Analysis of Stress Concentrations in ANSYS APDL

Computer Homework 5

MEMS1047, Finite Element Analysis

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Introduction:

The aim of this homework was to perform a finite element analysis of a square plate with a circular hole to determine the stress concentration factor. The loading scenario for this problem is shown in Figure 1 below.

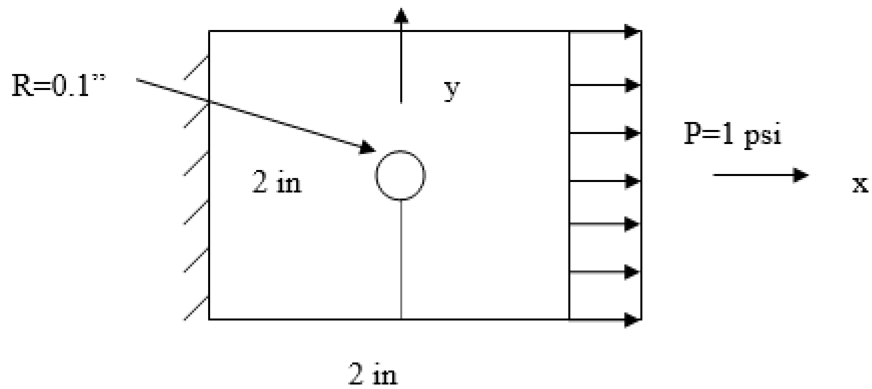

Figure 1 – Loading Scenario

We will use E = 29e06 psi and Poisson's Ratio, $\upsilon = 0$. We will use both a coarse mesh (element size = 0.1) and a fine mesh (element size = 0.005) to solve the problem. The element type used for this problem is PLANE182 (4 node quadrilateral element). We will examine σ_{xx} for the stress concentration and compare our analytical results to the theoretical results. For each mesh size, we will include contour plots of both nodal and elemental stresses as well line plots which depict the stress variation along the x-axis.

Problem Statement:

Stress concentrations arise in materials due to holes, cracks, sharp corners, etc. I.e., we expect there to be a significant rise in the stress near the hole at x = 0" from Figure 1. The stress concentration for an infinite plate according to Timoshenko and Goodier (*Theory of Elasticity*, p. 78-83) is 3.0. From a much more involved formula given by Howland (*Trans. Royal Society (London) Series A, Vol. 229*, p. 49), the stress concentration for a plate with the given dimensions is 3.14. As stated above, we will perform an analysis using both a coarse mesh and a fine mesh and comment on the differences in results. Due to the restrictions of the Finite Element Method, we hypothesize that the maximum analytical stress will be lower than that of the theoretical stress, due to the use of linear elements and the lack of truly infinitesimal elements.

Calculations:

Let us first derive the anticipated theoretical stress due to the stress concentration. We can treat the pressure load as a singular, concentrated load and calculate the associated axial stress. This is given by equation 1, shown below.

$$\sigma_{xx} = \frac{Pwt}{A} = \frac{(1)(2)(1)}{(2)(1)} = 1\,psi \tag{1}$$

Where a unit thickness, *t* = 1 in., is used for this problem. The result from equation 1 represents the anticipated theoretical axial stress. The formula for the stress concentration, *k*, is given by equation 2.

$$k = \frac{\sigma_{max}}{\sigma_{nom}} \tag{2}$$

In this case, σ_{nom} is given by the result obtained from equation 1, i.e., $\sigma_{nom} = 1$ psi. Then using the Howland stress concentration factor, we can calculate the anticipated maximum axial stress:

$$\sigma_{max} = k\sigma_{nom}$$

$$\sigma_{max} = (3.14)(1) = 3.14\,psi \tag{3}$$

This result will be used to examine our analytical results.

Results:

Let us first examine the coarse mesh. An element size of 0.1 was used for this case. The corresponding mesh is shown in Figure 2 on the following page. Note that the coarse mesh size seems to interfere with the geometry of the problem, i.e., it seems to distort the circular hole at the center of the plate. We will keep this in mind when analyzing our results. From Figure 1, we have applied a zero-displacement boundary condition on the left edge of the plate and a pressure load of 1 psi on the right edge of the plate. The loading scenario for the coarse mesh is shown in Figure 3 on the following page.

