Econ 2230: Public Economics

Lecture 8: Mechanism Design

Introduction

- Showed that private provision results in under provision of the public good
- Is it possible for the government to come up with rules that secure efficient provision of the public good
- Outline
  1. Lindahl Equilibrium
  2. Groves Clark
  3. Groves Ledyard

1: Lindahl equilibrium (1919)

- Objectives:
  - Can a Pareto optimal allocation be sustained in a decentralized manner?
  - Is it possible to solve the free rider problem through a market solution?
  - Simultaneously solving both the allocation and distribution problem

1: Lindahl equilibrium

- Intuition:
  - Private goods: Individuals maximize utility by letting marginal rates of substitutions equal the relative price. Since in a competitive equilibrium all individuals face the same relative prices, the implication is that marginal rates of substitutions are equalized across individuals, resulting in a Pareto efficient allocation. All individuals face the same price, but may consume different quantities of goods.
  - Public goods: Individuals have to consume the same amount of public good, thus individuals cannot equalize their marginal rates of substitutions to the common relative price. If consumption of the good must be the same is it possible to vary prices across individuals to get efficiency.
  - Private goods: consumption differs price the same
  - Want public goods: consumption same price differs

Lindahl Equilibrium

- Propose individualized prices set equal to the individuals $MRS^i_{G,x}$

- The Lindahl Equilibrium is a set of cost shares $\{t_1, t_2, \ldots, t_n\}$ and a public good provision G^* such that
  - $\sum_i t_i = 1$
  - $G^* = h^*_1(t_1) = h^*_2(t_2) = \ldots = h^*_n(t_n)$

- Provision decision effectively secured through unanimity all want the same provision

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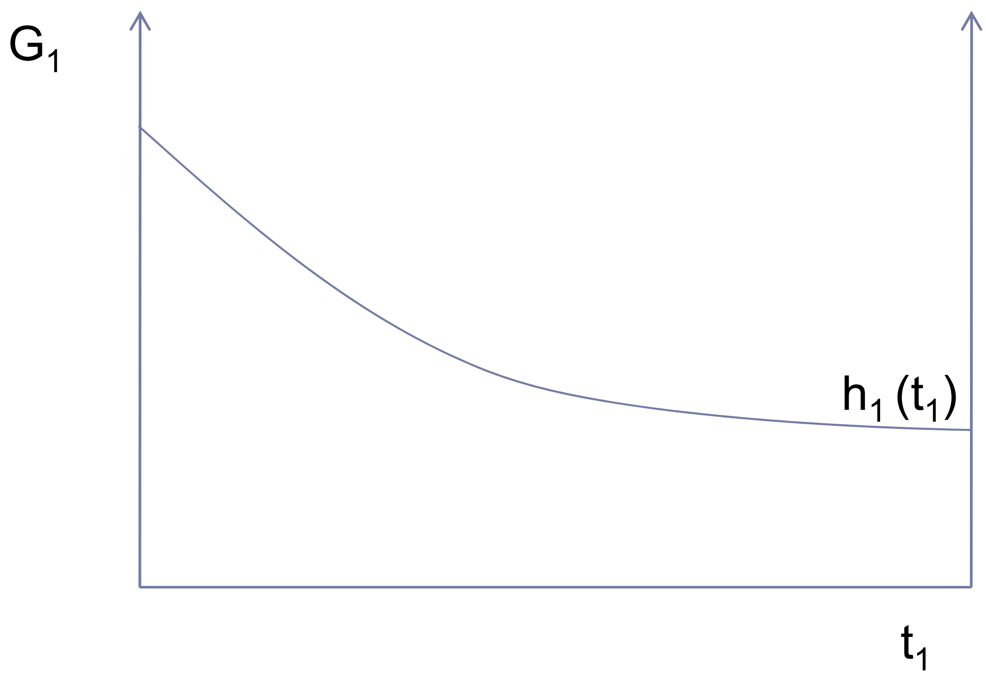

