Supplementary Material for: Belief Updating in Sequential Games of Two-Sided Incomplete Information: An Experimental Study of a Crisis Bargaining Model

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1 Overview

In these supplementary materials we present the equilibrium analyses in more detail, a discussion of an Agent QRE version of the model, and a discussion of player strategies.

2 Perfect Bayesian Equilibrium

We solve for perfect Bayesian Equilibria using the standard backward induction method and assuming risk neutrality of the players. A summary of results is presented in Table 1. At the last node, Player 1 must choose between F and not F. Player 1 will Fight only if the payoff from F is greater than that from not Fighting:

$$W_1 + \mu_1 > B_1 \tag{1}$$

Let $\mu_1^F = B_1 - W_1$ be the cutoff private value above which Player 1 would choose Fight. Define $\Pr(F|E) = 1 - G(\mu_1^F)$ as the probability Player 1 Fights given that she has already Entered.

Next consider Player 2's decision node where he must choose between R and Not R. He will only choose R if the expected payoff from Resisting, $\Pr(F|E)[W_2 + \mu_2] + [1 - \Pr(F|E)]D_2$, is greater than the payoff from Not Resisting, B_2:

$$\Pr(F|E)W_2 + \Pr(F|E)\mu_2 + D_2 - \Pr(F|E)D_2 > B_2 \tag{2}$$

Let $\mu_2^R = \frac{B_2 - \Pr(F|E)W_2 - D_2 + \Pr(F|E)D_2}{\Pr(F|E)}$ be the cutoff private value above which Player 2 would choose Resist. Define $\Pr(R|E) = 1 - G(\mu_2^R)$ as the probability that 2 will Resist given that 1 has chosen to Enter.

Now consider 1's initial decision to Enter or Not Enter (E or Not E). She will choose E if the expected payoff from Entering, $[1 - \Pr(R|E)]D_1 + \Pr(R|E)\max[W_1 + \mu_1, B_1]$, is greater than the payoff from the status quo (Not Entering), S_1:

$$\max[W_1 + \mu_1, B_1] > \frac{S_1 - D_1}{\Pr(R|E)} + D_1 \tag{3}$$

If $B_1 > \frac{S_1 - D_1}{\Pr(R|E)} + D_1$, Player 1 would choose to Enter regardless of her private value, according to (3). Let $\mu_1^E = \frac{S_1 - D_1}{\Pr(R|E)} + D_1 - W_1$ be the cutoff private value above which Player

1 would choose to Enter if it is not the case that Player 1 chooses to Enter regardless of private value. Define $\Pr(E) = 1 - G(\mu_1^E) I(B_1 < \frac{S_1 - D_1}{\Pr(R|E)} + D_1)$ as the probability that Player 1 Enters at the initial decision node (where I is an indicator function).

We are now able to fully specify $\Pr(F|E) = \Pr(\mu_1 > \mu_1^F \,|\, \max[W_1 + \mu_1, B_1] > \frac{S_1 - D_1}{\Pr(R|E)} + D_1)$. Using the definition of conditional probability and then simplifying, we can rewrite this as

$$\Pr(F|E) = \frac{\Pr[(\mu_1 > \mu_1^F) \cap (\mu_1 > \mu_1^E)]}{\Pr(\mu_1 > \mu_1^E)}$$

We now solve for the equilibrium strategies and beliefs as a function of B_1, the cost to Player 1 of choosing to Not Fight and our treatment variable in the experiment. The players' types are drawn from a logistic distribution with mean 0 and variance 1. The CDF is $G(x) = \frac{e^x}{1+e^x}$ with a corresponding PDF of $g(x) = \frac{e^x}{(1+e^x)^2}$. The payoff parameters are $S_1 = S_2 = 0$, $D_1 = D_2 = W_1 = W_2 = 0.5$, and $B_2 = -0.3$. The equilibrium strategies are characterized by equilibrium cutpoints which players 1 and 2 use to decide between the binary choices at each of the decision nodes. Given the cutpoints, we can find the expected equilibrium belief that each player has about the other player's type after observing each possible action.

There are three possible action profiles that Player 1 could have: Not Enter, Enter and Not Fight, Enter and Fight. Note that Player 1's private value does not affect the comparison of payoffs in the Not Enter vs. Enter and Not Fight cases because the private value only influences the payoff when Fight is chosen. Therefore Player 1 has the same preference over the two action profiles regardless of private value.

Proposition 1 *(i) There exists a threshold B_1^H such that for all $B_1 > B_1^H$, Player 1 always chooses to Enter in the unique PBE.*
(ii) There exists a threshold B_1^L such that for all $B_1 < B_1^L$, Player 1 always choose Fight given Entry in the unique PBE.
(iii) For $B_1^L \leq B_1 \leq B_1^H$, there exists a PBE where a Player 1 in the low private value region chooses to Not Enter, a Player 1 in the intermediate private value region chooses to Enter then Not Fight, and a Player 1 in the high private value region chooses to Enter

then Fight.

