Lab 3: Use of Strain Gages to Determine the Strain in Cantilever Beams

MEMS1041, Mechanical Measurements I

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Objective:

The objective of this experiment was to measure mechanical strain in a cantilever beam using strain gages and to compare the results with theoretical strain values calculated from an equation derived from solid mechanics.

Theory:

Strain gages are made of thin metal wires whose resistance changes whenever it is strained. As the wire is strained, its length L and its cross-sectional area A changes, which leads to a change in resistance R given by the formula

$$R = \frac{\rho L}{A} \tag{1}$$

We can see that if the wire is stretched, L will increase, A will decrease, and resistance R will increase. Note that the wires resistivity, ρ, will also change when the wire is strained, but we will not take that into account here. As we will see below, if we can measure the change in resistance, say ΔR, then we can infer the strain and ultimately the stress. If we take the derivative of equation 1 with respect to each variable, we have that (derivation omitted)

$$\frac{dR}{R} = \frac{dL}{L} + \frac{d\rho}{\rho} - \frac{dA}{A} \tag{2}$$

From equation 2, we can define gage factor, GF, as

$$GF = \frac{dR/R}{\varepsilon} \tag{3}$$

Where ε is strain in the specified direction. Finally, we find that the relationship between strain and the change in resistance of the wire is given by

$$\varepsilon = \frac{\Delta R/R}{GF} \tag{4}$$

Consider the beam shown by Figure 1 on page 4. This a simple cantilevered beam subject to a force F at the end of the beam. In this case, the top of the beam will experience tension and the bottom of the beam will experience compression. In this way, the wires in the strain gage on top of the beam will be stretched, inducing a positive strain and thus positive ΔR. The wires in the strain gage on the bottom of the beam will be compressed, inducing a negative strain and thus negative ΔR. We will use this process to determine the strain in this cantilever beam.

Let us first determine the theoretical equation for strain in the beam using our knowledge of beams in bending. For a cantilever beam with a single force *F* concentrated at the end of the beam (see Figure 1), the deflection at the end of the beam is given by

$$\delta = \frac{FL_2^3}{3EI} \tag{5}$$

(from Riley, *Mechanics of Materials*, 6*th* *ed.*). Then we can calculate the theoretical force *F* as

$$F = \frac{3EI\delta}{L_2^3} \tag{6}$$

Consider the bending stress experienced at the location of the strain gages. This is given by

$$\sigma = \frac{My}{I} = \frac{FL_1 y}{I} \tag{7}$$

Where L_1 is the distance from the end of the beam to the strain gages. Now, with (6), we have

$$\sigma = \frac{3E\delta L_1 y}{L_2^3} \tag{8}$$

The gages are mounted at the top/bottom of the beam, i.e., $y = t/2$. Thus,

$$\sigma = \frac{3E\delta L_1 t}{2L_2^3} \tag{9}$$

Consider the stress-strain relationship given by

$$\epsilon = \frac{\sigma}{E} \tag{10}$$

Then, with (9), we have

$$\epsilon = \frac{3\delta L_1 t}{2L_2^3} \tag{11}$$

Thus, we have produced an equation for the theoretical strain on the surface of the beam at the location of the gages. These values for various deflections are given in Tables 1, 2, and 3.

Consider the beam arrangement shown in Figure 1 below:

L_1 = 9 11/16" L_2 = 10"   W = 1"   t = 1/8" or 1/4"

