Term Project

MEMS1041, Mechanical Measurements I
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Contents

List of Parameters

A - cross-sectional area $[m^2]$
b - cross-sectional base $[m]$
$c = h/2$ - maximum distance from neutral axis $[m]$
$[d, \delta]$ - displacement $[m]$
ϵ - strain $[\epsilon]$
E - energy $[J]$
E - Young's Modulus $[Pa]$
F - force $[N]$
f - frequency $[Hz]$
g - gain $[-]$
GF - Gage Factor $[-]$
h - cross-sectional height $[m]$
I - Second Moment of Area $[m^4]$
k - Bride Constant $[-]$
K - Sitffness $[N/m]$
L - length $[m]$
M - moment $[N \cdot m]$
P - load $[N]$
R - resistance $[\Omega]$
σ - stress $[Pa]$
V - velocity $[m/s]$
V - voltage $[V]$
z - height $[m]$

Chapter 1

Abstract

The majority of people check their phone hundreds of times every day. Just think about how many times you send a text, email, make a call, or use an app daily. You quickly realize that phone glass is one of the most important materials we use in daily life. All this interaction creates a unique design challenge that is critical to the function of our phones. In this report, the breakage stress of the Corning Gorilla Glass aftermarket iPhone 6 glass screen is measured using strain gages and simply supported beam theory. The testing apparatus utilized in this experiment uses a full Wheatstone Bridge to measure the strain resulting from simply supported bending of the glass due to a falling mass. The results of our experiment confirm the accuracy of the Young's Modulus provided by Corning and estimates the breakage stress to be close to the expected value. Breakage stress was calculated to be 245 [MPa] which is nearly within the range of expected value of 200 $\pm$ 30 [MPa]. Young's Modulus was calculated to be 68.9 [GPa] and is almost exactly the expected value of 69.3 [GPa]. Calculation of the drop height at the breakage point was the one thing that stood out as flawed during this experiment. The theoretically calculated breakage height was 0.23748 [m] using a 0.066068 [kg] steel ball. However, the experiment showed that three steel balls dropped from a height of 1.905 [m] was necessary to reach the breakage stress.

Uncertainty of the measurement system used in this experiment is 11.4% which seems to be reasonable given the multiple factors that contribute to error in the experiment. Factors that contribute to error in this experiment include inconsistencies in the glass dimensions, error associated with the actual resistance and capacitance values, strain gage uncertainty, and multimeter measurement error, among several others. Test results significantly varied from the theoretically calculated drop height at the breakage stress. It is believed that this inconsistency is a result of the highly dynamic loading conditions present during the drop test. Since the glass is simply supported, it absorbs the impact of the falling mass like a spring and therefore doesn't see the full potential of the impact. Furthermore, the theory assumes that all of the kinetic energy of the falling mass was transferred into the glass screen upon impact, which is likely not the case. For these reasons, the experimental drop height did not agree with the theoretical drop height. A possible change that could correct this would be altering the test apparatus to support the glass across the entirety of its area. Using this apparatus would more closely represent the actual conditions phone glass sees during use. However, the simply supported beam model used in this experiment would no longer be applicable. Hertzian contact would be the proper theoretical model to use under the newly proposed loading conditions. Ultimately, the experimental results detailed in this report show that the simply supported beam testing apparatus accurately measures the properties of Corning Gorilla Glass.

Chapter 2

Theory

2.1 Strain Gage Theory

Strain gages are made of thin metal wires whose resistance changes whenever it is strained. As the wire is strained, its length, L, and cross-sectional area, A, changes, which leads to a change in resistance R given by the formula:

$$R = \frac{\rho L}{A} \tag{2.1}$$

When the wire is stretched, L will increase, A will decrease, and resistance R will increase. Note that the wires resistivity, ρ, will also change when the wire is strained, but will not be taken into account here. As shown below, if the change in resistance can be measured, then strain and stress can be calculated. Taking the derivative of equation (2.1) with respect to each variable (derivation omitted):

$$\frac{dR}{R} = \frac{dL}{L} + \frac{d\rho}{\rho} - \frac{dA}{A} \tag{2.2}$$

From equation (2.2), we can define gage factor, GF, as:

$$GF = \frac{dR/R}{\epsilon} \tag{2.3}$$

Where ϵ is strain in the specified direction. Finally, the relationship between strain and the change in resistance of the wire is given by:

$$\epsilon = \frac{\delta R/R}{GF} \tag{2.4}$$

In this way, the wires in the strain gage on top of a beam will be compressed, inducing a negative strain and thus negative δR. The wires in the strain gage on the bottom of a beam will be stretched, inducing a positive strain and thus positive δR. A similar process is used to determine the strain in our cantilever beam model. If the strain and Young's Modulus, E, of the beam is known, then stress can be solved for using Hooke's Law:

$$\sigma = E\epsilon \tag{2.5}$$

(Section Reproduced from MEMS 1041 Lab #3, via Seth Strayer)

2.2 Beam Theory used for Gage Placement

The phone screen is approximated by a simply supported beam with a central load. The bending stress along the length of the beam is given by:

$$\sigma(x) = \frac{M(x)(h/2)}{I} \tag{2.6}$$

Plugging in for the moment at the center of the glass:

$$\sigma_{max} = \frac{P(L_{bend}/2)(h/2)}{I} \tag{2.7}$$

Using the stress strain relationship, the strain along the length of the beam is given by:

$$\epsilon(x) = \frac{Pxh}{4EI} \tag{2.8}$$

Equation (2.8) calculates the theoretical strain at any location along the length of the phone glass. This is used to decide the placement of strain gages for the experiment.