Proof.

We first define the indifference condition and solve for the probability of Resist given Entry and Fight given Entry that would sustain indifference between Not Enter and Enter then Not Fight.

Player 1 is indifferent between Not Entering and Entering but Not Fighting if

$$[1 - \Pr(R|E)]D_1 + \Pr(R|E)B_1 = 0$$

Which simplifies to

$$\Pr(R|E) = \frac{D_1}{D_1 - B_1} \tag{4}$$

From the definition of $\Pr(R|E)$ in Section 3.1, we know that

$$\begin{aligned} \Pr(R|E) &= \Pr(\mu_2 > \frac{\Pr(F|E)(D_2 - W_2) + B_2 - D_2}{\Pr(F|E)}) \\ &= 1 - \frac{1}{1 + e^{\frac{0.8}{(F|E)}}} \end{aligned} \tag{5}$$

From this, we can solve for the probability of Fight given Entry that would sustain the indifference condition.

$$\Pr(F|E) = \frac{0.8}{ln(\frac{Pr(R|E)}{1 - Pr(R|E)})} \tag{6}$$

Part (i)

For all Player 1s who Enter, those with $\mu_1 > B_1 - W_1$ will Fight so

$$\begin{aligned} \Pr(E \cap F) &= \Pr(W_1 + \mu_1 > B_1) \\ &= \int_{-0.6}^{\infty} \frac{e^x}{(1 + e^x)^2} dx = 0.65 \end{aligned}$$

We first note that when $\Pr(E) = 1$, $\Pr(F|E) = \frac{\Pr(E \cap F)}{\Pr(E)} = 0.65$ which is the lowest value that $\Pr(F|E)$ can have. According to (5), $\Pr(R|E) = 0.774$ which is the highest possible value for $\Pr(R|E)$. In this case, the expected loss from Entering then encountering Resist and choosing Not Fighting, $\Pr(R|E)B_1$, is maximized for a given B_1 and the expected gain from Entering then encountered Not Resist, $[1-\Pr(R|E)]D_1$, is minimized. Still, there is a B_1^H above which the indifference condition cannot be sustained and Enter and Not Fight is always preferred to Not Enter: $[1 - \Pr(R|E)]D_1 + \Pr(R|E)B_1 > 0$.

$$B_1^H = \frac{[\Pr(R|E) - 1]D_1}{\Pr(R|E)} = -0.145$$

If Enter and Not Fight is always preferred to Not Enter, then Enter is always preferred at the first node. If Fight is chosen at the last node, then it must have been preferred to Not Fight. Since we have always established that the action profile Enter and Not Fight is always preferred to Not Enter, then the action profile Enter and Fight must also be preferred to Not Enter when it's chosen. Thus choosing Enter is always optimal at the first node. For $B_1 > B_1^H$, the unique PBE is then for Player 1 to always choose Enter then choose between Fight and Not Fight using the unique cutpoint pinned down by (1).

Part (ii)

We first note that when $\Pr(F|E) = 1$, the highest value possible, $\Pr(R|E) = 0.690$ according to (5). This is the lowest possible value for $\Pr(R|E)$. In this case, the expected loss from Entering then encountering Resist and choosing Not Fighting, $\Pr(R|E)B_1$, is minimized for a given B_1 and the expected gain from Entering then encountered Not Resist, $[1 - \Pr(R|E)]D_1$, is maximized. Still, there is a B_1^L below which the indifference condition cannot be sustained and Not Enter is always preferred to Enter and Not Fight: $[1 - \Pr(R|E)]D_1 + \Pr(R|E)B_1 < 0$.

$$B_1^L = \frac{[\Pr(R|E) - 1]D_1}{\Pr(R|E)} = -0.225$$

If Not Enter is always preferred to Enter and Not Fight, then if Player 1 chooses to Enter, she must also choose to Fight or else she would have chosen Not Enter in the first place. For $B_1 < B_1^L$, the unique PBE is then for Player 1 to decide between Enter and Not Enter using the unique cutpoint pinned down by (3) then to always choose Fight if Enter has been chosen.

Part (iii)

For this intermediate region of B_1 values, $B_1^L \leq B_1 \leq B_1^H$, we construct an equilibrium with three partitions of Player 1 private values, where those with the lowest private values will choose to Not Enter, those in the middle region of private values choose to Enter and Not Fight, and those with the highest private values will Enter and Fight. The indifference condition, thus equations (4) and (6), must hold in this equilibrium.