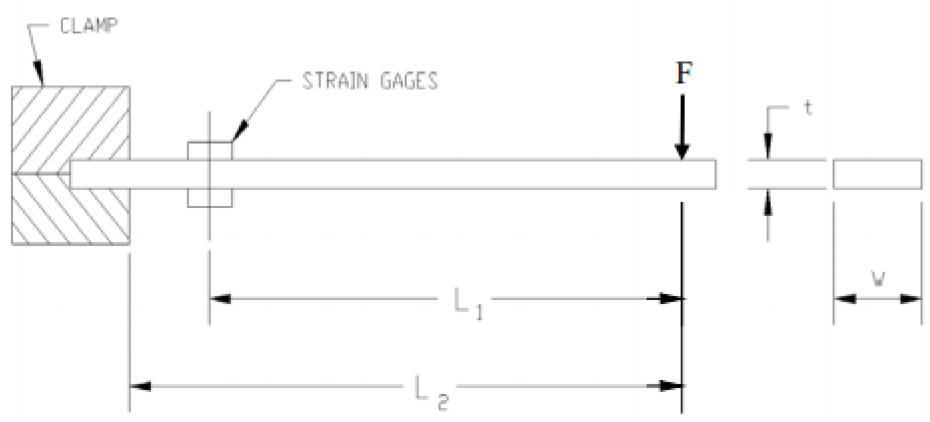

Figure 1: Schematic of the Cantilever Beam used throughout the Experiment

When mounting the strain gages on the beam, we can use either a Full Bridge, Half Bridge, or Quarter Bride configuration. Let us first examine the Full Bridge configuration shown in Figure 2 below.

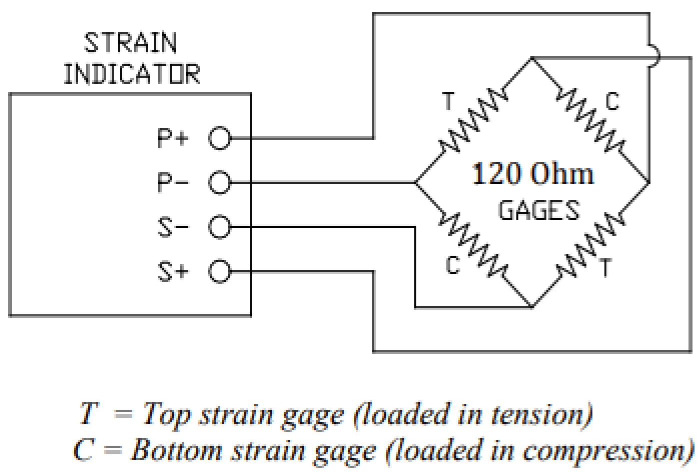

Figure 2: Wiring Configuration for Full Bridge

In this configuration, there are two strain gages mounted on top of the beam, labeled as a resistor T, and two strain gages mounted on the bottom of the beam, labeled as a resistor C. These are wired to a strain indicator which will measure the strain of each gage, given by equation 4. As the wire is stretched/compressed, we will be able to read off the values from the strain indicator and determine the strain of the cantilever beam. Now consider the Half Bridge configuration given by Figure 3 on the following page.

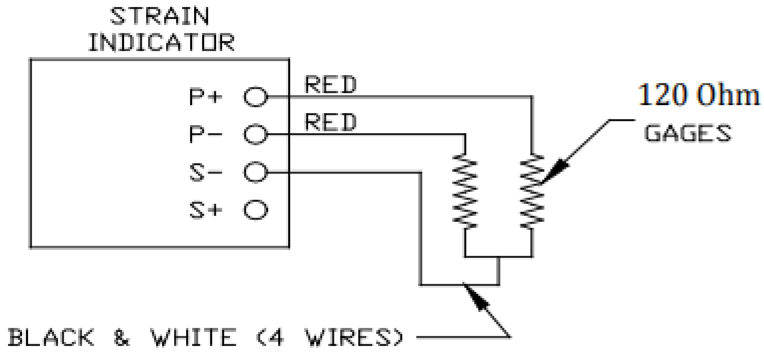

Figure 3: Wiring Configuration for Half Bridge

In this configuration, there is one strain gage mounted on the top of the beam and one strain gage mounted on the bottom of the beam. Finally, consider the quarter bridge configuration shown in Figure 4 below.

