St. Venant's Principle

Computer Homework 1

MEMS1047, Finite Element Analysis

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Introduction:

The aim of this homework was to demonstrate St. Venant's principle by doing a plane stress finite element analysis of the problems shown below.

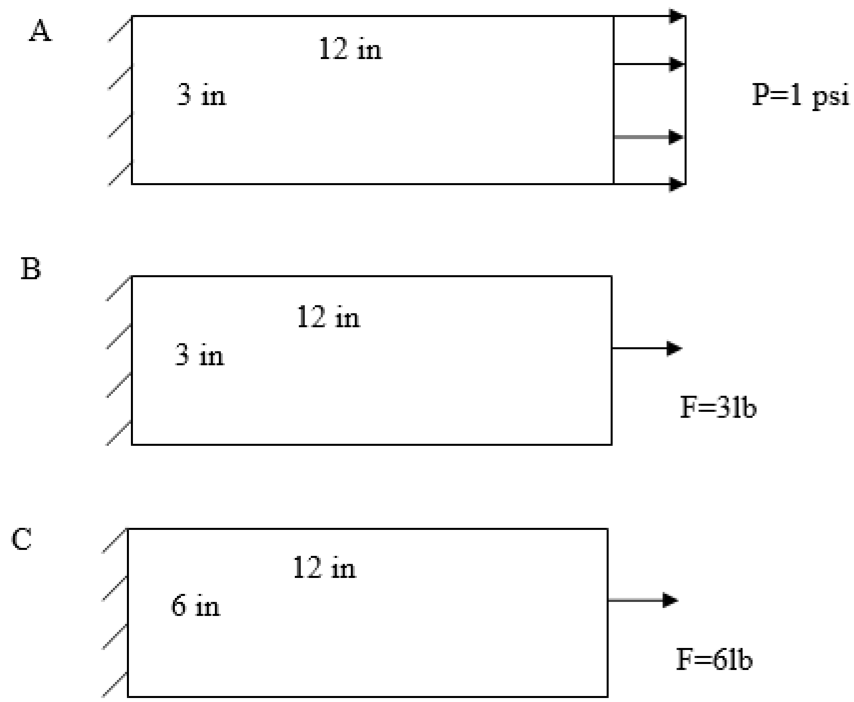

Figure 1 – Loading Scenarios

We wish to analyze the above problems, and, considering σ_{xx}, to comment on the differences in results and how St. Venant's principle applies. We will use E = 29e06 psi and Poisson's Ratio (υ) = 0. We will also analyze Problem A with $\upsilon = 0.25$ and comment on how σ_{xx} changes. Also, we will compare σ_{xy} with the two different Poisson's Ratios.

St. Venant's principle was first formulated in 1855 by Barré de Saint-Venant. His formulation was as follows:

> "If the forces acting on a small portion of the surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of the surface, this redistribution of loading produces substantial changes in the stresses locally, but has a negligible effect on the stresses at distances which are large in comparison with the linear dimensions of the surface on which the forces are changed."

Essentially, this statement tells us that stress distributions located far away (relatively) from the point of load application are unaffected by how the load is applied. However, the stress distributions local to the point of load application *are* affected by how the load is applied. St. Venant's principle in relation to this problem will be discussed in the problem statement section of this document (see below).

Note that throughout this document, the use of the word "stress" will refer to axial stress, σ_{xx}, unless otherwise noted.

Problem Statement:

As stated in the introduction, we wish to demonstrate St. Venant's principle by doing a plane stress finite element analysis of the problems shown in Figure 1. In this problem we will consider stresses in the axial direction, namely σ_{xx}, and comment on the differences in the problems and how St. Venant's principle applies. We wish to look at the axial stress since this is direction in which the loads are applied. Furthermore, we only wish to examine how the axial stress distribution is affected as we move away from the point of load application.

There will be a singularity in Problems B and C where a point load is used (stress concentrated at a singular node). Thus, we expect to see a large spike in σ_{xx} where the load is applied in these problems. For Problem A, we expect a constant axial stress since the body is experiencing a uniform, distributed load.