2.3 Wheatstone Bridge Theory

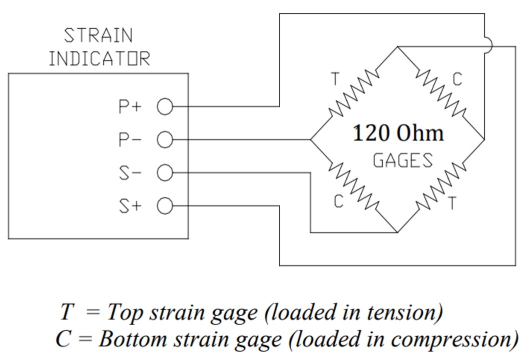

Figure 2.1: Full Wheatstone Bridge Diagram
(Reproduced from ME 1041 Lab #3 handout with permission of the ME Dept., University of Pittsburgh)

In Figure 2.1 the diagram of a full Wheatstone bridge is shown. A full Wheatstone bridge uses a total of four strain gages to detect strain in a body. It is important to note that for a full bridge to operate accurately, all gages must have equal resistances. If each gage has equal resistance, the bridge will be balanced and yield accurate results. Two strain gages are placed on both the top and bottom of the simply supported glass beam to create the desired arrangement. The full bridge is widely considered to be the more accurate than quarter and half bridges because the greater amount of strain gages averages the strain and compensates for temperature affects. Below is the equation relating the bridge output to the measured strain:

$$\frac{\delta E}{E_{out}} = \frac{GF}{4}(\epsilon_{t1} - \epsilon_{c2} + \epsilon_{t3} - \epsilon_{c4}) = GF \cdot \epsilon_{avg} \tag{2.9}$$

(Section Reproduced from MEMS 1041 Lab #3, via Noah Sargent)

2.4 Signal Conditioning Theory

Signal conditioning is necessary to accurately measure the voltage output from a Wheatstone bridge. The circuit in this experiment utilizes a differential amplifier and a low pass filter to yield a signal without high-frequency noise and a large gain. A schematic of this circuit and a description of the theory used to design the differential amplifier and low pass filter used in this experiment is provided below.

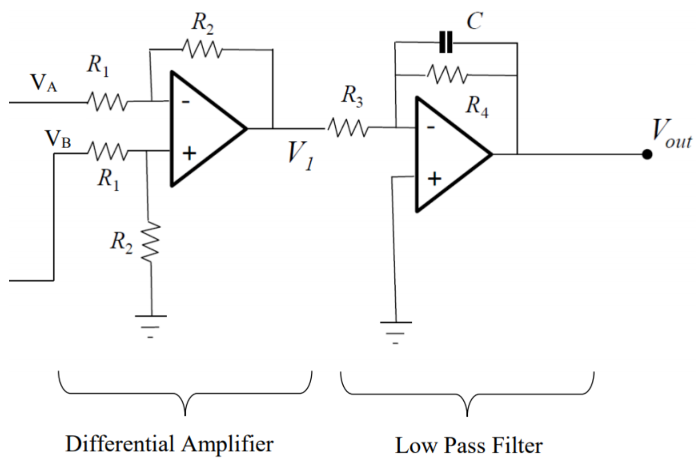

Figure 2.2: Differential Amplifier and Low Pass Filter Layout
(Reproduced from ME 1041 Semester Project - Circuitry Guidelines with permission of the ME Dept., University of Pittsburgh)

A differential amplifier calculates the voltage difference between two signals and amplifies it. In this circuit, the differential amplifier calculates the difference of the output voltages from the Wheatstone bridge according to Equation (2.10) as provided below:

$$\frac{V_1}{g_{amp}} = V_A - V_B \tag{2.10}$$

The calculated voltage difference is then amplified by the ratio of the resistors shown in Equation (2.11):

$$g_{amp} = R_2/R_1 \tag{2.11}$$

The amplified difference of the inputted signal is then passed to the low pass filter.

Low pass filters reduce or eliminate frequencies above the cutoff frequency and amplify the output voltage. The low pass filter in this circuit filters out frequencies above the cutoff frequency which is calculated using Equation (2.12), shown below:

$$f_c = \frac{1}{2\pi C R_4} \tag{2.12}$$

The filter then amplifies the portion of the signal below the cutoff frequency by the ratio of the resistors shown in Equation (2.13) and outputs the signal:

$$g_{filter} = R_4/R_3 \tag{2.13}$$

Lastly, the total gain of the circuit can be calculated by finding the product of the two gains according to Equation (2.14), shown below:

$$g_{total} = g_{amp} \cdot g_{filter} \tag{2.14}$$

Chapter 3

Procedure

3.1 Relating Forces to Strain Gage Output

Consider the model shown in Figure 3.1, shown below. In this orientation, the glass screen can be modeled as a simply-supported beam. The ball drop can be modeled as a point force localized at the center of the beam. The supports will *not* be placed at the edges of the glass to eliminate any possible stress concentrating effects of the holes in the glass. Thus, the bending length of the beam model will be $L_{bend} = 100$ [mm].

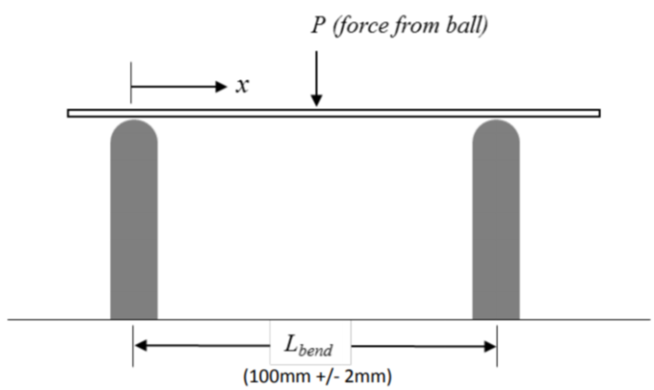

Figure 3.1: Idealized Beam Orientation

Using this setup, the strain at the breakage point, displacement at the center of the glass, and maximum force on the glass due to dropping the steel ball can be calculated. From the Corning Gorilla Glass 3 Spec Sheet, the Elastic Modulus of the glass is $E = 69.3$ [GPa], and the breakage stress is predicted to be $\sigma_{ult} = 200$ [MPa]. Thus, the ultimate strain at the point of breakage is given by:

$$\epsilon_{ult} = \frac{\sigma_{ult}}{E} = \frac{200 \cdot 10^6}{69.3 \cdot 10^9} = 2886\mu\epsilon \tag{3.1}$$

Next, solve for the maximum force on the glass due to dropping the steel ball. This result will be used to calculate the theoretical displacement at the center of the glass. The maximum stress in the beams cross-section is given by:

$$\sigma_{max} = \frac{M_{max}(h/2)}{I} = \frac{(P_{max}/2)(x)(h/2)}{I} \tag{3.2}$$

Let:

$\sigma_{max} = \sigma_{ult} = 200$ [MPa]
$x = L_{bend}/2 = 100/2 = 50$ [mm] $= 50 \cdot 10^{-3}$ [m]
$h = 0.75$ [mm] $= 0.75 \cdot 10^{-3}$ [m]
$b = 64$ [mm] $= 64 \cdot 10^{-3}$ [m]

The second moment of area, I, is given by:

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(64 \cdot 10^{-3})(0.75 \cdot 10^{-3})^3 = 2.25 \cdot 10^{-12}[m^4] \tag{3.3}$$