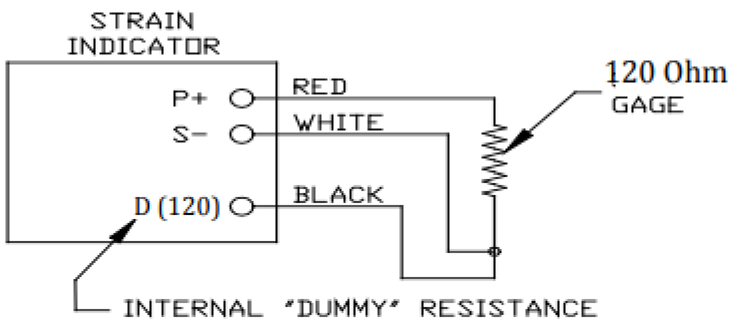

Figure 4: Wiring Configuration for Quarter Bridge

In this configuration, there is only one strain gage which we mounted to the top of the beam; this strain gage will be in tension. As we increase the number of strain gages, we should receive more accurate results. This is since we will receive temperature compensation (theory of temperature compensation omitted from this report), and more accurate measurements with more gage measurements.

Procedure:

In this experiment, a micrometer was used to apply a deflection to the end of a beam shown in the arrangement given by Figure 1. Before starting the experiment, the dimensions of our cantilever beam were measured using a ruler and a micrometer. The exact dimensions of the beam are given in Table A below.

Table A – Beam Measurements		
Quantity	Value	Units
L_1	9 11/16	in.
L_2	10	in.
w	1	in.
t	¼	in.

We next verified the nominal resistance of our strain gages using a digital multimeter. The measured values of these resistors are given in Table B below, where R_1 is the upper left (T) resistance, R_2 is the bottom right (T) resistance, R_3 is the top right (C) resistance, and R_4 is the bottom left (C) resistance, according to Figure 1.

Table B – Resistance Measurements		
Quantity	Value	Units
R_1	348	Ω
R_2	349	Ω
R_3	350	Ω
R_4	350	Ω

Note: for the full bridge assembly, all four resistors (gages) were used; for the half bridge assembly, R_1 and R_3 were used; and for the quarter bridge assembly, only R_1 was used.

The cantilever beam was then installed in the clamp, making sure that the micrometer head was not yet touching the beam. We then wired the strain gages to the strain indicator according to the diagrams given by Figures 2, 3, and 4 for the desired bridge.

For this process, we first set the gage factor for each of the strain gages in use. In each case, the gage factor (*GF*) was found to be *GF* = 2.11. We next balanced the bridge by adjusting the knobs on the strain indicator until it displayed a value of zero. In doing so, we assured that there was still no contact between the micrometer and the beam.

Finally, we adjusted the micrometer until the strain indicator display read approximately 10 to 20 micro strain. This indicates that contact has been established between the tip of the micrometer and the top surface of the cantilever beam; this measurement was recorded in each case. This location was defined as the zero deflection of the beam and the starting point for recording the data. These values have been omitted from the report. We continued to record strain measurements at increments of beam deflection as given in Tables 1, 2, and 3.

Results:

Using the micrometer, the beam was deflected in 0.050" increments up to a total deformation of 0.5" and the respective strains were recorded from the strain indicator. This data was compared to the theoretical strain given by equation 11 and percent error was calculated for each data point. A plot containing the experimental and theoretical strains versus beam displacement was plotted for each bridge configuration. The next several pages are reserved for these results.

Table 1: Recorded Data – Average Strain vs. Theoretical Strain for Full Bridge Configuration

Part A - Full Bridge Configuration					
Deformation (in.)	Strain (με) - Trial 1	Strain (με) - Trial 2	Average Strain (με)	Theoretical Strain (με)	Percent Error
0	1	-4	-1.5	0	-100.00
0.05	158	155	157	182	16.06
0.1	317	319	318	363	14.24
0.15	478	483	481	545	13.41
0.2	632	646	639	727	13.70
0.25	779	811	795	908	14.24
0.3	925	976	951	1090	14.66
0.35	1074	1142	1108	1271	14.75
0.4	1225	1309	1267	1453	14.69
0.45	1376	1475	1426	1635	14.68
0.5	1526	1643	1585	1816	14.64