To find the cutpoint for Entry, C, we first note that

$$\Pr(E) = \Pr(\mu_1 > C) = \frac{\Pr(E \cap F)}{\Pr(F|E)}$$

After plugging in $\Pr(\mu_1 > C), \Pr(E \cap F)$ and $\Pr(F|E)$, we have

$$1 - \frac{1}{1 + e^{-C}} = \frac{0.8(1 + e^{W_1 - B_1})}{e^{W_1 - B_1} ln(\frac{-D_1}{B_1})}$$

We solve for the cutpoint

$$C = -ln(\frac{\alpha}{1 - \alpha})$$

Where $\alpha = \frac{0.8(1+e^{W_1 - B_1})}{e^{W_1 - B_1} ln(\frac{-D_1}{B_1})}$

Thus, in this equilibrium, Player 1 with $\mu_1 \geq B_1 - W_1$ Enters and Fights, Player 1 with $C \leq \mu_1 < B_1 - W_1$ Enters and Not Fights, and Player 1 with $\mu_1 < C$ chooses to Not Enter.

To illustrate this equilibrium solution, we use the example $B_1 = -0.2$ which lies in the intermediate region. First, we know that under the indifference condition

$$\Pr(R|E) = \frac{D_1}{D_1 - B_1} = 0.714$$

Next, we solve for probability of Fight given Entry using (4)

$$\Pr(F|E) = \frac{0.8}{ln(\frac{-D_1}{B_1})} = 0.873$$

Using the cutoff value for Enter and Fight that follows from the incentive compatibility condition at the terminal node, $\mu_1 > B_1 - W_1 = -0.7$, we solve for the probability of Enter and Fight

$$\Pr(E \cap F) = 1 - \frac{1}{1 + e^{0.7}} = 0.668$$

Finally, we find the cutoff value for Entry

$$C = -ln(\frac{\frac{0.873}{0.668}}{1 - \frac{0.873}{0.668}}) = -1.182$$

Thus, in this example, Player 1 with $\mu_1 \geq -0.7$ Enters and Fights, Player 1 with $-1.182 \leq \mu_1 < -0.7$ Enters and Not Fights, and Player 1 with $\mu_1 < -1.182$ chooses to Not Enter.

■

The B_1 we chose for our low cost treatment, -0.1, is higher than B_1^H. The B_1 we chose for our high cost treatment, -0.3, is lower than B_1^L.

2.1 PBE with Experimental Parameters

We now solve for the equilibrium cutpoints, action probabilities, and beliefs/expectations in the our two treatments: $B_1 = -0.1$ or -0.3

Low Cost: $B_1 = -0.1$

In the low cost condition, Player 1 always chooses to Enter regardless of private value.

$$\Pr(E) = 1$$

From (1), we know that 1 chooses F if $\mu_1 > B_1 - W_1 = -0.6$. With this cutpoint,

$$\Pr(F|E) = 1 - G(\mu_1^F) = \int_{-0.6}^{\infty} \frac{e^x}{(1 + e^x)^2} dx = 0.65$$

Similarly, Player 2 chooses R if $\mu_2 > \frac{-.8}{\Pr(F|E)} = -1.23$ according to (2). Given this cutpoint,

$$\Pr(R|E) = \Pr(\mu_2 > \mu_2^*) = \int_{-1.23}^{\infty} \frac{e^x}{(1 + e^x)^2} dx = 0.77$$

We can now calculate the equilibrium beliefs that each player holds after observing each possible action on the equilibrium path. 2's expectation of 1's type does not change from the prior of 0 when 2 observes E because in equilibrium, 1 always challenges regardless of μ_1.

$$E_1(\mu_1|E) = 0$$

The equilibrium solution is silent on 2's expectation of 1's type if 1 chooses Not E because this action is off the equilibrium path. In equilibrium, 2 chooses R if $\mu_2 > -1.23$ so 1's expectation of 2's type if 2 chooses R is

$$E_2(\mu_2|R) = \frac{\int\limits_{-1.23}^{\infty} \frac{xe^x}{(1+e^x)^2} dx}{\int\limits_{-1.23}^{\infty} \frac{e^x}{(1+e^x)^2} dx} = 0.69$$

Similarly, 1's expectation of 2's type if 2 chooses Not R is

$$E_2(\mu_2|\text{Not R}) = \frac{\int\limits_{-\infty}^{-1.23} \frac{xe^x}{(1+e^x)^2} dx}{\int\limits_{-\infty}^{-1.23} \frac{e^x}{(1+e^x)^2} dx} = -2.36$$