Substituting this result into equation (3.2) and rearranging, the maximum anticipated force necessary to break the glass, say P_{ult}, is:

$$P_{ult} = \frac{2\sigma_{ult}I}{x(h/2)} = \frac{2(200 \cdot 10^6)(2.25 \cdot 10^{-12})}{(50 \cdot 10^{-3})(0.75 \cdot 10^{-3}/2)} = 48[N] \tag{3.4}$$

This result can be used to solve for the maximum displacement. The maximum deflection of the beam occurs at the center of the glass and is given by:

$$\delta = \frac{P_{ult}L_{bend}^3}{48EI} \tag{3.5}$$

Substituting the values from above, the deflection at the point of breakage is:

$$\delta = \frac{(48)(100 \cdot 10^{-3})^3}{48(69.3 \cdot 10^9)(2.25 \cdot 10^{-12})} = 6.4133[mm] \tag{3.6}$$

Next, solve for the displacement at the center of the glass, say, d, as a function of the ball drop height, z. By doing so, we are able to determine the height at which we expect the glass to break, given our result from equation (3.6). The ball is a 1.0" (25.4 mm) nominal diameter stainless steel sphere. The impact with the glass will cause the glass to bend elastically until the kinetic energy of the ball is stored in the "spring" deflection of the glass (or until the glass breaks). Thus, the glass is modeled as a linear spring with stiffness, K, given by:

$$K = \frac{48EI}{L_{bend}^3} \tag{3.7}$$

The force of the linear spring is a function of the displacement, d:

$$F = Kd \tag{3.8}$$

Its stored energy, when compressed, is given by:

$$E = \frac{1}{2}Kd^2 \tag{3.9}$$

The kinetic energy of the ball when it impacts the glass is equal to the potential energy of the ball before it is dropped. Thus, if the ball is dropped from some height, z:

$$E = \frac{1}{2}mV^2 = mgz \tag{3.10}$$

Combinining equations (3.9) and (3.10), and assuming that the glass "spring" will absorb all of the kinetic energy upon impact:

$$E = \frac{1}{2}Kd^2 = mgz \tag{3.11}$$

The mass of the ball, m, is given by:

$$m = \rho_{ss}V = \rho_{ss}(\frac{4}{3}\pi R^3) \tag{3.12}$$

Substituting values into equation (3.12):

$$m = (7700)(\frac{4}{3}\pi(12.7 \cdot 10^{-3})^3) = 0.066068[kg] \tag{3.13}$$

Then, from equations (3.7) and (3.11):

$$z = \frac{Kd^2}{2mg} = \frac{48EId^2}{2mgL_{bend}^3} = \frac{24EId^2}{mgL_{bend}^3} \tag{3.14}$$

Substituting values, the ball drop height as a function of displacement is given by:

$$z = \frac{24EId^2}{mgL_{bend}^3} = \frac{24(69.3 \cdot 10^9)(2.25 \cdot 10^{-12})d^2}{(100 \cdot 10^{-3})^3(0.066068)(9.81)} = 5773.9d^2 \tag{3.15}$$

Theoretically, the glass should break at the maximum displacement given by the result in (3.6). Thus, the ball drop height necessary to break the glass can be calculated by substituting $d = 6.4133$ [mm] into equation (3.15):

$$z_{break} = 5773.9(6.4133 \cdot 10^{-3})^2 = 0.23748[m] \tag{3.16}$$

3.2 Gage Placement Procedure

To ensure that the strain gages are sensitive enough to detect the strain at the breakage force, the measured strain should exceed $40\mu\epsilon$ while also being below $3000\mu\epsilon$. The decision to place our strain gages 25 [mm] away from the central impact location was made by using the estimated breakage force and equation (2.8). We have:

$$\epsilon(x) = \frac{(48)(0.050 - 0.025)(0.75 \cdot 10^{-3})}{4(69.3 \cdot 10^{9})(2.25 \cdot 10^{-12})} = 1443[\mu\epsilon] \tag{3.17}$$

The predicted theoretical strain of $1443[\mu\epsilon]$ at 25 [mm] from the impact location is within the sensitivity range of the strain gage and far enough away from the impact location to avoid contact with the steel ball. Thus, 25 [mm] away from the center of the glass is an acceptable location for strain gage placement.

To compensate for temperature differences resulting from electrical heat generation and achieve an average strain value a full bridge arrangement is used. Four gages are placed on the top and bottom of the glass 25 [mm] away from the impact location in both directions. The strain gages were placed along the centerline of the glass to limit the effects caused by three dimensional bending. A diagram of the full bridge setup is shown below in Figure 3.2.

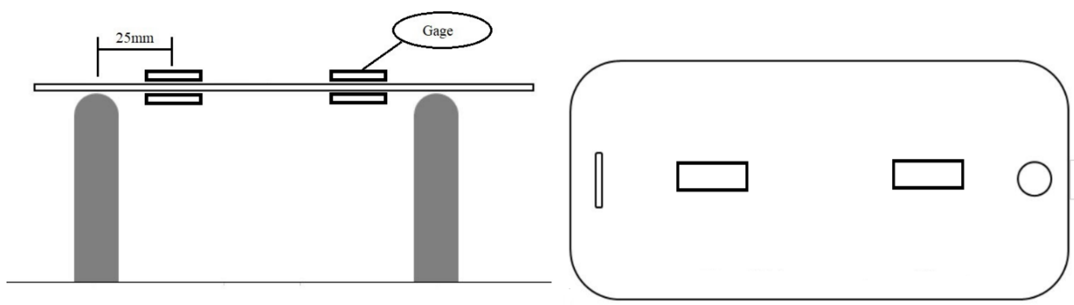

Figure 3.2: Gage Placement Diagram

3.3 Mounting and Wiring of Strain Gages

Achieving a proper bond between the glass and the strain gage is critical to achieving an accurate strain measurement. First, a neutralizer is applied to the glass to ensure the application area is clean. Second, the gage is placed on a piece of scotch tape which is then attached to the screen such that someone can apply a catalyst to the bottom. Third, a catalyst is applied to the bottom of the strain gage and allowed to dry for aproximately one minute. Finally, the adhesive is applied to the application area and the strain gage is pressed to the glass and held until the bond is secure. Careful consideration must be given to the location of the application area so that it closely matches the designed gage placement described previously.

The strain gage leads must be soldered to wire extensions so that the full bridge can be wired into the breadboard circuit. Strain gages in tension must be wired opposite each other in the full bridge to create the adding effect shown in Equation (2.9) and strain gages in compression must also be wired opposite each other. In this wiring arrangement, the signs of the resistance changes result in a bridge constant, k, of 4, temperature effects cancel out, and the average of the four strain gage outputs is measured, as shown in Equation (2.9).