In relation to St. Venant's principle, we wish to show that in all three cases, the axial stress distribution located far away from the point of load application is unaffected by how the load is applied and that the stress distributions local to the point of load application *are* affected by how the load is applied. Thus, our hypothesis is that we will see a large spike in axial stress in Problems B and C where the point load is applied combined with an even, consistent distribution of stress as we move away from the point load. And, in Problem A, we would expect that the stress distributions are completely unaffected by the loading since it is an even, distributed load – i.e., there are no stress concentrations at a singular node).

Results:

σ_{xx} and σ_{xy} for Problem A with $\upsilon = 0$ are plotted in Figure 2 shown below.

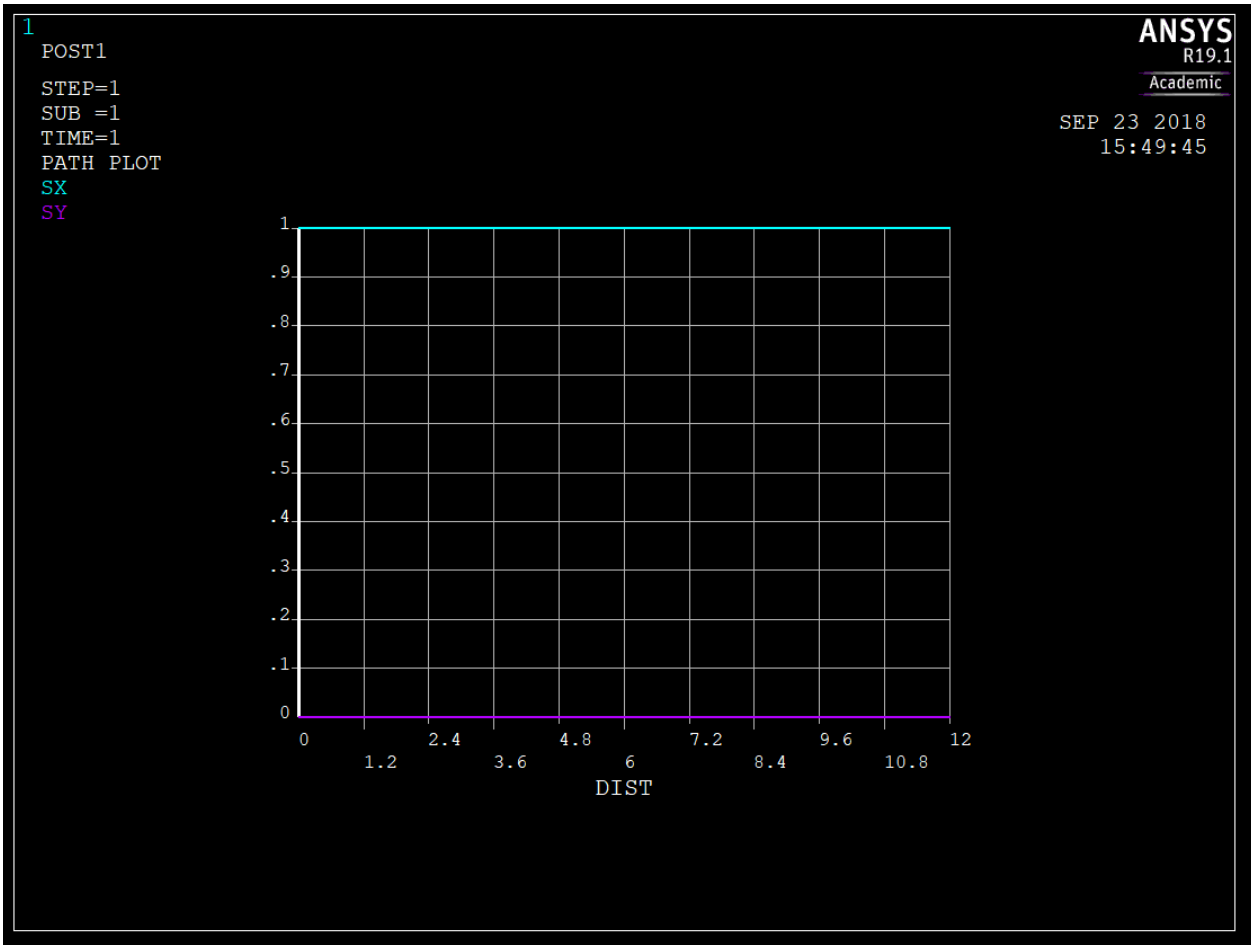

Figure 2 – Stress Distribution for Problem A with $\upsilon = 0$

The stresses in this case are constant. There are no stress concentrations because there is only a constant, distributed load of P = 1 psi. I.e., the load is not being applied at a singular node. The expected axial stress for this case is given by equation 1 shown below.