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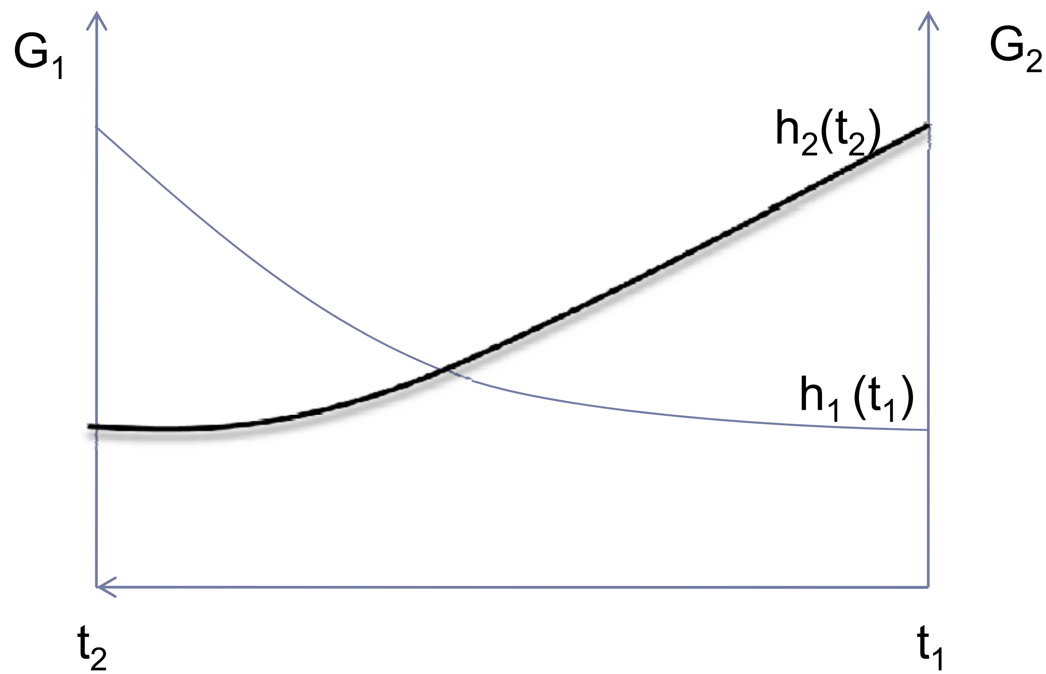

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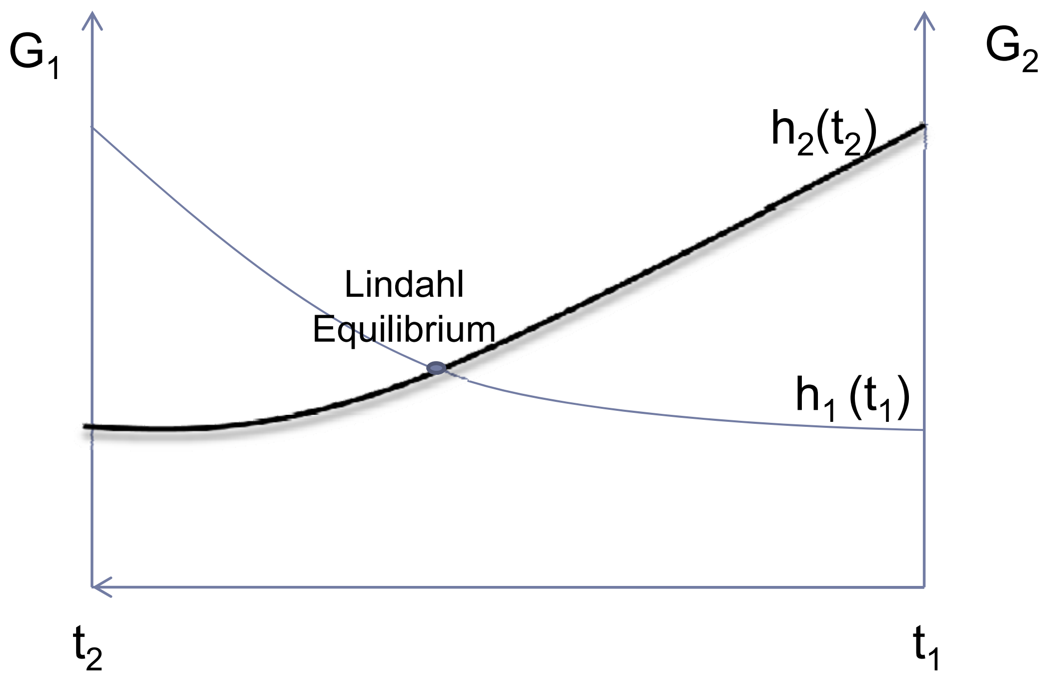