3.4 Amplify & Condition Signal from Strain Gages

Signal amplification and conditioning are necessary to yield a measurable signal that is free of high-frequency noise. In Theory Section 2.4, the design calculations for a circuit using a differential amplifier and low pass filter combination to achieve the proper amplification and signal conditioning was laid out. Through mixing and matching the available resistors and capacitors it is possible to design a circuit that fits the following criteria. First, the gain of the low pass filter and differential amplifier should be similar in value. Second, the product of the two gains should be around 200 to ensure the signal is large enough to measure. Lastly, high-frequency noise must be eliminated by setting the cutoff frequency close to 100 [Hz]. Using the multimeter to measure the real values of the selected resistors and capacitors, it is possible to calculate the overall gain and cutoff frequency of the circuit. The design calculations for the circuit used in this experiment are shown below:

Measured values:

$R_1 = 3.266[k\Omega]$
$R_2 = 47.27[k\Omega]$
$R_3 = 3.263[k\Omega]$
$R_4 = 47.25[k\Omega]$
$C = 43.30[nF]$

From Equation (2.11):

$$g_{amp} = R_2/R_1 = \frac{47.27}{3.266} = 14.47$$

From Equation (2.12):

$$f_c = \frac{1}{2\pi C R_4} = \frac{1}{2\pi(47.25 \cdot 10^3)(43.30 \cdot 10^{-9})} = 77.8[Hz]$$

From Equation (2.13):

$$g_{filter} = R_4/R_3 = \frac{47.25}{3.266} = 14.60$$

From Equation (2.14):

$$g_{total} = g_{amp} \cdot g_{filter} = 14.47 \cdot 14.60 = 214.6$$

3.5 Calibrate Sensor & Calculate Actual Young's Modulus

Calibration using known static loads was conducted on the test apparatus prior to testing. Weights of 200 [g], 400 [g], and 600 [g] were used to simulate loads during the drop test in order to calculate the actual Young's Modulus of the glass. This calibration step is necessary because the loads during the ball drop test are unknown and highly dynamic, thus static testing must be performed to calculate the actual Young's Modulus. From Equation (2.9), the strain can be calculated from the output voltage:

$$\epsilon_{avg} = \frac{\delta E}{E_{in}} \cdot \frac{1}{TotalGain \cdot GF}$$

From Equation (2.7):

$$\sigma = \frac{P(L_{bend}/2)(h/2)}{I}$$

Using the measured strain and theoretical stress, the Young's Modulus can be calculated using Equation (2.5).

3.6 Create Data Acquisition Program

A screenshot of the MATLAB script used to record and display the sensor output is shown in Figure 6.2, page 22, Section 6. The basic functionality of the code is to establish connection with the data acquisition (DAQ) unit, establish an input channel, and aquire its data. For this script, a sampling frequency of $f_s = 1000[Hz]$ was used over a two second time period. Thus, the total number of captured data points is given by:

$$N = f_s \cdot t = (1000)(2) = 2000 \tag{3.18}$$

Note that while calibrating the sensor, a 15 second time period was used, as to allow time to place the weights on the glass screen and record the measurements. The voltage range was set according to which trial was being run. For instance, with smaller loads applied, there is a smaller voltage output and thus a smaller voltage range was used; for larger loads (increased ball drop height), higher voltage outputs were expected, so a larger voltage range was used. This ensured that all of the voltage outputs were being recorded by the program.

Not every line of code will be explained here, but in essence, the script acquires all of the data from the DAQ unit and proceeds to plot the voltage as a function of time. Note that in order to plot strain vs. time, which will be done in Results Section 4.2, the "Brush/ Select Data" capability in MATLAB was used. This allowed us to choose a specified section of data from the plots, extract the data, and take it into Microsoft Excel to calculate strain at each point based on the voltage output.

3.7 Breakage Force

The breakage force was easily calculated after we obtained one, the average Young's Modulus based on our calibration trial (results for this trial shown in Section 4.1, and two, the breakage strain, which is calculated in Section 4.2. Given these results, the breakage stress is:

$$\sigma_{break} = \epsilon_{break} E_{avg} \tag{3.19}$$

Relating to moment by the bending equation:

$$\sigma_{break} = \frac{M_{break} c}{I} \tag{3.20}$$

Such that:

$$M_{break} = \frac{\sigma_{break} I}{c} \tag{3.21}$$

Force can be related to moment by the following:

$$P_{break} = \frac{M}{x} \tag{3.22}$$

Thus, not only the breakage force, but the force for any trial during the ball drop test, can be calculated. Note that by definition of elementary mechanics, the force P is the reaction force experience at either support of the beam. Thus, the actual force submitted by the ball drop will be twice that as calculated above.

These force values were verified by making a plot of Force vs. Stress, based on our experimental ball drop data. This plot is shown in Figure 6.1, page 21, Section 6. By plotting a line of best fit, the anticipated force for any value of stress can be calculated. Thus, the process can essentially be "reverse engineered". Indeed, by plugging in a value of $\sigma = 200[MPa]$, we receive a value of $P = 48[N]$, as anticipated in Section 3.1. These calculations are not directly stated, but they can be easily verified by the graph.

3.8 Uncertainty Analysis

Uncertainty analysis is a tool used to estimate the measurement error associated with a measurement system. During this experiment, multiple factors such as the error associated with electrical components, dimensional error in the glass, differences in the gage factor, and multimeter measurement error introduce uncertainty into the system. All these sources of error propagate through the measuring system. It is possible to estimate the combined uncertainty of the voltage output from the measurement system in this experiment by calculating the square root of the sum of squared uncertainties. The calculated uncertainty of the voltage output from the entire measurement system is $\pm 11.4\%$. A detailed calculation of the combined uncertainty is shown in the appendix section of this report (Section 6).

Chapter 4

Results

4.1 Calibration

A plot of stress vs. time for the calibration portion of the experiment is shown in Figure 4.1.