$$\sigma_{xx} = \frac{Pwt}{A} = \frac{(1)(3)(1)}{(3)(1)} = 1\, psi \tag{1}$$

Where P is the load, w is the width, t is the thickness, and A is the cross-sectional area. We see that this agrees with the results obtained in Figure 1. Furthermore, we note that $\sigma_{xy} = 0$ for this case since $\upsilon = 0$ – i.e., there is no strain or stress in the transverse direction.

σ_{xx} and σ_{xy} for Problem A with $\upsilon = 0.25$ are plotted in Figure 3 shown below.

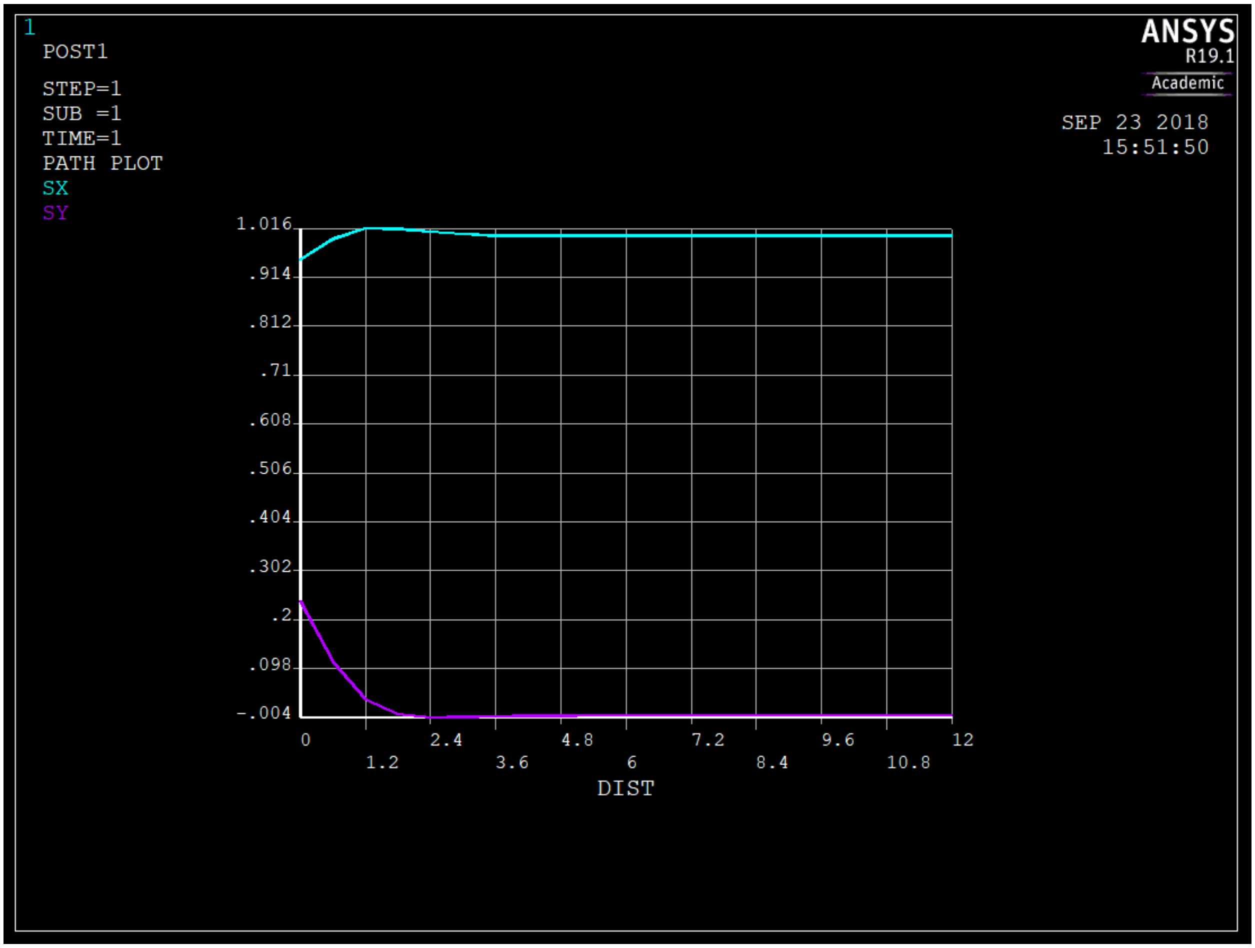

Figure 3 – Stress Distribution for Problem A with $\upsilon = 0.25$

There is a variation in the stresses experienced at x = 0 in. in this case. At this point, we have $\sigma_{xx} \approx .930$ psi and $\sigma_{xy} \approx .225$ psi. With a non-zero value for Poisson's Ratio, there is some strain (and hence, stress) experienced in both the axial and transverse direction. What is interesting is that this variation occurs at the point x = 0 in. and not x = 12 in., the point of loading application. The variation in stress goes away as we move away from the point x = 0 in., somewhat in conjecture with St. Venant's principle. However, usually with this principle we would expect to see some type of stress variations at the point of load application, i.e., x = 12 in. Nonetheless, the axial stress in this problem agrees with the expected axial stress of $\sigma_{xx} = 1$ psi, given by equation 1.

The axial stress for Problem B is plotted in Figure 4 shown below.