High Cost: $B_1 = -0.3$

All Player 1s who Enter decide to Fight regardless of private value so that the probability of Fight given Enter is 1:

$$\Pr(F|E) = 1$$

From (2), we have that $\mu_2 > \frac{-0.8}{\Pr(F|E)} = -0.8$ and

$$\Pr(R|E) = \Pr(\mu_2 > \mu_2^R) = \int\limits_{-.8}^{\infty} \frac{e^x}{(1+e^x)^2} dx = 0.69$$

Since $B_1 < \frac{-0.5}{\Pr(R|E)} + 0.5 = -0.22$, Player 1 chooses E if $\mu_1 > \mu_1^E = -0.5 - 0.22$ according to (3), so that

	Backdown payoff	
	Low Cost: $B_1 = -0.1$	High Cost: $B_1 = -0.3$
Cutpoints		
μ_1^E	$\forall \mu_1$	-0.72
μ_2^R	-1.23	-0.80
μ_1^F	-0.60	-0.80
Beliefs		
E_2: Enter	0	0.94
E_2: Not Enter	*N/A*	-1.93
E_1: Resist	0.69	.9
E_1: Not Resist	-2.36	-2
E_1: N/A	0	0

Table 1: Equilibrium Predictions

$$\Pr(E) = 1 - G(\mu_1^E)I(B_1 < \frac{-D_1}{\Pr(R|E)} + D_1) = \int_{-.72}^{\infty} \frac{e^x}{(1+e^x)^2} dx = 0.67$$

The equilibrium belief calculations in the high cost case are analogous to those in the low cost case. In equilibrium, 2 chooses R if $\mu_2 > -0.8$ so 1's expectation of 2's type if 2 chooses R, $E_2(\mu_2|R)$, is 0.90. Similarly, 1's expectation of 2's type if 2 chooses Not R, $E_2(\mu_2|\text{Not R})$, is -2.00. 1 chooses E if $\mu_1 > -0.72$ so 2's expectation of 1's type if 1 chooses E, $E_2(\mu_2|E)$, is 0.94. Similarly, 2's expectation of 1's type if 1 chooses Not E, $E_2(\mu_2|\text{Not E})$, is -1.93.

For both the Low Cost and High Cost treatments, we report in Table 1 the expectations one player should have (E_i) about the other player's private value after every possible action along with the cutpoints (μ_1^F, μ_2^R, and μ_1^E) at each decision node for both cost treatments. The cutpoints give us the truncated distribution from which the players form their equilibrium beliefs about the expectation of the other player's value. Note that we also elicit expectations of Player 1 about Player 2's type when Player 1 chooses Not E and Player 2 does not take an action (N/A).

3 Agent Quantal Response Equilibrium

To explore the deviations from PBE strategies that we have documented, we also derive the Agent Quantal Response Equilibrium (AQRE) (McKelvey and Palfrey, 1998) of this sequential game. In the AQRE, boundedly rational agents quantal respond rather than best respond. That is, they choose actions that yield higher payoff with higher probability but do not choose the optimal response, the action that yields the highest payoff, with certainty. We impose the usual error structure so that the logit function with a single parameter, λ, maps expected action payoffs to probabilities of action choices. [1] When $\lambda = 0$, agents are completely random and choose all actions with equal probabilities and as $\lambda \rightarrow 0$, agents approach full rationality. Under the AQRE framework, a different agent is assumed to choose the action at each decision node. Thus, Agent 1 chooses between Enter and Not Enter at the initial decision node, Agent 2 chooses between Resist and Not Resist, and Agent 3 chooses between Fight and Not Fight.[2]

At the final decision node, Agent 3 has the following utilities (assuming risk neutrality) from Fighting:

$$U_1^F = W_1 + \mu_1$$

and Not Fighting:

$$U_1^{NF} = B_1$$

Following the AQRE logit specification, the probability that Player 1 chooses to Fight is:

$$\begin{aligned} p_1(F) &= \frac{e^{\lambda_1 U_3^F}}{e^{\lambda_1 U_1^F} + e^{\lambda_1 U_1^{NF}}} \\ &= \frac{1}{1 + e^{\lambda_1 [B_1 - (W_1 + \mu_1)]}} \end{aligned} \tag{7}$$

At the second decision node, Agent 2 has the following expected utilities (assuming risk neutrality) from Resisting:

$$U_2^R = E_{\mu_1} p_1(F)(W_2 + \mu_2) + (1 - E_{\mu_1} p_1(F)) D_2$$

[1]Lewis and Schultz (2003), on the other hand, assume the normal distribution for the random utility error in their QRE formulation.