Lindahl Equilibrium

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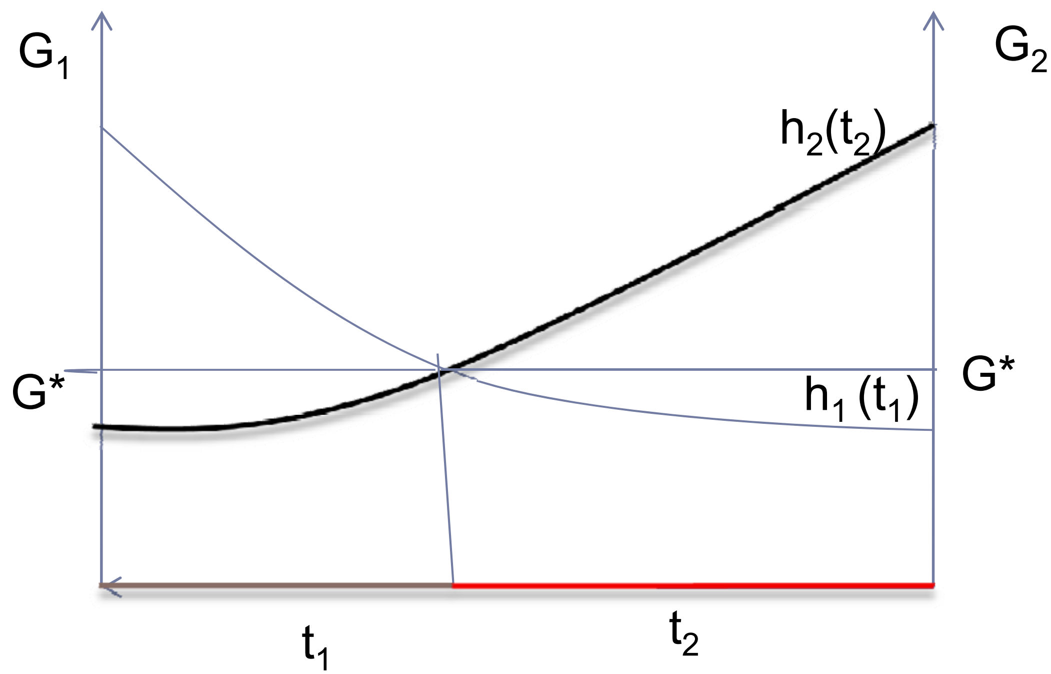


Lindahl equilibrium

- Standard voluntary provision problem:
  - Max $U(x_i, G)$ s.t. $x_i + q\, g_i = w$
  - Each i selects (x_i, g_i) such that $MRS^i_{G,x} \le q$ (with = for some i)
  - Thus $\sum MRS^i > q$
- Lindahl equilibrium:
  - Personalized price: $t_i q = MRS^i$ and $\sum_i t_i = 1$
  - Max $U(x_i, G)$ s.t. $x_i + t_i q \cdot g_i = w$
  - Each i selects (x_i, g_i) such that $MRS^i_{G,x} = t_i q$
  - Thus $\sum_i MRS^i = \sum_i t_i q = q \sum_i t_i = q$

Lindahl equilibrium

- Lindahl equilibrium results in a Pareto efficient provision of the public good
- Suppose public good is normal, preferences identical and individuals have different endowments – how does t_i vary with wealth?
  - Lindahl price increasing with wealth. Richer individual would prefer higher provision level, to secure that all demand the same provision in equilibrium t_i must be increasing in wealth
- Difference between Lindahl equilibria and standard competitive equilibrium: No decentralized mechanism for deriving prices; no market forces that will generate the right price vector (Foley)
- Example:

Lindahl Example

- Two individuals A, B
  - $U_A = x_A^{1-\alpha} G^{\alpha}$
  - $U_B = x_B^{1-\beta} G^{\beta}$
  - B.C. $x_i + t_i \, q \, G = w_i$