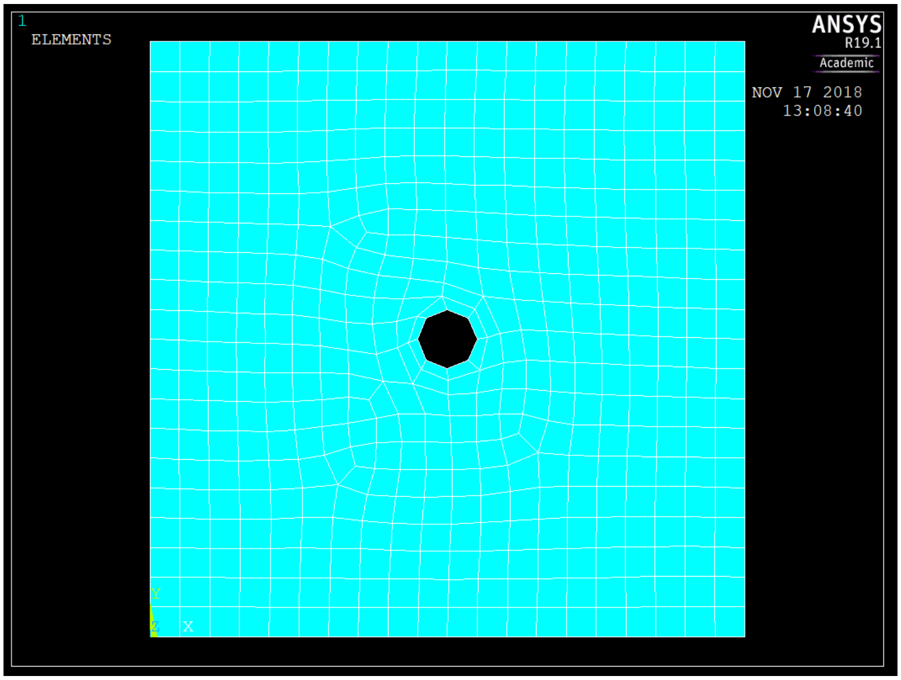

Figure 2 – Coarse Mesh

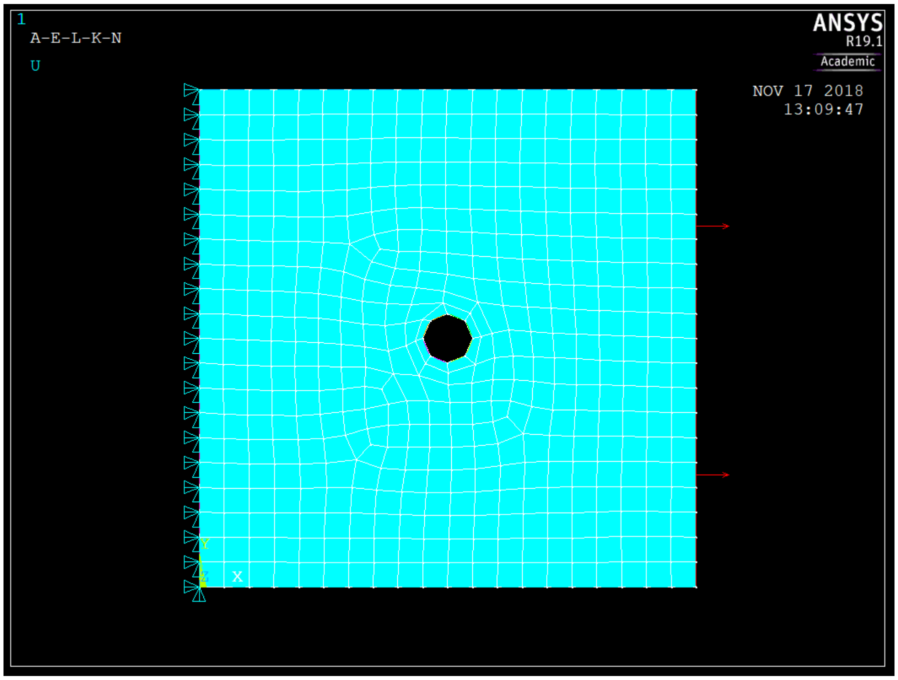

Figure 3 – Loading Scenario for Coarse Mesh

The nodal and elemental contour plots for the coarse mesh are shown in Figures 4 and 5, respectively, on the following page. These are given for reference but will not be used to analyze our results.