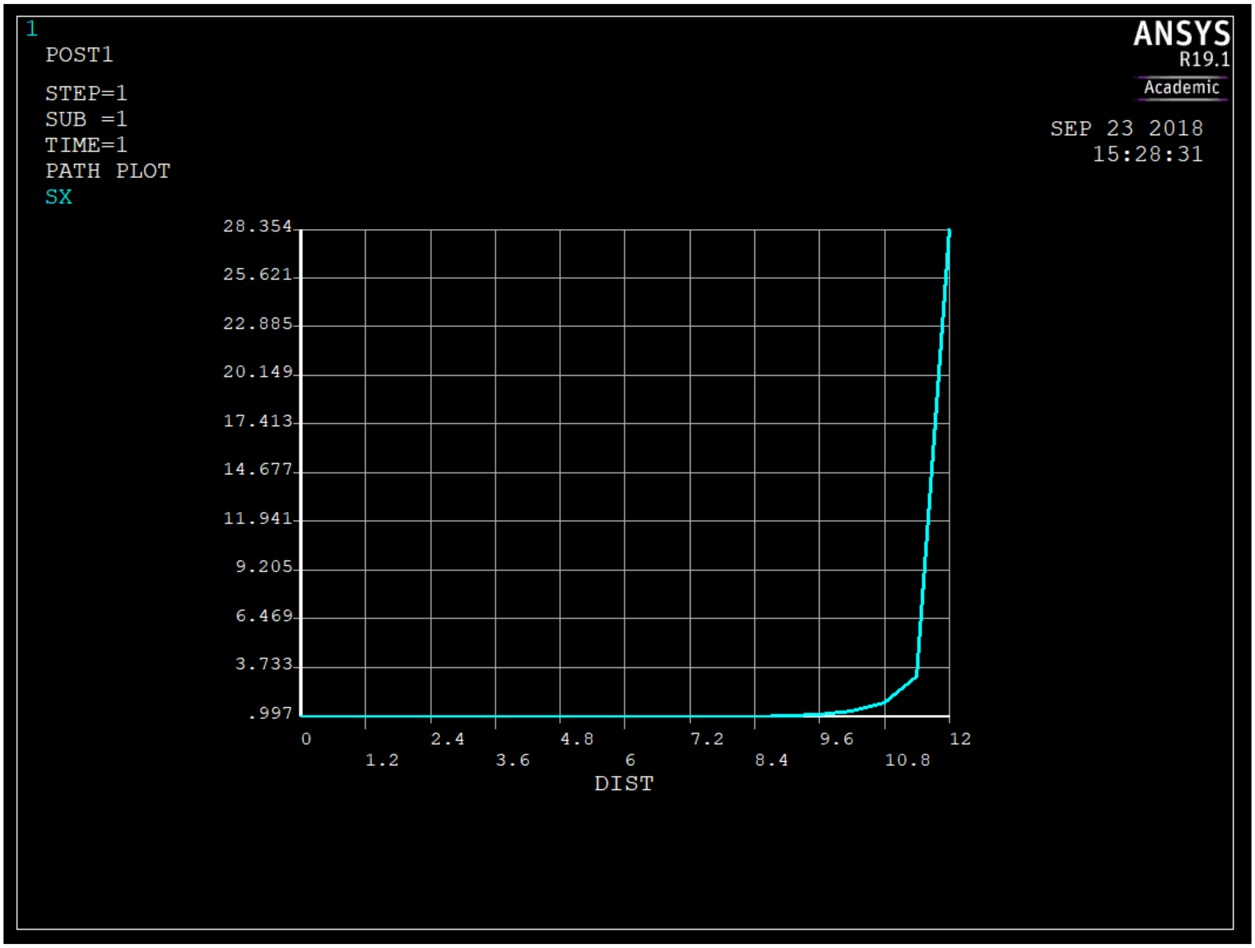

Figure 4 – Axial Stress Distribution for Problem B

As expected by St. Venant's principle, we see a large spike in the axial stress at x = 12 in., the point at which the load is applied. This is due to the stress being concentrated at a singular node. As we move away from the point of load application, the stress decreases exponentially to its nominal value of 1 psi, as expected by equation 2 shown below.

$$\sigma_{xx} = \frac{P}{A} = \frac{3}{(3)(1)} = 1\,psi \tag{2}$$

More specifically, it takes approximately 2 in. from the point of load application for the stress to decrease to its nominal value. The peak stress experienced (at the node) is σ_{max} = 28.354 psi.

The axial stress for Problem C is plotted in Figure 5 shown below.