- Find the Lindahl Equilibrium

Lindahl equilibrium

- Do we know that a Lindahl equilibrium exists? Can we always find a price vector where everyone wants to consume the same amount of the public good?
  - Foley (Econometrica 1970). Insight in an Arrow-Debreu environment we know a competitive market exist). Construct a private good economy such that we can apply the results from a competitive market. View public good like a technology of fixed proportion, simultaneously producing the same output for every individual => Lindahl Equilibrium exists

Lindahl equilibrium

- Efficiency:
  - Crucial for efficiency is that we know the individuals demands for the public good

- Lindahl mechanism?

Mechanism Design

- Engineering part of economic theory. Start with goals – find mechanisms that can implement it. Normative/prescriptive
- Let
  - A – alternatives
  - i observes preferences θ_i over A
  - $\theta_i \in \Theta_i$
  - Let f denote the social choice function: f: Θ_1x Θ_2x ….x $\Theta_n \rightarrow$A
  - That is f assigns a collective choice for each profile of agents
- Mechanism definition:
  - A mechanism $\Gamma = (S_1, S_2, \ldots, S_n, g(\,))$ is a game where $S = S_1 \times S_2 \times \ldots \times S_n$ is the message space and $g(\,): S \rightarrow A$ the outcome rule

Mechanism Design

- We say that a mechanism Γ implements f if there is an equilibrium of Γ that yields the same outcome as f for each possible profile ($\theta_1, \theta_2, \ldots, \theta_n$)
- Gibbard's revelation principle
  - Any outcome achievable by a mechanism is achievable by a direct mechanism

$$
\begin{tikzcd}
\Theta \arrow[rr, "f"] \arrow[dr] & & A \\
& S \arrow[ur, "g"'] &
\end{tikzcd}
$$

Example of an indirect mechanism

- N=1,
- Productivity type Θ_i = {L, H}
- Allocations A={T(ough), E(easy), wage}
- Individual cost of job

		Productivity	
		Low	High
Job	Tough	10	5
	Easy	4	4

- Social choice function: f(H) = T, f(L) =E
- Can we just ask them about type? Suppose pay $6
- Indirect mechanism ask which job they want:
  - S= {T, E}, g: S →{T, E, wage}
- What could an indirect mechanism look like?
- g(T) = (T, $8), g(E) = (E, $4)
- What would the direct mechanism look like?

Lindahl Mechanism

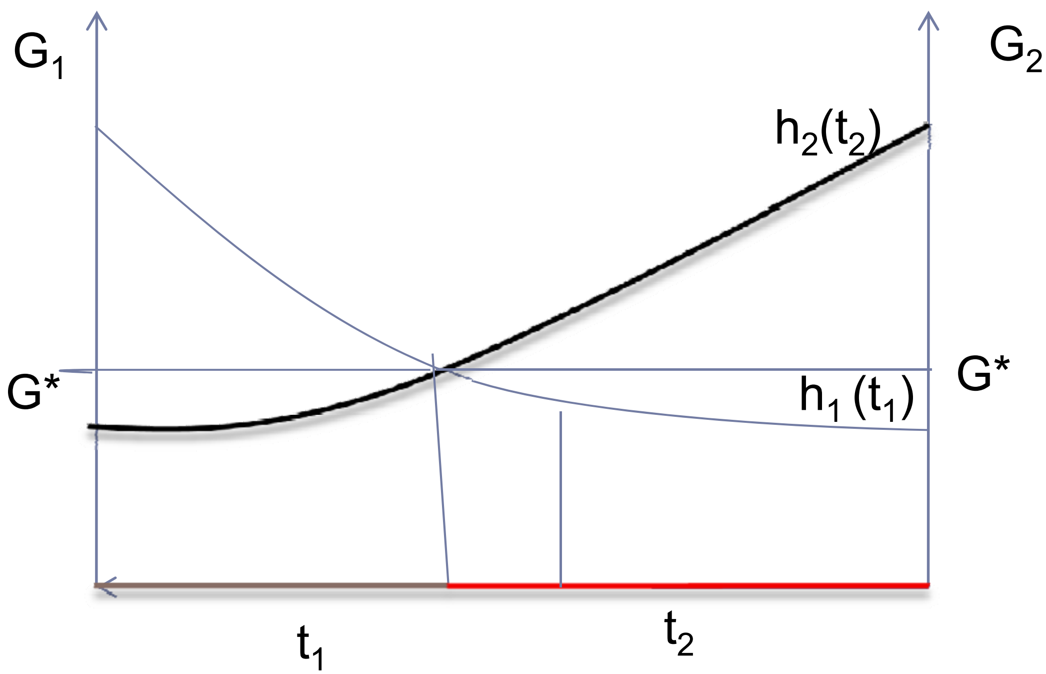
- Need to learn demand functions
- Lindahl mechanism
  - Message: $s_i(g)$ (marginal willingness to pay schedule)
  - Allocation Rule: choose g such that $\Sigma_i\, s_i(g) = q$
  - Tax Rule: $t_i = s_i(g(s)) \cdot q \cdot g(s)$

