3976	1.625	72.915	7.94	27.7	70.3	143	253
34	2.4242	1.3552	1.625	70.705	7.94	27.7	70.3	143	253
35	2.3529	1.3153	1.625	68.626	7.94	27.7	70.3	143	253
36	2.2857	1.2778	1.625	66.665	7.94	27.7	70.3	143	253
37	2.2222	1.2423	1.625	64.813	7.94	27.7	70.3	143	253
38	2.1622	1.2087	1.625	63.062	7.94	27.7	70.3	143	253
39	2.1053	1.1769	1.625	61.402	7.94	27.7	70.3	143	253
40	2.0513	1.1467	1.625	59.828	7.94	27.7	70.3	143	253
41	2.0000	1.1180	1.625	58.332	7.94	27.7	70.3	143	253

Table 2

We can now calculate the cost associated with various combinations of piles and planks. We start with the piles, since their cost is somewhat more intuitive than the planks. There is a cost for concrete footing needed to support each pile given by:

$$c = 40n$$

where n is the number of piles. Standard structural timbers cost $14 per cubic foot (dressed size). We will consider only 10 ft. long piles, as the piles must extend an even 5 ft. below grade and 5 ft. above grade. Total pile cost (C_{pile}) can be calculated using the following variables and constants: c, the cost of the concrete footings, n, the number of piles, h_d, the dressed height of the piles, t_d, the dressed thickness of the piles, and 10, the length of each pile, in feet. We have:

$$C = c + 14n\left(\frac{h_d}{12} * \frac{t_d}{12} * 10\right) = c + \left(\frac{35h_d t_d n}{36}\right)$$

Cost Analysis for the Piles					
Concrete Cost	Concrete Cost + Timber Cost (10 ft long piles)				
C ($)	4 × 4	6 × 6	8 × 8	10 × 10	12 × 12
$ 320.00					
$ 360.00					
$ 400.00					
$ 440.00					$1,854.34
$ 480.00					$2,022.92
$ 520.00					$2,191.49
$ 560.00					$2,360.07
$ 600.00					$2,528.65
$ 640.00					$2,697.22
$ 680.00					$2,865.80
$ 720.00				$2,299.38	$3,034.38
$ 760.00				$2,427.12	$3,202.95
$ 800.00				$2,554.86	$3,371.53
$ 840.00				$2,682.60	$3,540.10
$ 880.00				$2,810.35	$3,708.68
$ 920.00				$2,938.09	$3,877.26
$ 960.00				$3,065.83	$4,045.83
$ 1,000.00				$3,193.58	$4,214.41
$ 1,040.00				$3,321.32	$4,382.99
$ 1,080.00				$3,449.06	$4,551.56
$ 1,120.00				$3,576.81	$4,720.14
$ 1,160.00				$3,704.55	$4,888.72
$ 1,200.00				$3,832.29	$5,057.29
$ 1,240.00				$3,960.03	$5,225.87
$ 1,280.00				$4,087.78	$5,394.44
$ 1,320.00				$4,215.52	$5,563.02
$ 1,360.00				$4,343.26	$5,731.60
$ 1,400.00			$3,314.06	$4,471.01	$5,900.17
$ 1,440.00			$3,408.75	$4,598.75	$6,068.75
$ 1,480.00			$3,503.44	$4,726.49	$6,237.33
$ 1,520.00			$3,598.13	$4,854.24	$6,405.90
$ 1,560.00			$3,692.81	$4,981.98	$6,574.48
$ 1,600.00			$3,787.50	$5,109.72	$6,743.06
$ 1,640.00			$3,882.19	$5,237.47	$6,911.63

Table 3

We now analyze the cost for the planks. We will only use planks of nominal height size 8 in., since this is the only size that will divide evenly into 5 ft. (see *Table 4* below). This reduces waste and simplifies our process.

Nominal height, h (in)	Dressed Height, h_d (in)	Number of planks to reach 5 ft.
4	3.625	16.55
6	5.625	10.67
8	7.5	8
10	9.5	6.32
12	11.5	5.22
14	13.5	4.44

Table 4

Since standard structural timbers are available in only 8, 10, and 12 ft. lengths, there will obviously some amount of scrap leftover for most of the plank sizes. To minimize this waste, we use a simple excel algorithm that will calculate the minimum number of planks we must buy, given the spacing between piles and the lengths of the beams bought from the store.