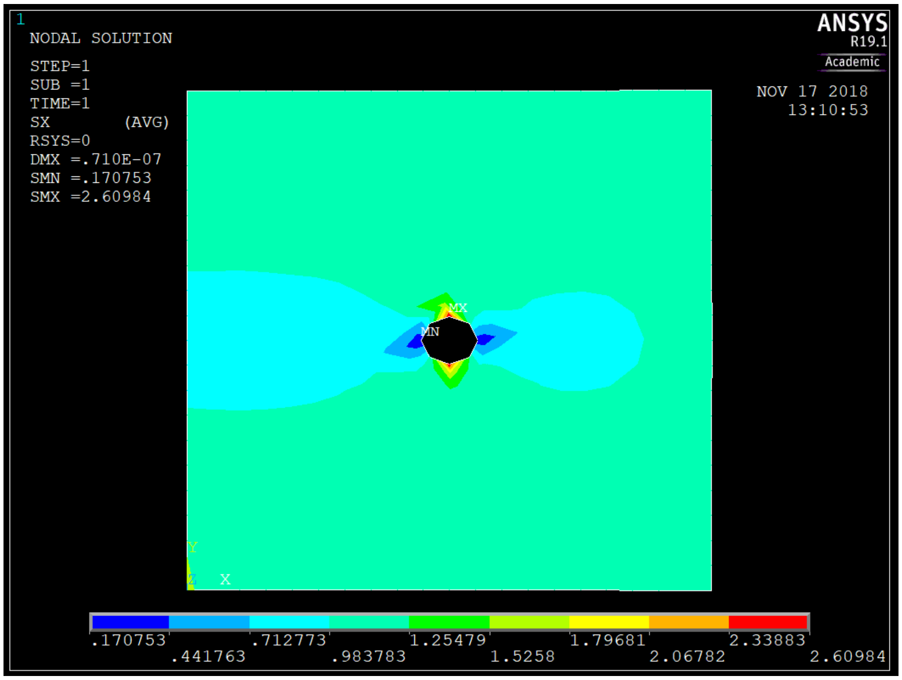

Figure 4 – Nodal Contour Solution for Coarse Mesh

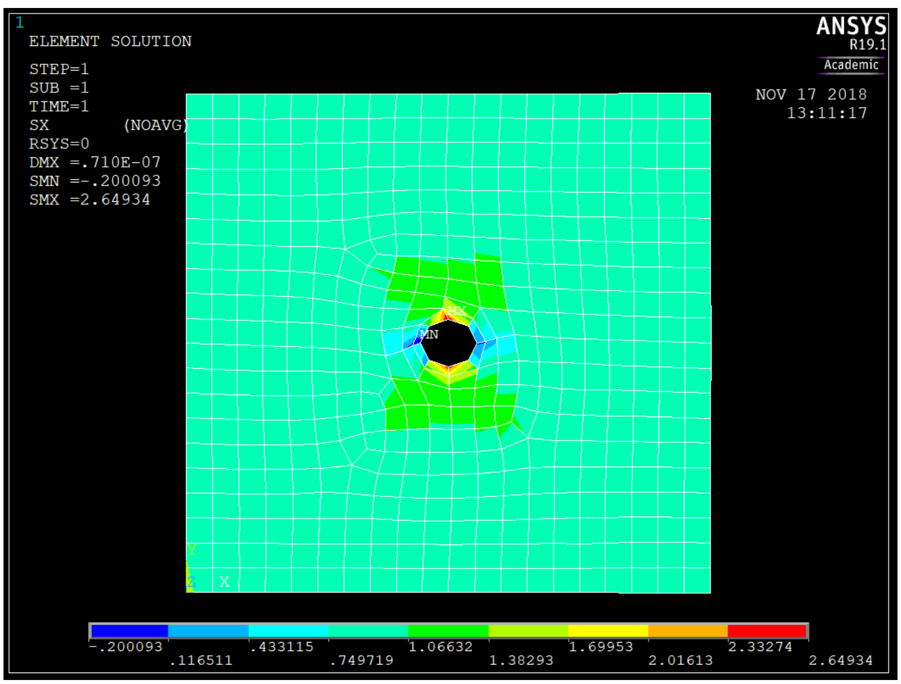

Figure 5 – Elemental Contour Solution for Coarse Mesh

Let us examine the stress distribution using the line plot given by Figure 6 on the following page. This graph plots the axial stress distribution as a function of location x along the plate. Note that for our analysis we have defined the origin at the bottom left corner of the plate.

