Giusy Mazzone

On the motions of a liquid-filled pendulum *

Abstract We consider the motions of a coupled system constituted by a heavy pendulum with an interior cavity entirely filled by a viscous liquid. We give a complete description of the long-time behavior of this coupled system, and answer some questions regarding attainability and nonlinear stability of the equilibrium configurations. In the particular case of a liquid-filled physical pendulum, we show that weak solutions corresponding to initial motions having arbitrary finite total energy converge, for large times, to the rest state, no matter the shape of the cavity, the physical properties of the solid, and the initial motion imparted to the whole system. The rest state is characterized by only two equilibrium configurations corresponding to zero velocity and the center of mass at its lower and highest position, respectively. Concerning the nonlinear stability of these equilibria, we show that the one with the center of mass at its highest position is unstable, while the other one is stable, in the sense of Lyapunov. These results show that the liquid has a "stabilizing" effect on the motion of the rigid body, thus providing a rigorous mathematical evidence of some existing and newly performed numerical and experimental tests.

* Department of Mechanical Engineering and Materials Science, University of Pittsburgh. Email: gim20@pitt.edu.