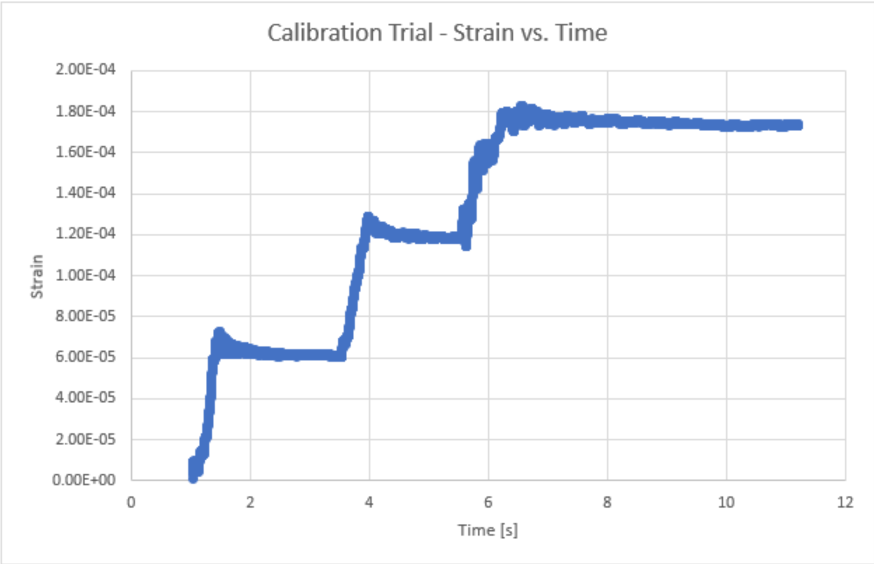

Figure 4.1: Calibration Trial - Strain vs. Time

Based on the equations provided in Section 3.5, Sensor Calibration, the actual Young's Modulus at each weight value can be calculated. These results are summarized in Table 1.
The average Young's Modulus based on all three calibration points is calculated to be:

$$E_{avg} = 6.89 \cdot 10^{10} = 68.9[GPa] \tag{4.1}$$

The result given by (4.1) is within 0.60 percent of the theoretical Young's Modulus of $E = 69.3[GPa]$. This result is used to calculate the experimental breakage stress, as shown in Section 4.3.

Weight [kg]	F [N]	Stress [Pa]	Volt [V]	Microstrain	Expected Microstrain	Young's Modulus [Pa]	Percent Error
0	0	0.00E+00	0.2015	---	---	---	---
0.2	1.962	4.09E+06	0.4765	61.05	58.98	6.70E+10	3.38
0.4	3.924	8.18E+06	0.7373	118.94	117.97	6.87E+10	0.82
0.6	5.886	1.23E+07	0.9798	172.78	176.95	7.10E+10	2.42
				Average Young's Modulus:		6.89E+10	0.60

Table 1 - Calibration Trial Results

4.2 Ball Drop Results

Since many more trials had to be run than what was expected, only three ball drop plots are shown in the report. Particularly, Trial 1 - 30 in. ball drop height with 1 ball, Trial 5 - 50 in. ball drop height with 2 balls, and Trial 7 - 75 in. ball drop height with 3 balls. Note that Trial 7 corresponded to the final breakage drop.

A plot of strain vs. time for Trial 1 is shown in Figure 4.2.

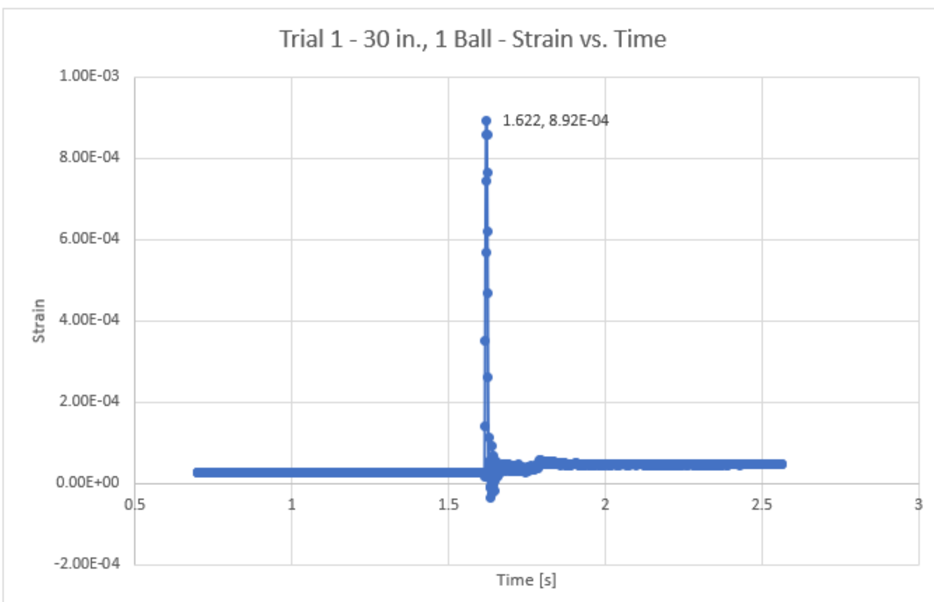

Figure 4.2: Trial 1 - Strain vs. Time

The maximum strain for this trial is:

$$\epsilon_1 = 8.92 \cdot 10^{-4} = 892\mu\epsilon \tag{4.2}$$

A plot of stain vs. time for Trial 5 is shown in Figure 4.3.
The maximum strain for this trial is

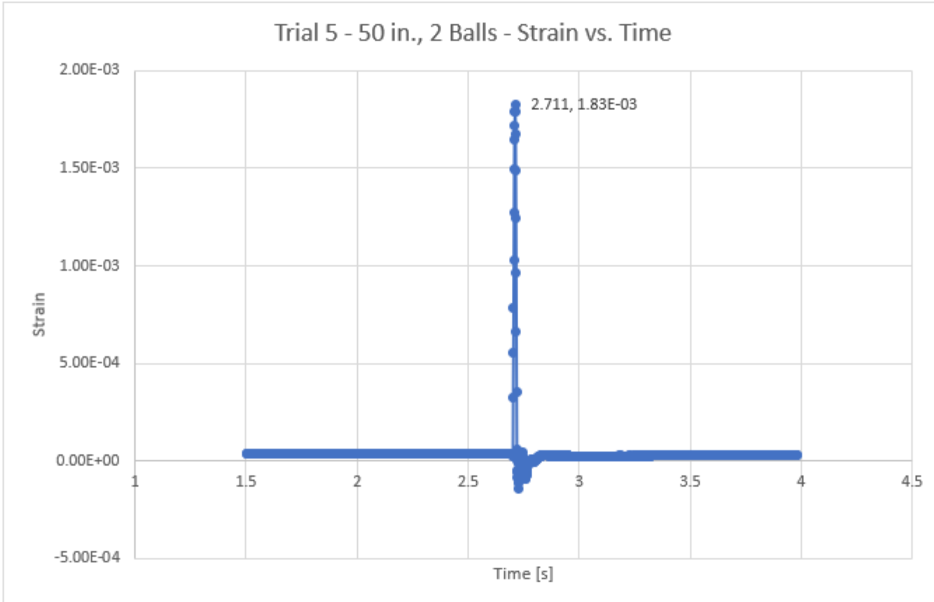

Figure 4.3: Trial 5 - Strain vs. Time

$$\epsilon_5 = 1.83 \cdot 10^{-3} = 1830\mu\epsilon \tag{4.3}$$

A plot of stain vs. time for Trial 7, the final breakage drop, is shown in Figure 4.4, page 16. The maximum breakage strain is gven by:

$$\epsilon_{break} = \epsilon_7 = 3.56 \cdot 10^{-3} = 3560\mu\epsilon \tag{4.4}$$

A table showing the results of all of the trials, including stress and force values, can be found in Table 2.