We utilize the ROUNDDOWN and ROUNDUP function in excel for this algorithm. Consider first the number of planks we can obtain from the actual length of the beams. We formulated the following algorithm:

$$N_i = ROUNDOWN\left(\frac{beam\ length}{plank\ length}\right)$$

where N_i is the number of planks that can be obtained from the given beam length, $beam\ length$ is the actual length of the beam bought from the store, and $plank\ length$ is the length of our planks (also the spacing between piles). Then we have the following algorithm to calculate the minimum number of beams we must buy from the store to meet our quota:

$$N = ROUNDUP\left(\frac{80}{N_i * plank\ length}\right)$$

This gives us the minimum number of beams, N, that we must purchase from the store. Taking into account that the timber costs \$14 per cubic foot, we can now calculate the total plank cost, C_{plank}:

$$C_{plank} = \left(\frac{7.5}{12}\right)\left(\frac{t_d}{12}\right)(L)(8)(14N)$$

Note this equation takes into account the volume of the plank $(h * t * L)$, the number of planks needed to reach the height of the wall, 8, the total planks needed to reach the length requirement of the wall, N, and the cost per cubic foot of timber, 14. Now since there is no trivial solution to what beam combinations will provide the optimum cost, we must analyze all three of the 8 ft., 10 ft., and 12 ft. lengths (see *Table 5* below).

Cost Analysis for the Planks									
12 ft. lengths			10 ft. lengths			8 ft. lengths			
Number of Planks	Number of Beams	Cost	Number of Planks	Number of Beams	Cost	Number of Planks	Number of Beams	Cost	Min Cost
1	10	$3,850.00	1	10	$3,208.33	1	10	$2,566.67	$2,566.67
1	11	$4,235.00	1	11	$3,529.17	1	11	$2,823.33	$2,823.33
1	12	$4,620.00	1	12	$3,850.00	1	12	$3,080.00	$3,080.00
1	13	$3,298.75	1	13	$2,748.96	1	13	$2,199.17	$2,199.17
2	7	$1,776.25	1	14	$2,960.42	1	14	$2,368.33	$1,776.25
2	8	$2,030.00	1	15	$3,171.88	1	15	$2,537.50	$2,030.00
2	8	$2,030.00	2	8	$1,691.67	1	16	$2,706.67	$1,691.67
2	9	$2,283.75	2	9	$1,903.13	1	17	$2,875.83	$1,903.13
2	9	$2,283.75	2	9	$1,903.13	1	18	$3,045.00	$1,903.13
2	10	$2,537.50	2	10	$2,114.58	1	19	$3,214.17	$2,114.58
3	7	$1,776.25	2	10	$2,114.58	2	10	$1,691.67	$1,691.67
3	7	$1,776.25	2	11	$2,326.04	2	11	$1,860.83	$1,776.25
3	8	$2,030.00	2	11	$2,326.04	2	11	$1,860.83	$1,860.83
3	8	$2,030.00	2	12	$2,537.50	2	12	$2,030.00	$2,030.00
3	8	$2,030.00	3	8	$1,691.67	2	12	$2,030.00	$1,691.67
3	9	$2,283.75	3	9	$1,903.13	2	13	$2,199.17	$1,903.13
3	9	$2,283.75	3	9	$1,903.13	2	13	$2,199.17	$1,903.13
4	7	$1,776.25	3	9	$1,903.13	2	14	$2,368.33	$1,776.25
4	7	$ 796.25	3	10	$ 947.92	2	14	$1,061.67	$ 796.25
4	8	$ 910.00	3	10	$ 947.92	2	15	$1,137.50	$ 910.00
4	8	$ 910.00	3	10	$ 947.92	3	10	$ 758.33	$ 758.33
4	8	$ 910.00	3	11	$1,042.71	3	11	$ 834.17	$ 834.17
4	8	$ 910.00	4	8	$ 758.33	3	11	$ 834.17	$ 758.33
4	9	$1,023.75	4	9	$ 853.13	3	11	$ 834.17	$ 834.17
5	7	$ 796.25	4	9	$ 853.13	3	12	$ 910.00	$ 796.25
5	7	$ 796.25	4	9	$ 853.13	3	12	$ 910.00	$ 796.25
5	8	$ 910.00	4	9	$ 853.13	3	12	$ 910.00	$ 853.13
5	8	$ 910.00	4	10	$ 947.92	3	13	$ 985.83	$ 910.00
5	8	$ 910.00	4	10	$ 947.92	3	13	$ 985.83	$ 910.00
5	8	$ 910.00	4	10	$ 947.92	3	13	$ 985.83	$ 910.00
6	7	$ 796.25	5	8	$ 758.33	4	10	$ 758.33	$ 758.33