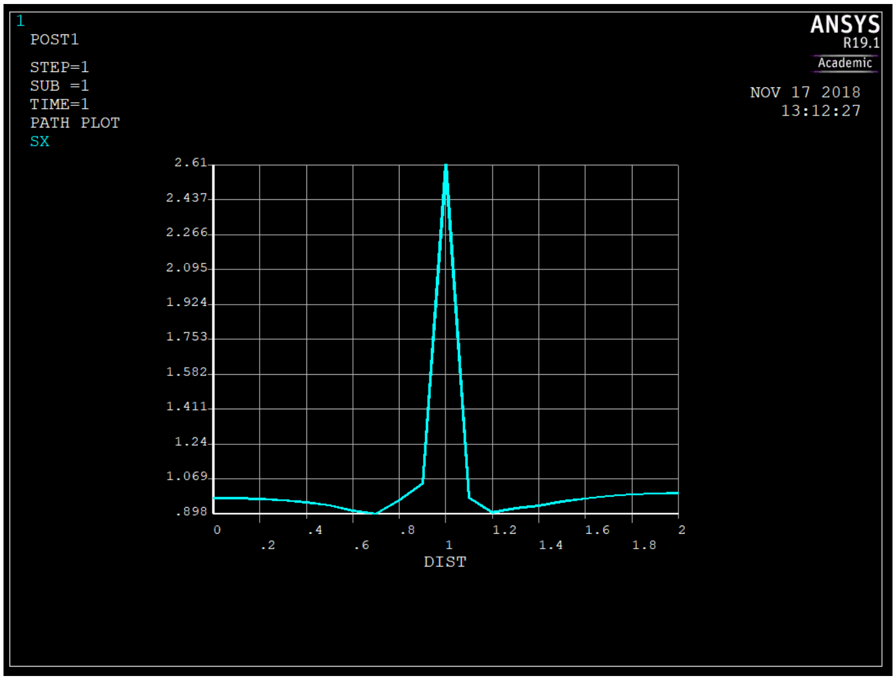

Figure 6 – Axial Stress Distribution for Coarse Mesh

This plot easily allows us to examine the stress distribution. Examining the figure, we see a large spike in the stress at the center of the plate where the circular hole is located. Namely,

$$\sigma_{max} = 2.61\ psi \tag{4}$$

From equation 3, the maximum theoretical axial stress is given by σ_{max} = 3.14 psi. Then we can calculate a percent error associated with the analytical and theoretical results:

$$Error = \frac{3.14 - 2.61}{3.14} \cdot 100 = 16.88\% \tag{5}$$

Thus, there is a 16.88% error associated with the maximum axial stress value. We explain this result by the fact that we are using very coarse elements and the solution is not converging very well to its theoretical value. We expect that by using a finer mesh, there will be less error associated with the solution.

An element size of 0.005 was used for the fine mesh. The mesh is shown in Figure 7, shown below. Note that, in this case, the mesh does not interfere with the circular geometry.

