<u>Design Project 2</u>

“Sizing of a Bicycle Chain Ring Bolt Set”

ENGR 0135

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Abstract

This report will analyze our calculations, data, and analysis used to redesign the design parameter ‘d’ for a competitive bicycle crank arm assembly. Our group determined at least two combinations of the radius of the bolts ‘d’ and the chain ring bolt options given in the table shown below by using known equations, a factor of safety value of 3, and given data. Assuming that the allowable shear stress must be less than the maximum shearing stress and given a factor of safety value of 3, our group was able to formulate an inequality equation for the cross-sectional area. In addition, we could safely assume that the total moment of the system is equivalent to five times the moment at one bolt. Subsequently, we were able to create an equation for force that could be used in the first inequality equation to determine design parameters for each option. Also, we had to calculate the bearing stress in the chain ring arm and the factor of safety value with respect to bearing failure. Using mathematical analysis, known equations, and the identified fact that bearing stress multiplied by factor of safety is equivalent to allowable shearing stress, our group was able to determine the factor of safety for the options. This report will document the process our group used to redesign the parameter ‘d’ and determine bearing stresses and factor of safeties for each option.

Option	*Diameter*	*Effective Area*
1	0.2 in	0.052 in^2
2	0.2 in	0.037 in^2
3	0.2 in	0.028 in^2

Introduction

The goal of this design project is to compute the most optimal design of a crank arm assembly by finding the most optimal sizing of the chain ring bolts and their distance from the central crank bearing. The assembly must be able to withstand an applied force of 300 pounds from the pedal which is set a fixed position L from the central axis. The moment must then be distributed to five bolts located a distance d between 0.8 and 1.5 inches away from the central axis, as shown in Figure 1 and 2.

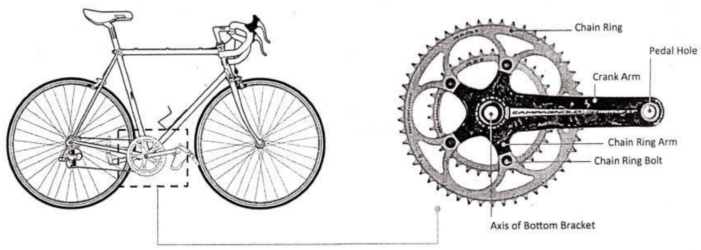


Figure 1. Crank Arm Assembly

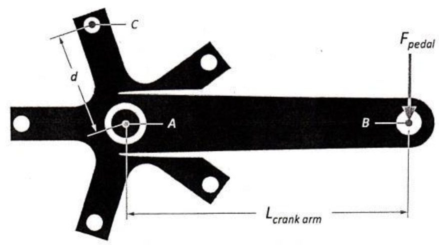


Figure 2. Idealized Crank Arm

The bicyclist will push down on the pedal at the pedal hole, resulting in the axis of the bottom bracket to rotate. As shown in Figure 3, the axis of the bottom bracket is connected to the chain ring with chain ring bolts.