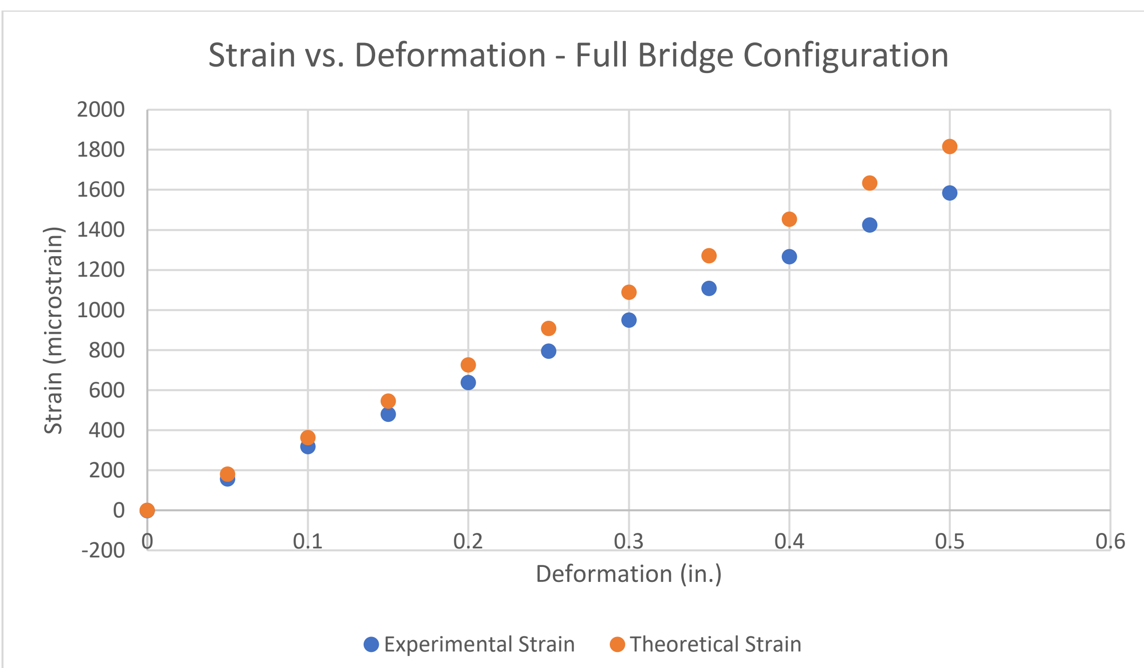

Figure 5: Strain vs. Deformation – Full Bridge Configuration

Table 2: Recorded Data – Average Strain vs. Theoretical Strain for Half Bridge Configuration

Part B - Half Bridge Configuration					
Deformation (in.)	Strain (με) - Trial 1	Strain (με) - Trial 2	Average Strain (με)	Theoretical Strain (με)	Percent Error
0	-0.1	-0.1	-0.1	0	-100.00
0.05	153	154	153.5	182	18.33
0.1	310	311	310.5	363	17.00
0.15	467	469	468	545	16.44
0.2	626	628	627	727	15.88
0.25	786	787	786.5	908	15.47
0.3	946	947	946.5	1090	15.14
0.35	1105	1108	1106.5	1271	14.91
0.4	1267	1267	1267	1453	14.69
0.45	1429	1429	1429	1635	14.40
0.5	1590	1592	1591	1816	14.17