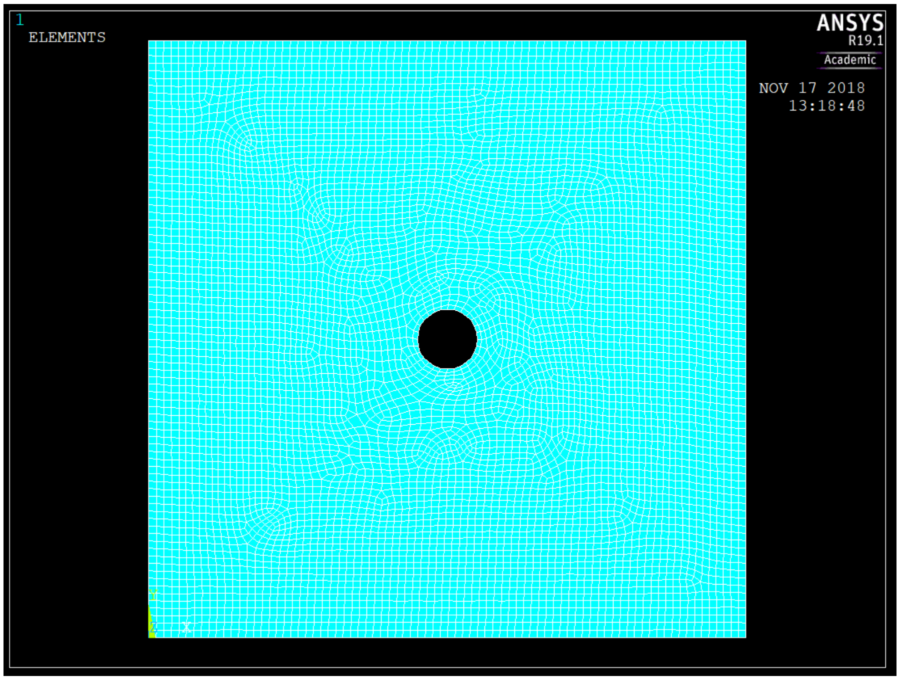

Figure 7 – Fine Mesh

The loading scenario is the same as that for the coarse mesh. We will omit this figure. The nodal and elemental contour plots for the fine mesh are shown in Figures 8 and 9, respectively.