Table 5

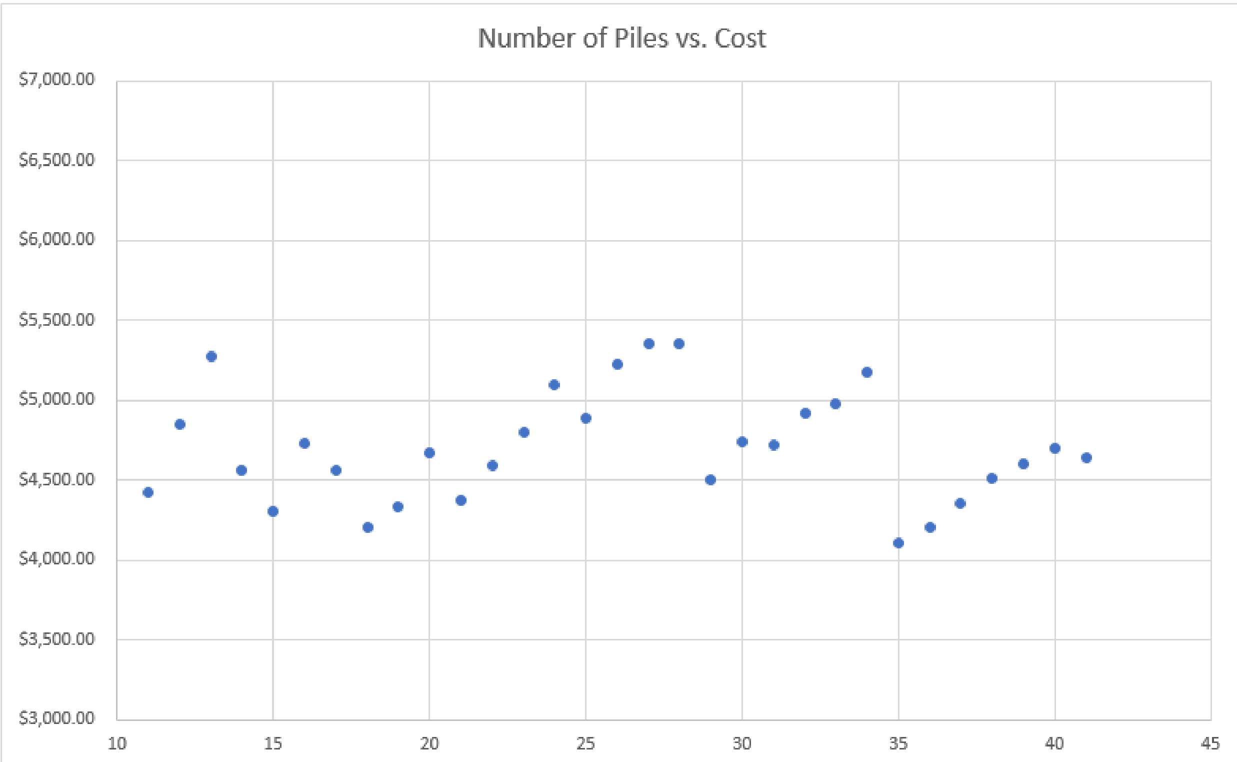
We obtain the minimum overall cost by adding together the minimum cost of the piles and the minimum cost of planks for any given value of n:

Number of Piles, n	Concrete Cost + Timber Cost (10 ft long piles)					Plank Cost	Cheapest Option
	4 × 4	6 × 6	8 × 8	10 × 10	12 × 12		
8							
9							
10							
11					$1,854.34	$2,566.67	$ 4,421.01
12					$2,022.92	$2,823.33	$ 4,846.25
13					$2,191.49	$3,080.00	$ 5,271.49
14					$2,360.07	$2,199.17	$ 4,559.24
15					$2,528.65	$1,776.25	$ 4,304.90
16					$2,697.22	$2,030.00	$ 4,727.22
17					$2,865.80	$1,691.67	$ 4,557.47
18				$ 2,299.38	$3,034.38	$1,903.13	$ 4,202.50
19				$ 2,427.12	$3,202.95	$1,903.13	$ 4,330.24
20				$ 2,554.86	$3,371.53	$2,114.58	$ 4,669.44
21				$ 2,682.60	$3,540.10	$1,691.67	$ 4,374.27
22				$ 2,810.35	$3,708.68	$1,776.25	$ 4,586.60
23				$ 2,938.09	$3,877.26	$1,860.83	$ 4,798.92
24				$ 3,065.83	$4,045.83	$2,030.00	$ 5,095.83
25				$ 3,193.58	$4,214.41	$1,691.67	$ 4,885.24
26				$ 3,321.32	$4,382.99	$1,903.13	$ 5,224.44
27				$ 3,449.06	$4,551.56	$1,903.13	$ 5,352.19
28				$ 3,576.81	$4,720.14	$1,776.25	$ 5,353.06
29				$ 3,704.55	$4,888.72	$ 796.25	$ 4,500.80
30				$ 3,832.29	$5,057.29	$ 910.00	$ 4,742.29
31				$ 3,960.03	$5,225.87	$ 758.33	$ 4,718.37
32				$ 4,087.78	$5,394.44	$ 834.17	$ 4,921.94
33				$ 4,215.52	$5,563.02	$ 758.33	$ 4,973.85
34				$ 4,343.26	$5,731.60	$ 834.17	$ 5,177.43
35			$ 3,314.06	$ 4,471.01	$5,900.17	$ 796.25	$ 4,110.31
36			$ 3,408.75	$ 4,598.75	$6,068.75	$ 796.25	$ 4,205.00
37			$ 3,503.44	$ 4,726.49	$6,237.33	$ 853.13	$ 4,356.56
38			$ 3,598.13	$ 4,854.24	$6,405.90	$ 910.00	$ 4,508.13
39			$ 3,692.81	$ 4,981.98	$6,574.48	$ 910.00	$ 4,602.81
40			$ 3,787.50	$ 5,109.72	$6,743.06	$ 910.00	$ 4,697.50
41			$ 3,882.19	$ 5,237.47	$6,911.63	$ 758.33	$ 4,640.52

Table 6

Results

From *Table 6* we see that our minimum overall cost occurs when we have 35 piles. We have the following relationship between the number of piles vs. overall cost:



We obtain the following bill of materials for the optimum retaining wall:

Bill of Materials:

Item	Size	Units	Total Cost
Pile	8”x8”x10’	35	$1914.06
Concrete Footing	N/A	35	$1400.00
Plank	2”x8”x12’	56	$796.25

The overall cost for this project will be:

$$\boxed{Final\ Cost = \$4110.31}$$

Although there is no trivial relationship between the number of piles and the overall cost, we do notice that when our minimum plank thickness decreases, our cost will decrease considerably (observe the change between 13 to 14 piles and 28 to 29 piles). This is because as thickness decreases, so too does the length of the planks (see equation 2). As length of the planks decreases, our minimum section modulus required for the piles also decreases (see equation 3), and we are able to use smaller sizes of timber. We note that it is no coincidence that our minimum overall cost occurs when we use 35 piles. Looking at *Table 2*, we see that the allowable section modulus changes from 143 in^3 to 70.3 in^3 at 35 piles. This allows us to use the 8 x 8 square cross section for piles, which decreases volume and in turn, overall cost for the piles. It happens that at 35 piles, we reach the optimum relationship between the number and piles and overall cost of the retaining wall.