4.3 Breakage Stress

With our measured breakage strain given in (4.4), along with our calculated Young's Modulus as in (4.1), the breakage stress can be calculated using Hooke's Law:

$$\sigma_{break} = \epsilon_{break} E_{avg} = (3.56 \cdot 10^{-3})(68.9 \cdot 10^9)$$

$$\sigma_{break} \approx 245[MPa] \tag{4.5}$$

Given that the theoretical breakage strain was $\sigma_{ult} = 200[MPa] \pm 30[MPa]$, we can use the nominal value to calculate the percent error in our breakage stress:

$$Error_\sigma = (\frac{245 - 200}{200}) \cdot 100 = 22.5\% \tag{4.6}$$

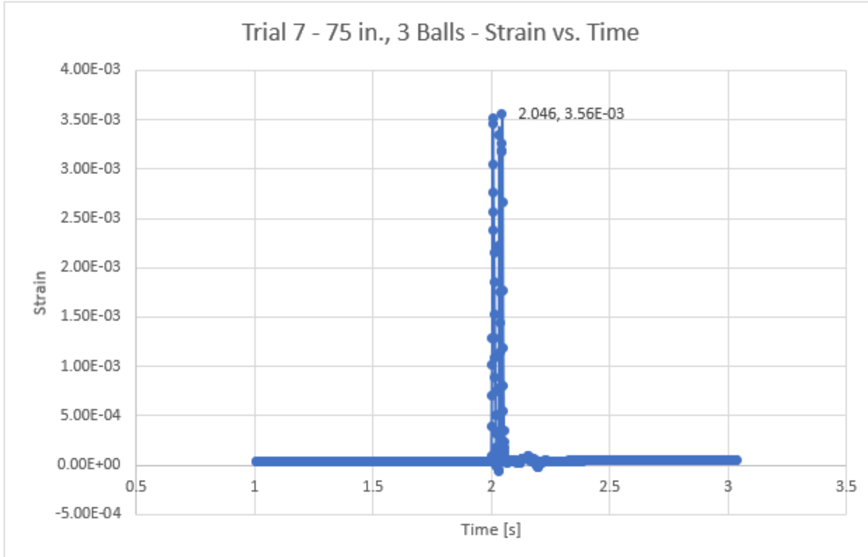

Figure 4.4: Trial 7 - Strain vs. Time

h [in]	h [m]	Voltage [V]	Strain	Stress [Pa]	Stress [MPa]	P [N]
30	0.762	4.219	8.92E-04	6.14E+07	61.44	14.745
50	1.2	5.289	1.13E-03	7.78E+07	77.80	18.672
70	1.778	6.288	1.35E-03	9.31E+07	93.08	22.338
30	**0.762	6.206	1.33E-03	9.18E+07	91.82	22.037
50	**1.2	8.448	1.83E-03	1.26E+08	126.11	30.266
70	**1.778	10.48	2.28E-03	1.57E+08	157.18	37.723
75	***1.905	16.23	3.56E-03	2.45E+08	245.11	58.827
***Represents two balls used | ***Represents three balls used*						

Table 2 - Trial Results

A picture of the broken glass screen assembly is shown in Figure 6.3, page 23, Section 6.

4.4 Breakage Height

It has become obvious that the actual breakage height is much larger than the expected breakage height, however the actual breakage stress is still somewhat within the range of the theoretical breakage stress. We can conclude that the glass "spring" is not absorbing all of the kinetic energy from the ball, and thus there is a lower force associated with each ball height. In particular, from (3.16), and the result given from Trial 7 (4.4), the percent difference in breakage height values is:

$$Error_{height} = \left(\frac{1.905 - 0.23748}{0.23748}\right) \cdot 100 \approx 702\% \tag{4.7}$$

Chapter 5

Discussion

5.1 Comparison to Theory

We wish to discuss how the expected strain due to known forces differs from the actual strain obtained from the experiment. This was partially discussed in the Calibration Results Section (4.1). The expected strain due to known forces was calculated using the stress values obtained from such known forces and the reported elastic modulus of the glass, namely, $E = 69.3[GPa]$. The actual strain obtained from measuerments was calculated by means of Equation 2.9, which relates voltage output, total gain, and the gage factor to the calculated strain. The results for these calibration trials are shown in Table 1. We note that there is very little error between any two values. In particular, by using the calculated strain values, we calculate an average Young's Modulus of

$$E_{avg} = 68.9[GPa]$$

which is within 0.60 % of the reported Young's Modulus value. Therefore, the strain gage system was very accurate in obtaining strain measurements.

5.2 Comparison of Breakage Stress

From the Breakage Stress Results Section (4.3), the calculated breakage stress is

$$\sigma_{break} \approx 245[MPa]$$

which maintains a 22.5% error with the theoretical breakage stress, namely $\sigma_{ult} = 200$[MPa]. Although this error is not negligable, if we consider the amount of uncertainty in our measurements, the results are fairly accurate.

Furthermore, when calculating the percent error in the stress, given by Equation (4.6), we used the nominal value for theoretical stress. If the upper bound was used, we would calculate a percent error of:

$$Error_{\sigma} = (\frac{245 - 230}{230}) \cdot 100 = 6.52\% \tag{5.1}$$

This result should be very inspiring, given all of the uncertainties in the measurements (which is rather plentiful for this large experiment). When taking into account the ± 11.4% uncertainty calculated for our measuring system, the 6.25% error associated with experimental and expected breakage stress is well within the expected range. From this we can confidently say that our calculation for uncertainty is in agreement with our experimental results when compared to the expected value for breakage stress.