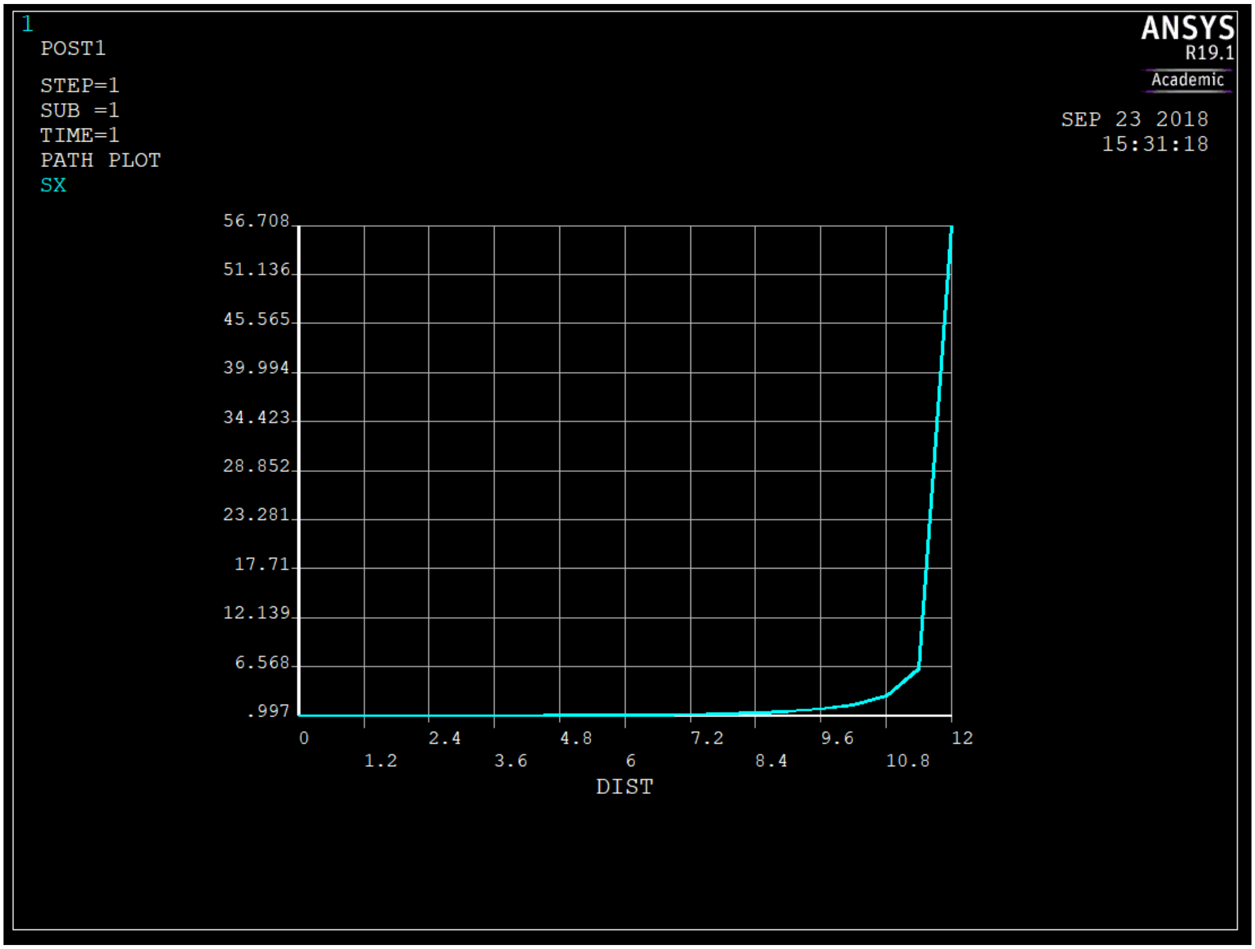

Figure 5 – Axial Stress Distribution for Problem C

Much like Problem B, we see a large spike in the axial stress at x = 12 in., the point at which the load is being applied. As we move away from the point of load application, the stress decreases exponentially to its nominal value of 1 psi, as expected by equation 3 shown below.

$$\sigma_{xx} = \frac{P}{A} = \frac{6}{(6)(1)} = 1\,psi \tag{3}$$

Again, we see that it takes approximately 2 in. from the point of load application to decrease to its nominal value. Here we see a peak stress, σ_{max} = 56.708 psi, that is double that of Problem B, σ_{max} = 28.354 psi. We agree with these results considering that the load applied in Problem C is exactly double that of the load applied in Problem B. Thus, the maximum stress experienced at the point of load application should correlate with the magnitude of the load.

Discussion:

This section will discuss, in further detail, the results obtained from our analyses. First, we will examine the difference in stress between Problems A and B. We note that, in general, the stress distribution is constant for Problem A. This is because there is an even distributed load being applied. For Problem B, we note that there is a stress concentration at the point where the load is being applied, namely x = 12 in. This is because there is a singularity at this point. However, the stress distributions located far away from the point of load application are completely unaffected by how the load is applied. As we move away from this point, the stress is constant and σ_{xx} = 1 psi. Thus, St. Venant's principle is upheld for this case.