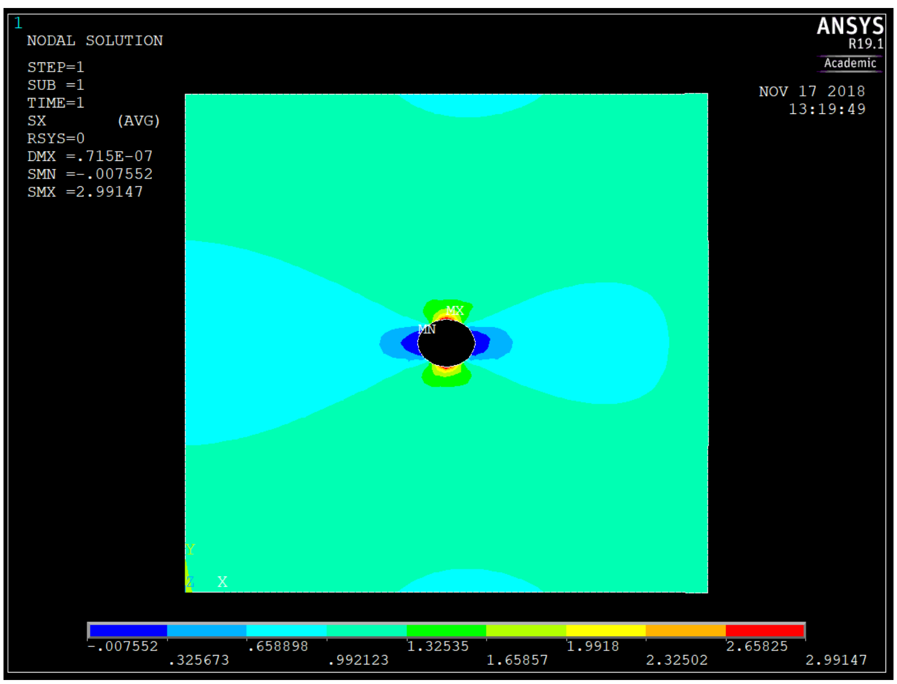

Figure 8 – Nodal Contour Solution for Fine Mesh

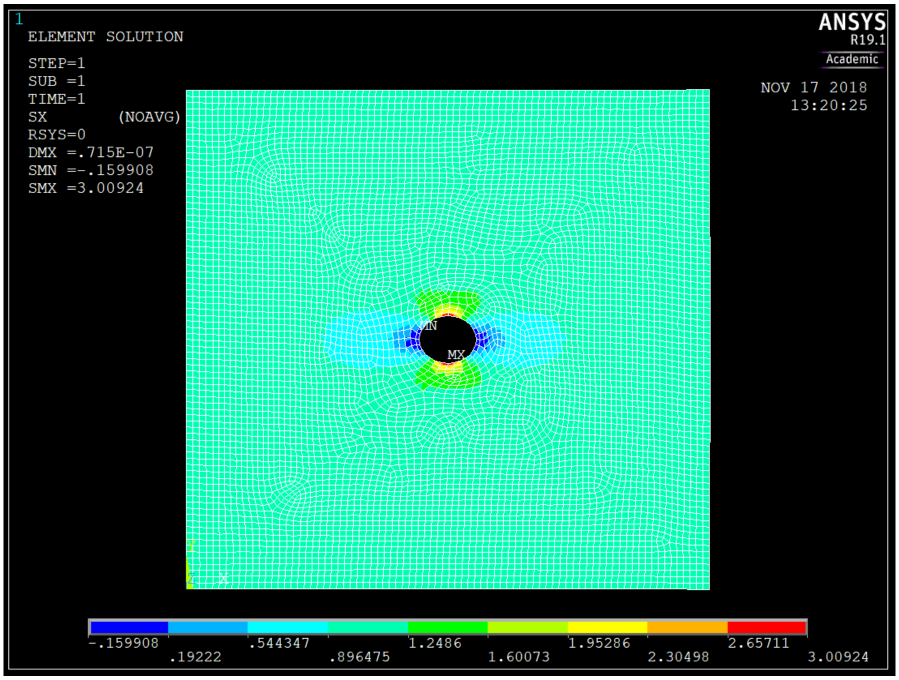

Figure 9 – Elemental Contour Solution for Fine Mesh

Let us now examine the stress distribution using the line plot given by Figure 10, shown below.

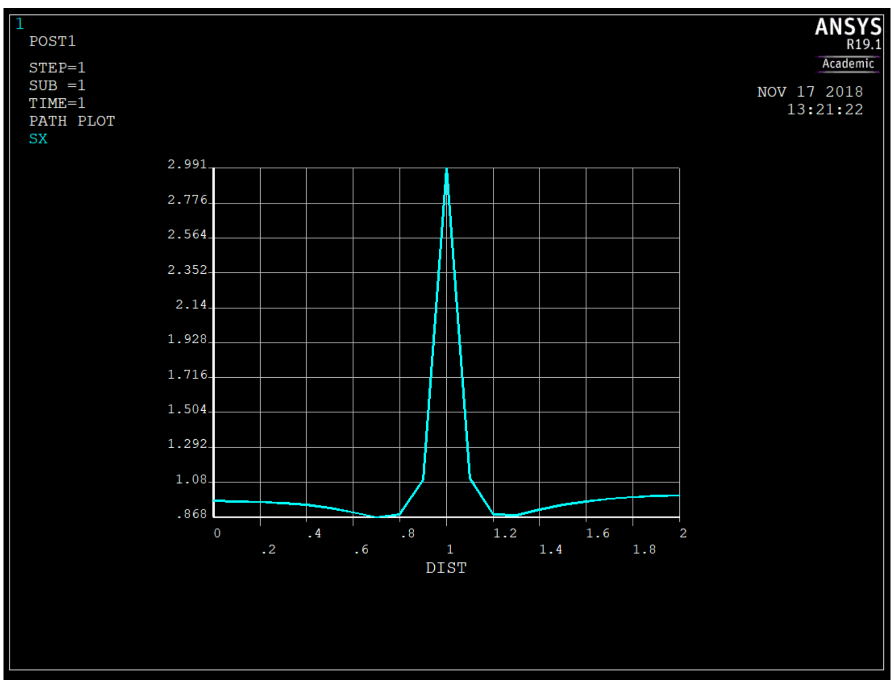

Figure 10 – Axial Stress Distribution for Fine Mesh

Again, we see a large spike in the axial stress at the center of the plate where the hole is located. Examining the figure, we see that the maximum analytical stress is

$$\sigma_{max} = 2.991\ psi \tag{6}$$

From equation 3, the maximum theoretical axial stress is given by σ_{max} = 3.14 psi. Then the percent error for the fine mesh solution is given by

$$Error = \frac{3.14 - 2.991}{3.14} \cdot 100 = 4.75\% \tag{7}$$

Discussion

In both cases, we see a large spike in the stress at the center of the plate where the circular hole is located. However, from equations 3 – 7, we see that our solution is much more accurate when a fine mesh is used over a coarse mesh. Namely, the error decreases by a factor of 4 when the fine mesh is used. This is explained by the fact that as we use smaller elements, infinitesimally sized elements, the analytical solution will converge towards the theoretical solution. We conclude that ANSYS APDL can be a valuable tool in analyzing simple stress concentration problems, provided that a fine enough element mesh is used.