We also wish to touch on the fact that our calculated breakage height was much higher than the anticapated breakage height. This was explained earlier in the report (Section 4.4) by the fact that the glass "spring" is not absorbing all of the kinetic energy from the ball, and thus there is a lower force associated with each ball height.

On another note, the experimental error calculated during this experiment is exceptional. We were able to predict the Young's Modulus within 1% error of the expected value and experimental breakage stress was within the calculated uncertainty. This clearly shows that the full bridge arrangement and circuit design used in this experiment was able to accurately determine the strain in the glass. Based off of the varied results of other groups, we believe that full bridge is necessary to capture the highly dynamic loading conditions during the drop test. If future experiments are done, the use of a full bridge should be required and special care should be taken to ensure the gages are located far enough away from the impact area to prevent unwanted collisions.

As previously stated, a potential improvement to this measurement system would be supporting the entire area of the phone glass. In this setup, simply supported beam theory would be replaced with Hertzian contact as the theoretical model. This change would more closely replicate the real conditions of an actual phone. Also, the breakage height would be easier to calculate given that phone glass is rigid and thus, will take the full brunt of the impact.

Overall, we are satisfied with our results. We have created a measurement system from scratch and been able to validate its results by applying the concepts of strain gage theory, beam theory, wheatstone bridge theory, signal conditioning theory, data acquisition, and uncertainty analysis. In conclusion, the aftermarket iPhone 6 glass screen bodes well as a replacement to that of an original screen, given the calculated breakage stress of this measurement system.

Chapter 6

Appendix

6.1 Uncertainty Analysis

Consider the strain equation given by Equation (2.8). We can manipulate this equation to derive the uncertainty in our beam strain measurement. We have that:

$$\epsilon = \frac{3Px}{Ebh^2} \tag{6.1}$$

Then the uncertainty in the beam strain measurement is given by:

$$\frac{u_\epsilon}{\epsilon} = [(\frac{u_x}{x})^2 + (\frac{u_P}{P})^2 + (\frac{u_E}{E})^2 + (\frac{u_b}{b})^2 + (2\frac{u_h}{h})^2]]^{.5}$$

We will assume zero uncertainty in the load, P, and Young's Modulus, E, since these are known quantities. Then

$$\frac{u_\epsilon}{\epsilon} = [(\frac{u_x}{x})^2 + (\frac{u_b}{b})^2 + (2\frac{u_h}{h})^2]]^{.5} \tag{6.2}$$

Equation (6.2) represents the uncertainty in our beam strain measurement. Next consider the uncertainty in the strain gages themselves. Since

$$\delta R = GF\epsilon R_0 \tag{6.3}$$

The strain gage uncertainty is given by

$$\frac{u_{\delta R}}{\delta R} = [(\frac{u_{GF}}{GF})^2 + (\frac{u_\epsilon}{\epsilon})^2 + (\frac{R_0}{R_0})^2]]^{.5} \tag{6.4}$$

Equation (6.4) represents the uncertainty in our strain gages. Next consider the uncertainty in our Wheatstone Bridge. We have that

$$E_0 = E_i \frac{k}{4} \frac{\delta R}{R_0} \tag{6.5}$$

where the bridge constant, k, is equal to 4 for our full-bridge setup. If we assume zero uncertainty for this value, we have that

$$\frac{u_{E_0}}{E_0} = [(\frac{u_{E_i}}{E_i})^2 + (\frac{u_{\delta R}}{\delta R})^2 + (\frac{R_0}{R_0})^2]]^{.5} \tag{6.6}$$

Equation (6.6) represents the uncertainty in our Wheatstone Bridge. Next conisder the uncertainty in our differential amplifier circuit. We have that

$$V_1 = \frac{R_2}{R_1}(E_{01} - E_{02}) \tag{6.7}$$

Then the uncertainty in our differential amplifier circuit is given by

$$\frac{u_{V_1}}{V_1} = [(\frac{u_{R_2}}{R_2})^2 + (\frac{u_{R_1}}{R_1})^2 + (\frac{E_0}{E_0})^2]]^{.5} \tag{6.8}$$

Finally, consider the uncertainty in the low pass filter. We have that

$$V_{out} = \frac{R_4}{R_3}V_1 \tag{6.9}$$

Then the uncertainty in the low pass filter is given by

$$\frac{u_{V_{out}}}{V_{out}} = [(\frac{u_{R_3}}{R_3})^2 + (\frac{u_{R_4}}{R_4})^2 + (\frac{V_1}{V_1})^2]]^{.5} \tag{6.10}$$

Using Equations (6.2), (6.4), (6.6), (6.8), (6.10), we can calculate the uncertainty at breakage. The uncertainty values are given by the following:

$R_1 = R_3 = 3.3[k\Omega] \pm 5\%$
$R_2 = R_4 = 47[k\Omega] \pm 5\%$
$GF = 2.105 \pm 0.5\%$
$R_0 = 120 \pm 0.3\%$
$E_i = 10 \pm 0.3\%$
$h = 0.75 \pm 0.02[mm]$
$b = 64 \pm 0.5[mm]$
$x = 100 \pm 0.5[mm]$

Then from (6.2),

$$\frac{u_\epsilon}{\epsilon} = [(\frac{0.5}{100})^2 + (\frac{0.5}{64})^2 + (2\frac{.02}{0.75})^2]]^{.5} = 5.41\% \tag{6.11}$$

Plugging this result into equation (6.4):

$$\frac{u_{\delta R}}{\delta R} = [(0.5)^2 + (5.41)^2 + (0.3)^2]]^{.5} = 5.44\% \tag{6.12}$$

Plugging this result into equation (6.6):

$$\frac{u_{E_0}}{E_0} = [(0.3)^2 + (5.44)^2 + (0.3)^2]]^{.5} = 5.46\% \tag{6.13}$$

Plugging this result into equation (6.8):

$$\frac{u_{V_1}}{V_1} = [(5)^2 + (5)^2 + (5.46)^2]]^{.5} = 8.93\% \tag{6.14}$$

Plugging this result into equation (6.10):

$$\frac{u_{V_{out}}}{V_{out}} = [(5)^2 + (5)^2 + (8.93)^2]]^{.5} = 11.4\% \tag{6.15}$$