Figure 3. Typical Chain Ring Bolt

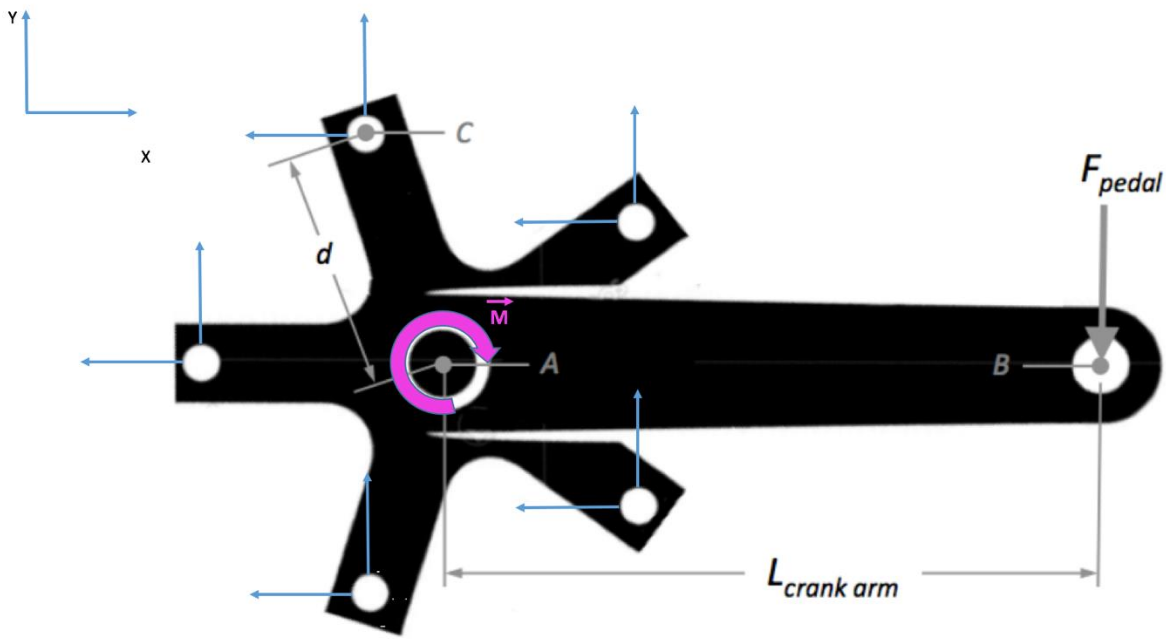
These bolts will undergo a shear stress which must be calculated in order to maintain a safety factor of 3. The main purpose of our design is to choose which options out of the three bolt sizes are possible given constraints on distance ‘d’ and then calculate the bearing stress at each bolt and estimate the factor of safety with respect to bearing failure. We hypothesize that the greater the effective area of the bolt is, the closer the bolt will have to be to the axis of the bottom bracket.

Analysis and Design

We start by noting some of the variables we were given:

$F_{pedal} = 300\ lb$, the force applied at the pedal
$L_{crank\ arm} = 8\ in$, the length of the crank arm
$\theta_{chain\ ring\ arms} = 72°$, the angle between the chain ring arms
$0.8" \leq d \leq 1.5"$, the radial distance from the bottom bracket axis to chain ring bolts
$t = 0.4\ in$, the thickness of the chain ring arm
$\tau_{max} = 35{,}000\ psi$, the maximum allowable shear stress of the chain ring bolt
$\sigma_{max} = 5{,}000\ psi$, the maximum allowable bearing stress of the crank arm

Relevant Free Body Diagrams



FBD 1

Calculations

We know that the equation for shearing stress is:

$$\tau_S = \frac{P}{A} \quad (1)$$

where P is the shearing force applied at the ring bolts and A is the cross sectional area of the bolts. To avoid failure of the system, we know that the shearing stress applied at the ring bolts must be less than or equal to the maximum allowable shearing stress of the ring bolt, τ_{max}:

$$\tau_S = \frac{P}{A} \leq \tau_{max} \quad (2)$$

Under our Design Objective 1, we wish to determine at least two combinations of the design parameter "d" and chain ring bolt, using a factor of safety against shear failure of 3. Substituting this in to (2), we have:

$$\tau_S = \frac{P}{A}\,(FS)$$

$$A \geq \frac{3P}{\tau_S} \quad (3)$$

We have found an equation for the minimum cross sectional area of the bolts. We now wish to find P, the shearing force applied at the ring bolts. Using the knowledge we have about moments, we can easily accomplish this. We have that the moment about point A, the center of the bottom bracket axis, is:

$$M_A = F_{pedal} \times L_{crank\ arm} = (300lb)(8in) = 2400(lb \cdot in) \circlearrowright$$

Since we know that the system must be at equilibrium, this moment must be offset by the moment of each of the arms about the center of the bottom bracket axis. Although the crank arms have both an x-direction force and a y-direction force, we can substitute these component forces with P, since the force we are trying to find is the single shear force. Since there are 5 crank arms, we assume:

$$M_A = 5M_C = 5Pd$$

Where M_C is the moment due to the crank arms, P is the shearing force applied at the ring bolts, and d is the radial distance from the bottom bracket axis to chain ring bolts, as previously defined. By recalling equation 3, we can obtain:

$$P = \frac{2400(lb \cdot in)}{5d} \quad (4)$$

$$A \geq \frac{7200(lb \cdot in)}{5d\tau_S}$$

Since we know that the maximum allowable shear stress of the chain ring bolt is $\tau_{max} = 35{,}000\ psi$, we have that:

$$A \geq \frac{7200(lb \cdot in)}{5(35000psi)d}$$

$$A = \frac{0.041143\ in^3}{d} \quad (5)$$

$$d = \frac{0.041143\ in^3}{A}$$

Since we know have an equation in terms of the distance d, we can analyze our Design Objective 1. Using the following table, we will plug in the different values of A to solve for d.