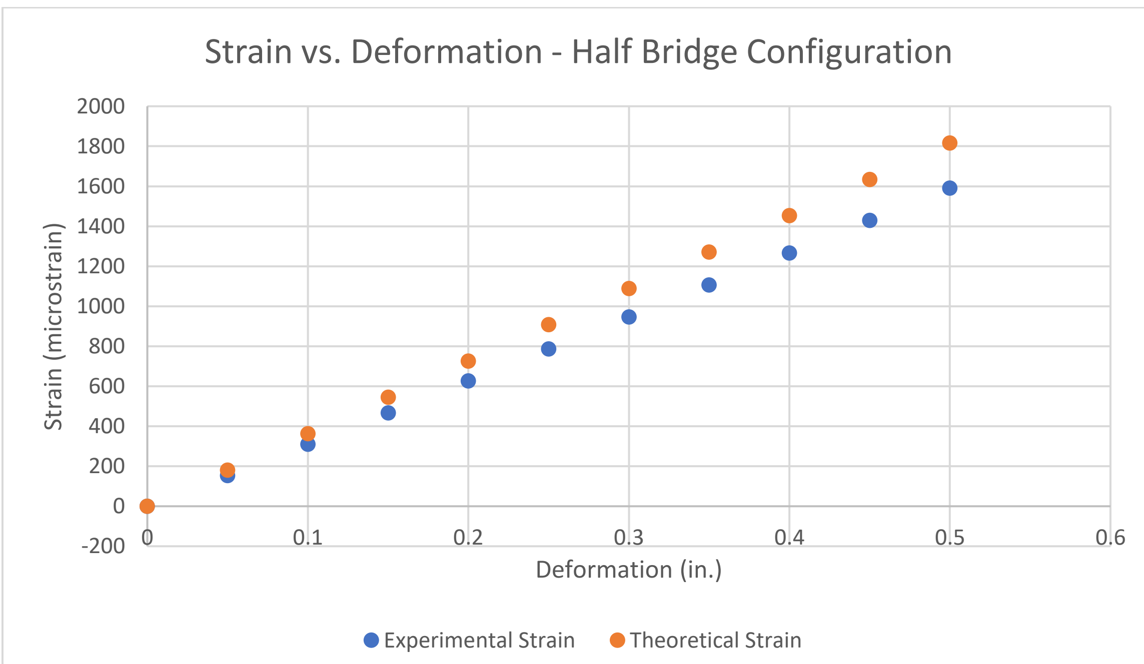

Figure 6: Strain vs. Deformation – Half Bridge Configuration

Table 3: Recorded Data – Average Strain vs. Theoretical Strain for Quarter Bridge Configuration

Part C - Quarter Bridge Configuration					
Deformation (in.)	Strain (με) - Trial 1	Strain (με) - Trial 2	Average Strain (με)	Theoretical Strain (με)	Percent Error
0	2	1	1.5	0	-100.00
0.05	157	154	155.5	182	16.81
0.1	314	311	312.5	363	16.25
0.15	471	469	470	545	15.94
0.2	630	628	629	727	15.51
0.25	790	787	788.5	908	15.18
0.3	950	948	949	1090	14.84
0.35	1111	1109	1110	1271	14.55
0.4	1272	1271	1271.5	1453	14.28
0.45	1434	1430	1432	1635	14.16
0.5	1594	1596	1595	1816	13.88