6.2 Additional Figures

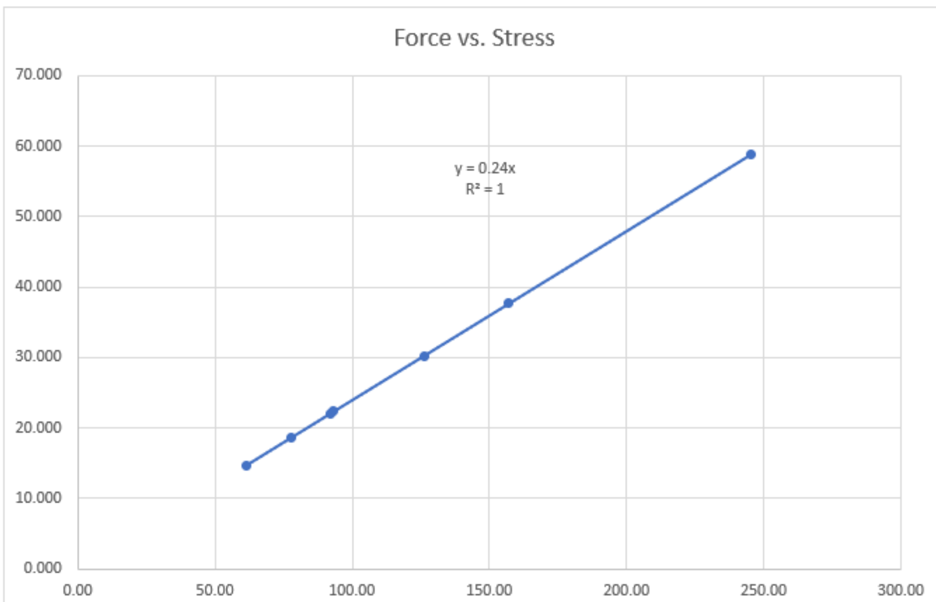

Figure 6.1: Force vs. Stress

```
%Seth Strayer, Noah Sargent
%11/29/18
%MEMS1041
%Term Project Data Acquisition
%This code is used to acquire data from a DAQ unit (NI USB-6008) that
%connects via USB to the computer.  The specifications for this device are
%up to 8 seperate input channels, up to 12 kHz sampling rate, and 12 bit
%resolution per sample.  The code will establish connection with the DAQ
%unit, establish an input channel, and acquire it's data.

%clearing workspace
clear
clc

%setting parameters
fs = 1000;
t = 2;
N = 2000;

%finding and setting up device
daq.getDevices
s = daq.createSession('ni')
[ch, idx] = s.addAnalogInputChannel('dev1', 'ai0', 'Voltage')

%setting device parameters
s.Rate = fs;
% s.DurationInSeconds = t;
s.NumberOfScans = N;

%setting channel range - voltage output will be in the range [-1,1]
ch(1).Range = [-5, 5];

%acquiring data
s.NotifyWhenDataAvailableExceeds = 40;
listen = s.addlistener('DataAvailable', @(s, event) plot(event.TimeStamps, event.Data));
[data, time] = s.startForeground();
% plot(time, data);

%saving and clearing data
save();
% delete(s);
% clear s;
```

Figure 6.2: Data Aquisition Script

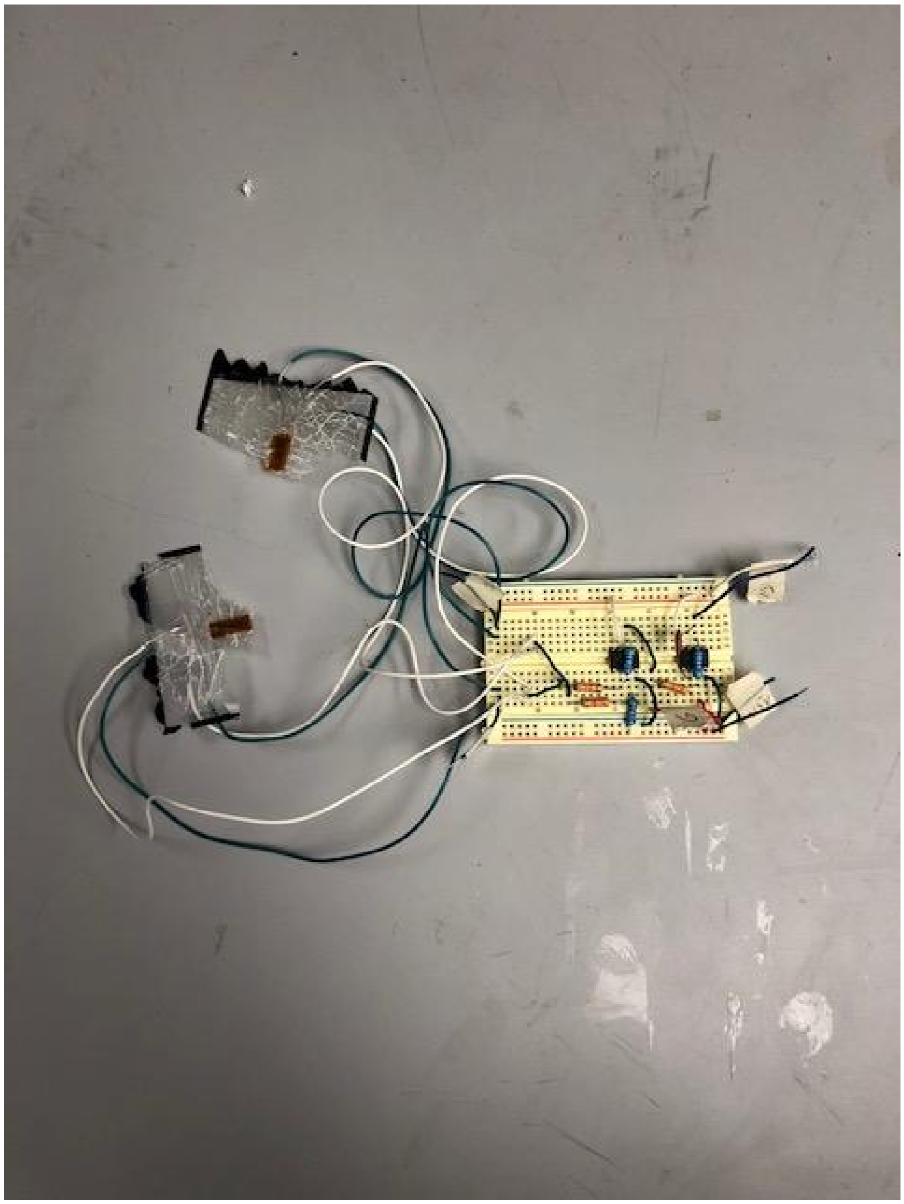

Figure 6.3: Broken Glass Screen Assembly

List of Figures