<u>Design Project 1</u>

<u>"Design of a Cheap Thermal Switch"</u>

ENGR 0135

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Abstract

This report will analyze our calculations, data, and analysis used to redesign a cheap thermal switch to activate with a temperature change of 100º F. The initial design has the switch activate at 180ºF. Our group initially approached this design by assuming that the aluminum strip must have different dimensions as opposed to the original switch. To calculate this, we used mathematical data from the original switch and defined an equation of how temperature relates to the stress, pressure, and Young's modulus. By using certain dimensional assumptions and theoretical analysis, we managed to calculate a realistic solution to redesign this cheap thermal switch. By utilizing conceptions of the switch and mathematical data analysis, this design report will show how we computed the new dimensions for the cheap thermal switch.

Introduction

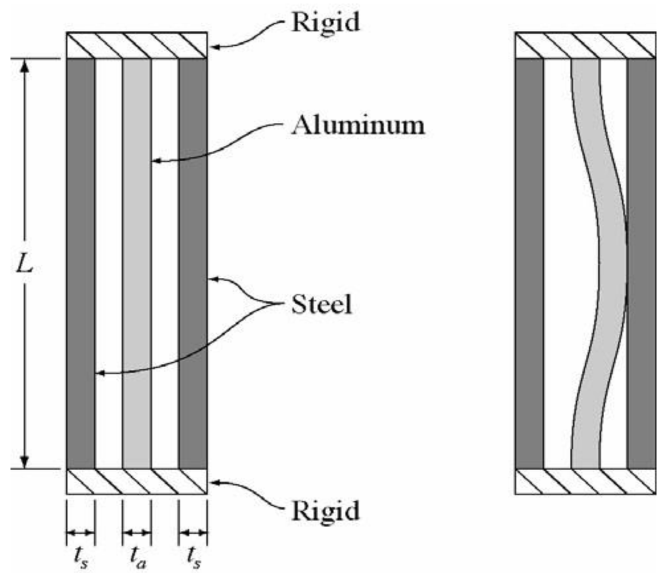
The project is to design an inexpensive thermal switch for use in a product that works at a temperature increase of 100ºF. The thermal switch is designed to close currently at a temperature increase of 180ºF. The switch is composed of three metal strips clamped together: two outer steel strips and one middle aluminum strip. No external mechanical loading exists on this switch. Subsequently, the only loads are caused by temperature changes. In addition, the switch has no initial stresses. As the temperature increases, the central aluminum strip will snap aside to make contact with one of the outer steel strips to conduct an electric current. This is due to the higher thermal coefficient of aluminum and lower Young's modulus, which allows for the aluminum strip to elongate faster and buckle easily.

We intend to approach this problem with a hypothesis that we will need a smaller cross sectional area than the original design. To achieve this, we could decrease the width, thickness, or both of the aluminum strip to allow the switch to activate at a temperature increase of 100ºF.

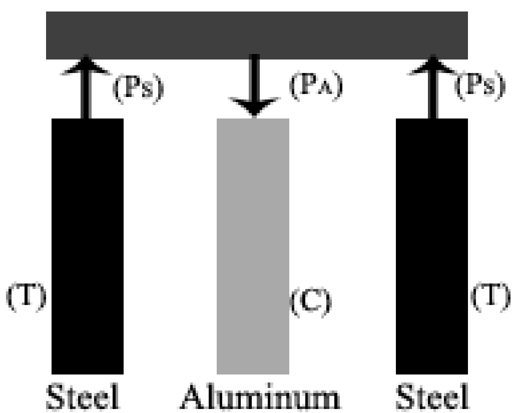
Analysis and Design

General Variables

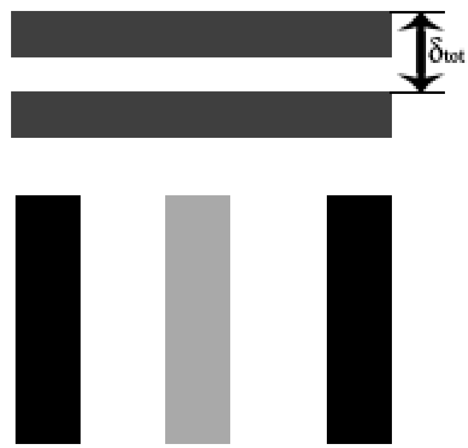
- E = Elastic Modulus
- P = Force
- L = Length
- t = Thickness
- w = Width
- ΔT = Change in Temperature
- α = Coefficient of Thermal Expansion
- A = Area



Relevant Free Body Diagrams



FBD 1



FBD 2

Calculating an Equation for the Force in the Aluminum Strip

Under the assumption that the forces in the system are in equilibrium, we can use free body diagram 1 to write the following equation.

$$\sum P_y = P_A + 2P_S = 0$$

- P_S is a tensile (+) force in both steel strips.
- P_A is a compressive (-) force in the aluminum strip.