[2]We follow the approach of McKelvey and Palfrey (1998) in assuming that a player cannot predict her future play so a separate agent decides the action at each node.

where $E_{\mu_1} p_1(F) = \int\limits_{-\infty}^{\infty} (\frac{e^{\mu_1}}{(1+e^{\mu_1})^2}) \frac{e^{\lambda_1(W_1+\mu_1)}}{(e^{\lambda_1(W_1+\mu_1)}+e^{\lambda_1 B_1})} d\mu_1$

and Not Resisting:

$$U_2^{NR} = B_2$$

In the Agent Quantal Response Equilibrium, Agent 2, who chooses between Resist and Not Resist, treats the player who decides between Enter and Not Enter and the player who decides between Fight and Not Fight as two separate agents rather than the same agent. Therefore, Agent 2 does not update his belief about the private value of the player who will choose between Fight and Not Fight at the next node conditional on whether the player before him chose Enter or Not Enter. The expected probability of Fighting being chosen at the next node used to calculate Agent 2's utility from Resisting is the unconditional probability of Fighting averaged over the entire distribution of μ_1.

Following the AQRE logit specification, the probability that Agent 2 chooses Resist is:

$$\begin{aligned} q_2(R) &= \frac{e^{\lambda_2 U_2^R}}{e^{\lambda_2 U_2^R} + e^{\lambda_2 U_2^{NR}}} \\ &= \frac{1}{1 + e^{\lambda_2 [B_2 - E_{\mu_1} p_1(F)(W_2+\mu_2) - (1-E_{\mu_1} p_1(F)) D_2]}} \end{aligned} \tag{8}$$

At the first decision node, Agent 1 has the following expected utilities (assuming risk neutrality) from Entering:

$$U_1^E = (1 - E_{\mu_2} q_2(R)) D_1 + E_{\mu_2} q_2(R)[p_1(F)(W_1 + \mu_1) + (1 - p_1(F)) B_1]$$

where $E_{\mu_2} q_2(R) = \int\limits_{-\infty}^{\infty} (\frac{e^{\mu_2}}{(1+e^{\mu_2})^2})(\frac{1}{1+e^{\lambda_2[B_2 - E_{\mu_1} p_1(F)(W_2+\mu_2) - (1-E_{\mu_1} p_1(F)) D_2]}}) d\mu_2$

and Not Entering:

$$U_1^{NE} = S_1$$

The player who chooses between Enter and Not Enter treats the player who will decide between Fight and Not Fight down the line as a separate agent who has the same private value. She uses the unconditional probability for Fighting for that private value as well as the expected probability of Resist across the entire distribution of Agent 2's private values in calculating her utility from Entering.

Following the AQRE logit specification, the probability that Player 1 chooses Enter is:

$$\begin{aligned} p_1(E) &= \frac{e^{\lambda_1 U_1^E}}{e^{\lambda_1 U_1^E} + e^{\lambda_1 U_1^{NE}}} \\ &= \frac{1}{1 + e^{\lambda_1 [S_1 - (1 - E_{\mu_2} q_2(R)) D_1 - E_{\mu_2} q_2(R)[p_1(F)(W_1 + \mu_1) + (1 - p_1(F)) B_1]]}} \end{aligned} \tag{9}$$

We solve for the probability of Fight as a function of the private value and then solve for the probability of Resist which is dependent on the Fight probability and the probability of Enter which is in turn dependent on both the Fight and Resistance probabilities. Below we plot the probabilities of Enter, Resist, and Fight as a function of the private value which ranges from -10 to 10 at 0.1 increments. We did this for a wide range of λ (we assume that $\lambda = \lambda_2 = \lambda_1$), and plot four representative cases ($\lambda = .1, 2, 5, 10$) in Figures 1-3. The variation in probabilities, especially the probability of Entry, are consistent with the comparative statistics of the Perfect Bayesian Equilibrium, especially as λ increases. However, there is one notable exception that underscores the potential weakness of the AQRE approach. Agent 2 does not update his beliefs about the private value of the agent who will be making the Fight or Not Fight choice after he moves based upon the Entry decision. Since Agent 1 knows that Agent 2's expected probability of encountering Fight does not depend on her choice of Enter or Not Enter at all, her expected utility only changes with respect to her private value through the impact of that private value on the probability of choosing Fight vs. Not Fight. This produces the non-monotonic dip in the probability of Entry as the probability of Fight increases for a region of negative private values that is not found in the Perfect Bayesian Equilibrium. Figure 1 plots the probability of Entry under AQRE in the two cost treatments as we set $\lambda = 1, 2, 5, 10$. Figures 2 and 3 do the same for the probabilities of Resistance and Fight respectively.