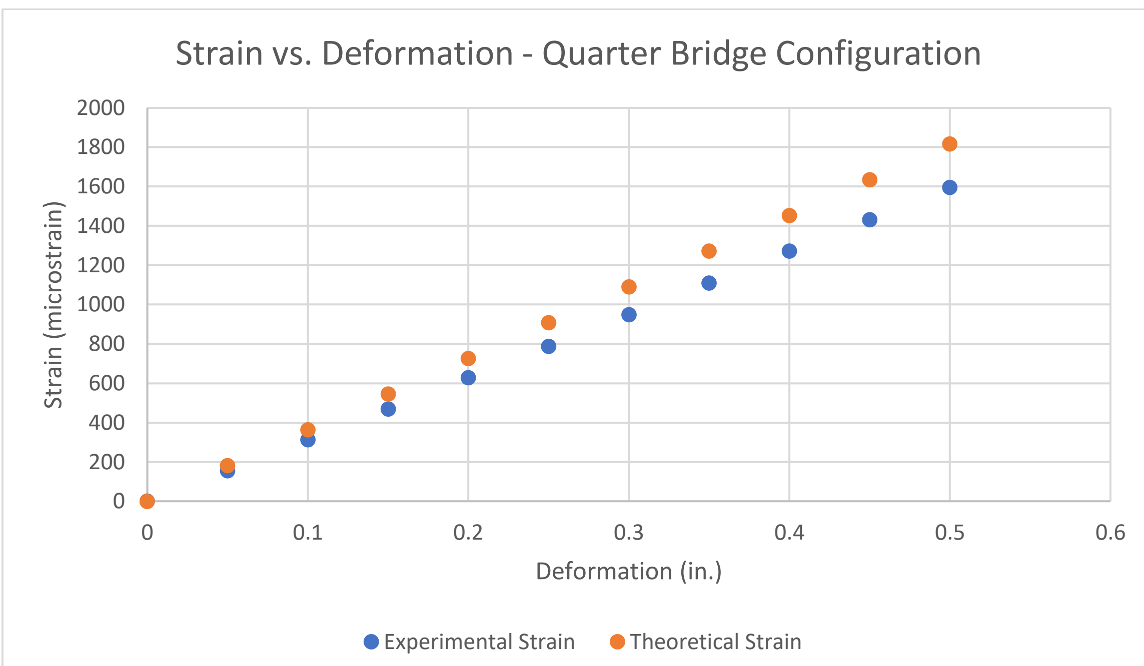

Figure 7: Strain vs. Deformation – Quarter Bridge Configuration

Discussion:

We see that there is a linear relationship between deformation and strain. I.e., as we continue to deform the beam, the wires in the strain gage continue to stretch. As the length of the wires in the strain gage changes, so too will the total resistance of the strain gage (see equations 1, 4). This change in resistance allows us to measure the strain using the strain indicator.

Looking at our data, we see that there is a maximum of 18.33% error between the experimental strain values and the theoretical strain values, where the percent error for this experiment is given by the equation below.

$$Percent\ Error = \frac{Theoretical - Experimental}{Experimental} \tag{12}$$

Note, we are taking the experimental value to be nominal for this experiment. The error is not discouraging considering that there will be a vast amount of error in measuring the strain using the strain gages. This error could be induced from a variety of sources, including, but not limited to, miscalibration of the strain indicator to the strain gages, miscalibration of the micrometer, inaccurate measurements from the strain indicator in measuring the change in resistance of the wires, etc.

We notice two immediate trends in the data. One, that total error tends to increase as we decrease the number of strain gages in our system, and two, that total error tends to decrease as we increase deformation. The former is explained in the theory section of this lab report. I.e., with more strain gages, we will receive temperature compensation (theory of temperature compensation omitted from this report), and more accurate measurements with more gage measurements. The latter can be explained by the fact that initially, the strain gages are relatively "unstretched". As we apply an initial deformation, the wires will see excitation in the form of stretching that may cause error in the measurements. Furthermore, there may be some initial calibration error present in the strain indicator, but as we apply more and more deformation, this miscalibration error diminishes. Both concepts can explain the fact that we receive more accurate measurements as we increase the deformation.

Note that for this discussion we are excluding the case where deformation is equal to 0, since theoretically we should have zero strain, but in the experiments, there is already some strain present due to calibration of the strain indicator. This will always lead us to have 100% error in the zero-deformation case, given our definition of percent error.

Conclusion:

Overall, we are satisfied with the results and say that these strain gages yield accurate measurements that are fairly close to the theoretical strain values. The maximum percent error between the experimental strain measurements and the theoretical strain measurements was 18.33%, with a range of approximately 14% - 18%. Depending on the application, these strain gages can be very valuable in obtaining strain measurements, thus being able to determine stress and estimating at which point the structure will experience permanent yielding.

References

Riley, William Franklin, et al. Mechanics of Materials, 6th Edition. J. Wiley and Sons, 2007.