If we assume that the plates on the ends of the switch are rigid, we can use free body diagram 2 to write the following equation:

$$\delta_A = \delta_{S1} = \delta_{S2} = \delta_{tot}$$

Using the equation we just wrote for δ, we can substitute for δ using the equation bellow:

$$\delta = {}^{PL}/_{AE} + \alpha\Delta TL$$

After substituting for δ the equation we get is:

$$\delta_A = \delta_S = {}^{P_AL}/_{A_AE_A} + \alpha_A\Delta TL = {}^{P_SL}/_{A_SE_S} + \alpha_S\Delta TL$$

We can then use our force equation to substitute for P_S in the deformation equation.
We first solve for P_S and get:

$$P_S = \frac{-1}{2}P_A$$

Then we substitute for P_S in the deformation equation and get:

$${}^{P_AL}/_{A_AE_A} + \alpha_A\Delta TL = {}^{\frac{-1}{2}P_AL}/_{A_SE_S} + \alpha_S\Delta TL$$

We can then solve for P_A. First we can factor out and cancel L:

$${}^{P_A}/_{A_AE_A} + \alpha_A\Delta T = {}^{\frac{-1}{2}P_A}/_{A_SE_S} + \alpha_S\Delta T$$

Then we solve for P_A and get the following equation:

$$\boldsymbol{P_A = -\Delta T(\alpha_A - \alpha_S)/(\frac{1}{2A_SE_S} + \frac{1}{A_AE_A})}$$

Validating the Mathematical Model

To validate our mathematical model, we used the values given for the initial thermal switch and calculated the force in the Aluminum at $180°F$. Since this is the temperature at which the Aluminum strip buckles, we can compare P_A given by our formula to P_{cr}. P_{cr} is calculated using Euler's formula for buckling columns. The validation of our formula is shown below.

Values of the original thermal switch

- $\alpha_A = 12.5 \times 10^{-6}/°F$
- $\alpha_S = 6.6 \times 10^{-6}/°F$
- $E_A = 10{,}000\ ksi$
- $E_S = 30{,}000\ ksi$

- $t_A = \frac{1}{16} in$
- $t_S = \frac{1}{16} in$
- $w_A = \frac{1}{4} in$
- $w_S = \frac{1}{8} in$
- $L = 4\ in$
- $\Delta T = 180°F$

Plugging in the values we get:

$$124.4\ lb = P_A = -\Delta T(\alpha_A - \alpha_S)/(\frac{1}{2w_S t_S E_S} + \frac{1}{w_A t_A E_A})$$

We then used Euler's formula to calculate the force required for the aluminum strip to buckle:

$$P_{cr} = {}^{4\pi^2 EI}/_{L^2}$$

First we substituted for I with $I = w_a t_a^3/12$ and then calculate:

$$125.5\ lb = P_{cr} = {}^{4\pi^2 E w_a t_a^3}/_{12L^2}$$

When we compare P_A to P_{cr} we get a 0.88% error. This tells us that our mathematical model for the force on the aluminum strip is correct.

$$125.5\ lb \approx 124.4\ lb$$

We now attempt to solve in terms of ΔT; we will plug in the value of P_{cr} for P_A since we care about the change in temperature when the force on the aluminum is at its critical value (hence when it buckles). Doing so, we obtain

$$\Delta T = \left(\frac{-P_{cr}}{(\alpha_A - \alpha_S)}\right)\left[\frac{1}{2w_S t_S E_S} + \frac{1}{w_A t_A E_A}\right]$$