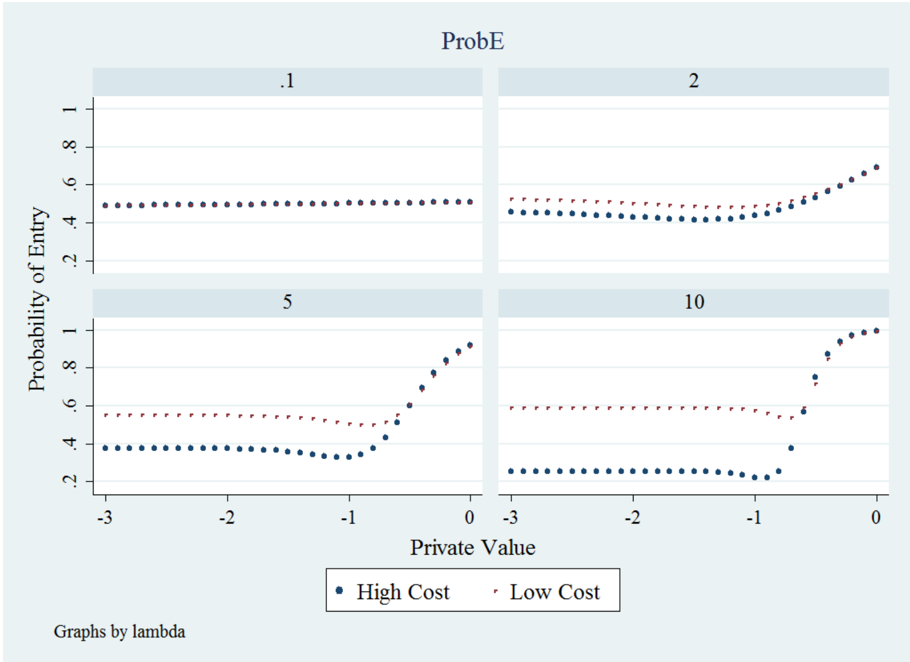

Figure 1: AQRE Probability of Entry by Treatment and λ

	λ	$-\ln L$
$B_1 = -0.1$	2.935	1153.7
$B_1 = -0.3$	5.052	812.5

Table 2: AQRE Estimation

We do maximum likelihood estimation of λ (assuming a common λ for all agents) separately for the low and high cost treatments. In contrast, Signorino (2003) and Signorino (1999) assume $\lambda = 1$ in characterizing the QRE solution rather than empirically estimating the best-fit λ. The likelihood function maximized is

$$L = \prod_{\mu=-10}^{10} p_1(E)^{n_E}(1-p_1(E))^{n_{NE}} q_2(R)^{n_R}(1-q_2(R))^{n_{NR}} p_1(F)^{n_F}(1-p_1(F))^{n_{NF}}$$

Where n_A is the number of times action A is chosen in the dataset.

We report the estimated λ for each treatment along with the negative log likelihoods in Table 2. The lower estimated λ in the low cost treatment compared to the high-cost treatment likely reflects the significant deviations from the always Enter PBE when the cost to Not Fighting is low.

Figure 4 graphs the predicted probabilities of Entry, Resistance, and Fight under