Option	*Effective Area*	d
1	0.052 in^2	0.79121 in
2	0.037 in^2	1.11197 in
3	0.028 in^2	1.46939 in

Design Objective 1: After applying the factor of safety against shear failure of 3, and applying the constrain that 0.8" $\leq d \leq$ 1.5", we obtain two combinations of the design parameter d.

Option	*Effective Area*	d
2	0.037 in^2	1.11197 in
3	0.028 in^2	1.46939 in

For Design Objective 2, we wish to determine the bearing stress in the chain ring arm of the crank and estimate the factor of safety with respect to bearing failure. We know that the equation for bearing stress is:

$$\sigma_{bearing} = \frac{P}{A_{bearing}} \quad (6)$$

where P is still the shearing force applied at the ring bolts, but $A_{bearing}$ is now the cross sectional area bearing the stress. By definition, we have that,

$$A_{bearing} = Dt$$

where D is the diameter of the chain ring bolt and t is the thickness of the chain ring arm. We will combine equations (4), (5), and (6), we obtain:

$$\sigma_{bearing} = \frac{2400(lb \cdot in)}{5dDt}$$

Since $d, D,$ and t are all given (see table), we can find the bearing stress of the respective arms to be:

Option	d	D	t	$\sigma_{bearing}$
2	1.11197 in	0.2 in	0.4 in	5395.83 psi
3	1.46939 in	0.2 in	0.4 in	4083.33 psi

Our last step is to find a relationship between the maximum allowable bearing stress and the factor of safety. By recalling equation (2) and utilizing equation (6), we find that:

$$\sigma_{bearing}(FS) = \sigma_{max}$$

For this case, we have that the maximum allowable bearing stress of the crank arm alloy is 5,000 psi. We can use this equation to estimate the factor of safety with respect to bearing failure:

Option	$\sigma_{bearing}$	*Factor of Safety*
2	5395.83 psi	.926641
3	4083.33 psi	1.22449

Discussion

The first goal of our design project was to determine two viable distances of ‘d’ using a safety factor of 3 for the allowable shear stress on the chain ring bolt. We hypothesized that the bigger the effective area of the bolt was, than the closer to the axis of the bottom bracket it would have to be. We were given a range of d values between $0.8'' \leq d \leq 1.5''$. In order to determine if the distance ‘d’ for each of the chain ring bolts was within our value range, we had to make an assumption about how the pedal force was distributed. First we stated that the system was in equilibrium and set the net moment equal to zero. Then we knew that the moment due to the pedal force about point A must be offset by the moments at each chain ring arm. Using this information, we made the assumption that force offsetting the moment at each of the bolts was equal. From this assumption we were able to calculate that option 2 and 3 had values of ‘d’ within our design requirements. We also proved our hypothesis was right by showing that the effective area of b was inversely proportional to ‘d’. Our next goal was to determine the bearing stress in each chain ring arm and determine the factor of safety with respect to failure. With the information given to us about the chain ring arm thickness of 0.4 inches, we were able to calculate the shear bearing stress for options 2 and 3. Then using the allowable bearing stress of 5,000psi for the crank arm alloy, we determined a factor of safety for each bolt.

Conclusion

Through mathematical analysis, we successfully modeled a formula for calculating possible options for different chain ring bolt sizes and their corresponding distances from the pivot. The cross sectional area of each bolt was inversely proportional to the distance ‘d’ from the axis of the bottom bracket. This mathematical relationship proved our hypothesis that a greater area of the bolt would result in a smaller distance from the axis. Taking these calculations into account, option 3 is the most effective chain ring bolt because it generates a distance that falls in the range of 0.8 and 1.5 inches and has the highest factor of safety with respect to bearing failure of the bolts meeting the distance constraint.