We will stay in terms of w_A and T_A, since that is ultimately what we are interested in finding. We have that:

$$\Delta T = \left(\frac{-4\pi^2 E_A w_A {t_A}^3}{12L^2(\alpha_A - \alpha_S)}\right)\left[\frac{1}{2w_S t_S E_S} + \frac{1}{w_A t_A E_A}\right]$$

$$\Delta T = \left(\frac{-4\pi^2 E_A w_A {t_A}^3}{12L^2(\alpha_A - \alpha_S)}\right)\left[\frac{w_A t_A E_A + 2w_S t_S E_S}{2w_S t_S E_S w_A t_A E_A}\right]$$

$$\Delta T = \left(\frac{-\pi^2 E_A w_A {t_A}^3}{6L^2(\alpha_A - \alpha_S)}\right)\left[\frac{w_A t_A E_A + 2w_S t_S E_S}{w_S t_S E_S w_A t_A E_A}\right]$$

$$\Delta T = \left(\frac{-\pi^2 {t_A}^2}{6L^2(\alpha_A - \alpha_S)}\right)\left[\frac{w_A t_A E_A + 2w_S t_S E_S}{w_S t_S E_S}\right]$$

Here, w_A and t_A are our only variables. We will plug in our desired change in temperature, 100°F, along with the other values we are already given, and attempt to solve for w_A and t_A.

$$\Delta T = 100°\text{F} = \left(\frac{-\pi^2 {t_A}^2}{6(4^2)(59 \times 10^{-7})}\right)\left[\frac{(10000 \times 10^3 w_A t_A + 2(\frac{1}{8})(\frac{1}{16})(30000 \times 10^3)}{(\frac{1}{8})(\frac{1}{16})(30000 \times 10^3)}\right]$$

$$\Delta T = 100°\text{F} = \left(743531.15\frac{°\text{F}}{in^4}\right)(\,w_A {t_A}^3) + \left(34850.3\frac{°\text{F}}{in^2}\right)({t_A}^2)$$

We noted that $w_A > t_A$ so that the central aluminum strip will buckle, and not the steel strips. For simplicity, we will assume the width of the aluminum strip to be twice that of the thickness ($w_A = 2t_A$). Note that this is not necessarily the only value that w_A can take on. There are many other valid solutions, providing that the width is greater than the thickness. Using this substitution, we obtain

$$\Delta T = 100°\text{F} = \left(1487062.3\frac{°\text{F}}{in^4}\right)\,{t_A}^4 + \left(34850.3\frac{°\text{F}}{in^2}\right){t_A}^2$$

Solving this equation requires us to apply the quadratic formula. We have that

$$\Delta T = \left(1487062.3\frac{°\text{F}}{in^4}\right)\,{t_A}^4 + \left(34850.3\frac{°\text{F}}{in^2}\right){t_A}^2 - 100°\text{F} = 0$$

Applying the quadratic formula reveals:

$${t_A}^2 = \left[\frac{-34850.3\frac{°\text{F}}{in^2} \pm \sqrt{(34850.3\frac{°\text{F}}{in^2})^2 - 4(1487062.3\frac{°\text{F}}{in^4})(-100°\text{F})}}{2(1487062.3\frac{°\text{F}}{in^4})}\right]$$

Here we take only the positive root (since that is the only one that makes sense in this situation), and we find that

$$t_A{}^2 = .002584414\ in^2$$

$$\boxed{t_A = .050837136\ in}$$

We substitute this value into our width restriction to ($w_A = 2t_A$) to obtain

$$\boxed{w_A = .101674272\ in}$$

Finally, we can find area using the fact that $A_A = w_A * t_A$.

$$\boxed{A_A = w_A * t_A = .005168829\ in^2}$$

Conclusion

In this design project, our task was to successfully design a cheap thermal switch to activate at 100ºF from 185ºF by modifying the design of a previously existing switch. To do this, we analyzed and reverse-engineered the mathematics behind the thermal switch and were able to compile a mathematical relation between the dimensions of the aluminum strip and how much expansion due to temperature change resulted in buckling. Since the temperature change we desired was lower than the initial model switch, we designed our variant by reducing the cross sectional area of the aluminum strip while keeping the width larger than the thickness to ensure it buckled towards the steel strips. Our hypothesis that smaller dimensions for the aluminum strip would result in activation at a lower temperature change proved to be supported by the calculations. The final dimensions in our design are reinforced by our mathematical model to successfully fit the requirements of this project and produce the activation of the switch at a lower change in temperature.