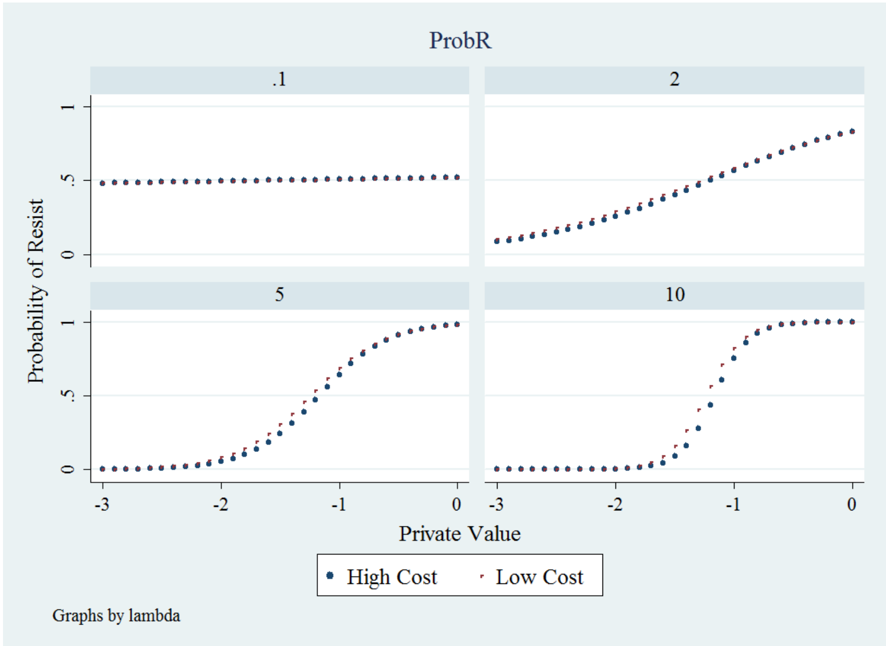

Figure 2: AQRE Probability of Resistance by Treatment and λ

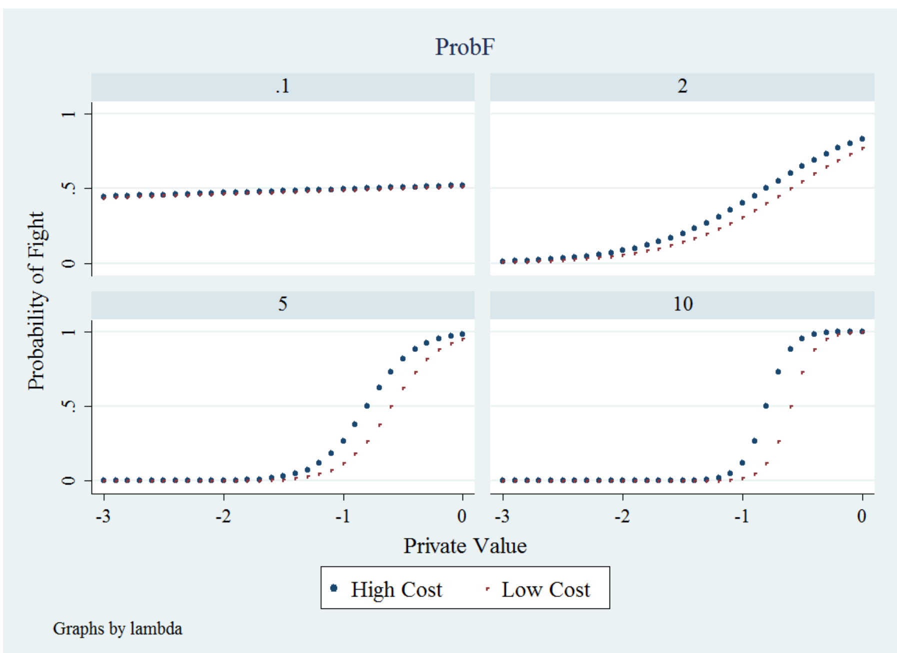

Figure 3: AQRE Probability of Fight by Treatment and λ

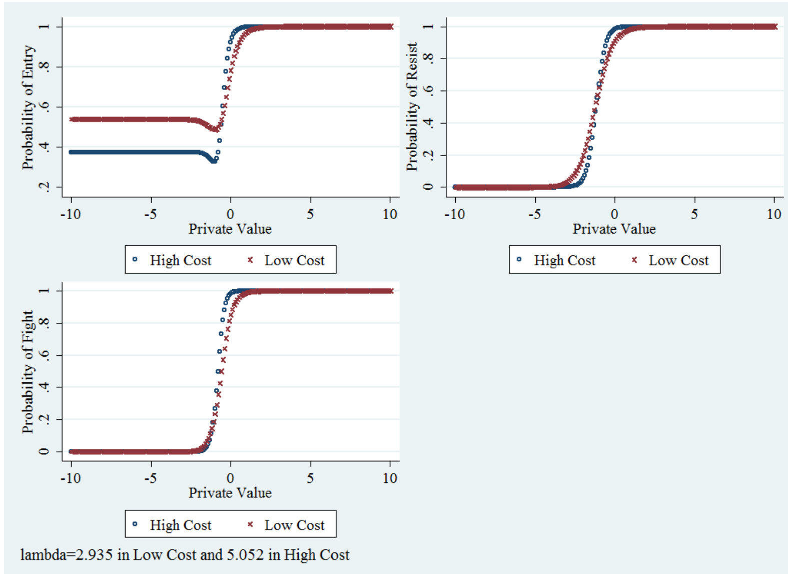

Figure 4: AQRE and Estimated Action Probabilities

the estimated as a function of the private values against the corresponding empirical frequencies. At the estimated λ's we see that most of the differentiation between the two treatments occurs with Entry decisions. This is consistent with the empirical results from the paper.

Finally, we solve for the equilibrium belief about the mean private value after observing each action under AQRE. The expectation that Player 2 has about Player 1's private value after observing Enter is

$$E_2(\mu_1|Enter) = \int_{-\infty}^{\infty} p(\mu_1|Enter)\mu_1 = \int_{-\infty}^{\infty} \frac{p(Enter|\mu_1)p(\mu_1)\mu_1}{\int_{-\infty}^{\infty} p(Enter|\mu_1)p(\mu_1)} \quad (10)$$

We approximate $E_2(\mu_1|Enter)$ by summing over μ_1 from -10 to 10 at discrete intervals of 0.1.