Next, let us examine the difference in stress between Problems B and C. These two cases are nearly identical. There is a stress concentration at the point where the load is being applied, x = 12 in. As we move away from this point, the stress decreases exponentially to its nominal value of σ_{xx} = 1 psi, given by equations 2 and 3. Since the loading type is the same in Problems B and C, we simply note that that the stress distribution is unaffected as we move away from the singularity of the load application. The maximum stress experienced at the singularity for Problem B is given by σ_{max} = 28.354 psi, and, for Problem C, σ_{max} = 56.708 psi. Thus, it appears as if the maximum stress experienced at the singularity is proportional to the magnitude of the applied load. This physically makes sense. There is twice as much load being applied to the node, so the stress experienced at this point should be double.

Finally, we would like to examine the differences in Problem A with υ = 0 and υ = 0.25. With υ = 0, there is no transverse strain and there is a constant stress distribution such that σ_{xx} = 1 psi (see equation 1) and σ_{xy} = 0 psi. With υ = 0.25, there is some strain in the transverse direction, so there is stress in both the x and y directions. We see from Figure 3 that there is a peculiar stress distribution for this case and the stress in the transverse direction, σ_{xy}, is only relevant near x = 0 in. Otherwise, there is a constant stress distribution – even at the point of load application. This somewhat contradicts St. Venant's principle and should lead us to further investigation of the stresses experienced in this case. Overall, however, the results are consistent and St. Venant's principle has been verified with this set of problems.

References

Sönnerlind, Henrik. "Applying and Interpreting Saint-Venant's Principle." COMSOL Multiphysics©, COMSOL INC., 22 Jan. 2018, www.comsol.com/blogs/applying-and-interpreting-saint-venants-principle/.