	High Cost			Low Cost		
Choice	Value	Belief	AQRE	Value	Belief	AQRE
Entry	.75(.05)	.77(.08)	0.57	.45(.06)	.58(.06)	0.42
No Entry	−1.87(.07)	−1.44(.10)	−1.72	−1.80(.06)	−1.16(.14)	−1.29
Resist	.86(.06)	.64(.10)	0.56	.72(.06)	.55(.05)	0.43
No Resist	−1.88(.05)	−1.36(.08)	−2.16	−1.93(.07)	−1.18(.11)	−1.96

Table 3: Equilibrium Beliefs in AQRE

For the low cost condition, $E_2(\mu_1|Enter) = \sum_{\mu_1=-10}^{10} \frac{0.1p(Enter|\mu_1)p(\mu_1)\mu_1}{\sum_{\mu_1=-10}^{10} 0.1p(Enter|\mu_1)p(\mu_1)}$

Unlike under PBE where the equilibrium strategy was to choose Enter regardless of the private value in the low cost condition, the probability of Entry is lower than 1 for a substantial range of private values in the low cost condition under AQRE. Thus, the mean private value of those who choose Enter is greater than 0, the prior, under AQRE.

Table 3 contains the equilibrium expectations after each action for both treatments. As expected, these equilibrium beliefs are closer to the empirical beliefs we observe. However, we still observe a clear asymmetry in deviations from equilibrium beliefs after Enter and Resist vs. No-Enter and No-Resist. Namely, the conservatism in belief updating is observed after the player observes a No-Enter or No-Resist decision.

4 Empirical Strategies

We explore subjects' strategy choices in several ways. First, we examine the average private value that subjects had when they choose a particular strategy (e.g., we find the average private value of subjects when they choose to Enter). Next, we compute the proportion of subjects who chose each strategy. Finally, we estimate a private value cutpoint for each subject that best predicts her decision at each node. In other words, we estimate the private value above which the subject choose one strategy (e.g., Enter) and below which the subject chose the other strategy (e.g., Not Enter) such that we recover the private value that optimally predicts her decision at each node (Carrillo and Palfrey, 2009). For example, if a cutpoint of −.3 correctly predicts 80% of a subject's Enter decisions, and −.5 predicts 60% of the subject's Enter decisions, then −.3 would be better. When several values optimally predict a strategy choice, we use the average of those values. We continue to use clustered standard errors in the analyses with repeated observations at the subject level.

4.1 Enter vs. Not Enter

The PBE solutions predict that there should be more Entry in the low cost treatment than in the high cost treatment. While there was more Entry in the low cost treatment, the prediction that all subjects would Enter is not observed. Nevertheless, we observe the proper ordering of Entry rates given the theory. Player 1s in the low cost treatment Entered 78% of the time, while the entrance rate was 71% in the high cost, with the difference being significant ($p < .05$). We also find that on average private values of those who Enter were significantly lower in the low cost case ($p < .01$).

4.2 Resist vs. Not Resist

In equilibrium, Player 2s who decide to Resist in the low cost treatment should have a lower mean private value than those who do so in the high cost treatment. The mean private value of those who do Resist is not significantly different between the low cost (.72) and high cost treatment (.86) ($p = .06$). However, we do observe that the proportion of subjects Resisting in the low cost case is higher (74%) versus the high cost case (70%) ($p < .05$), which is consistent with the lower equilibrium cutpoint.

4.3 Fight vs. No Fight

Player 1 should always Fight in the high cost case in equilibrium. While we do not observe this in the data, we do observe a very low rate of not Fighting, 9%. In addition, the proportion of subjects who chose Fight in the high cost case is significantly higher than in the low cost case ($p < .01$), with 83% fighting in the low cost case. Although the model predicts that the average private value should be lower in the high cost case where every player who chose to Enter should chose Fight in equilibrium, the average private value of players are not significantly different in the two treatments. The lack of sharp separation predicted by the equilibrium analysis at this decision node is a direct consequence of the not-as-sharp distinction in behavior across treatments at the initial node. If all Player 1s had chosen Enter in the low cost treatment, then the differences at the Fight vs. No Fight node would be more pronounced as well.

References

Carrillo, J. D. and Palfrey, T. R. (2009). The compromise game: Two-sided adverse selection in the laboratory. *American Economic Journal: Microeconomics*, **1**(1), 151–181.

Lewis, J. B. and Schultz, K. A. (2003). Revealing preferences: Empirical estimation of a crisis bargaining game with incomplete information. *Political Analysis*, **11**(4), 345–367.

McKelvey, R. D. and Palfrey, T. R. (1998). Quantal response equilibria for extensive form games. *Experimental Economics*, **1**, 9–41.

Signorino, C. (1999). Strategic interaction and the statistical analysis of international conflict. *American Political Science Review*, **93**(2), 279–298.

Signorino, C. (2003). Structure and uncertainty in discrete choice models. *Political Analysis*, **1**, 316–344.