- Alternatively
  - Message: $h_i(t_i)$ (report the amount of public good you want when cost share is t_i)
  - Allocation Rule: choose g such that
    - $h_1(t_1) = h_2(t_2) = .. = h_n(t_n) = g$
    - $\Sigma_i\, t_i(g) = 1$
  - Tax Rule: $t_i = t_i(g) \cdot q \cdot g$

- Will this mechanism implement the social choice function?

Lindahl Equilibrium: truthful reporting?

- The Lindahl Equilibrium is a set of cost shares $\{t_1, t_2, \ldots, t_n\}$ and a public good provision G^* such that
  - $\sum_i t_i = 1$
  - $G^* = h^*_1(t_1) = h^*_2(t_2) = \ldots = h^*_n(t_n)$



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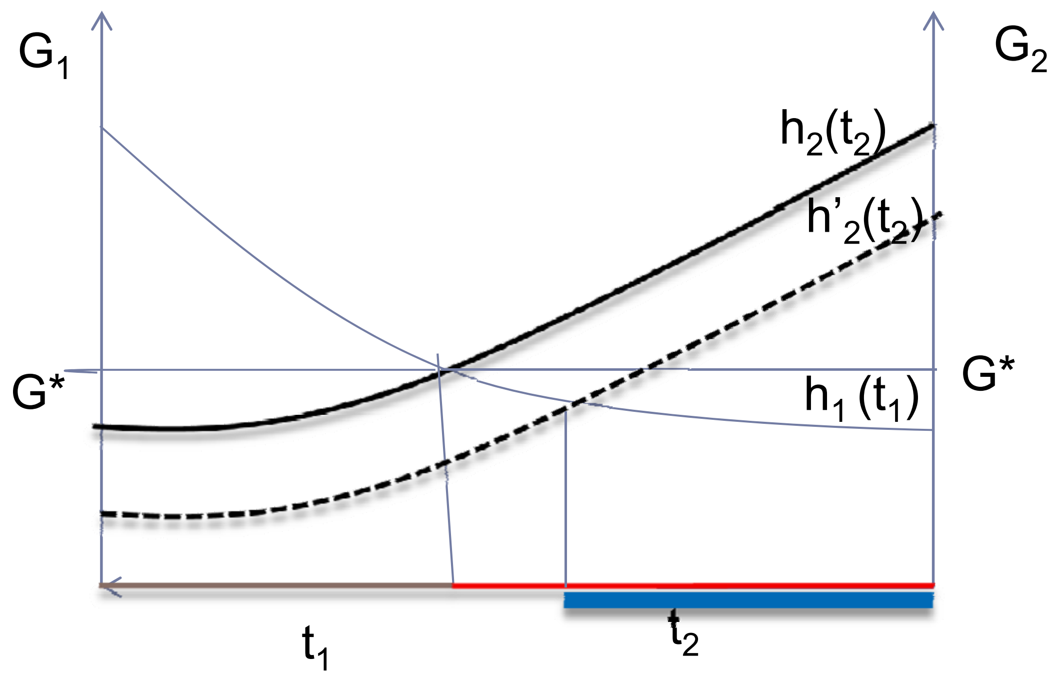

Lindahl Equilibrium

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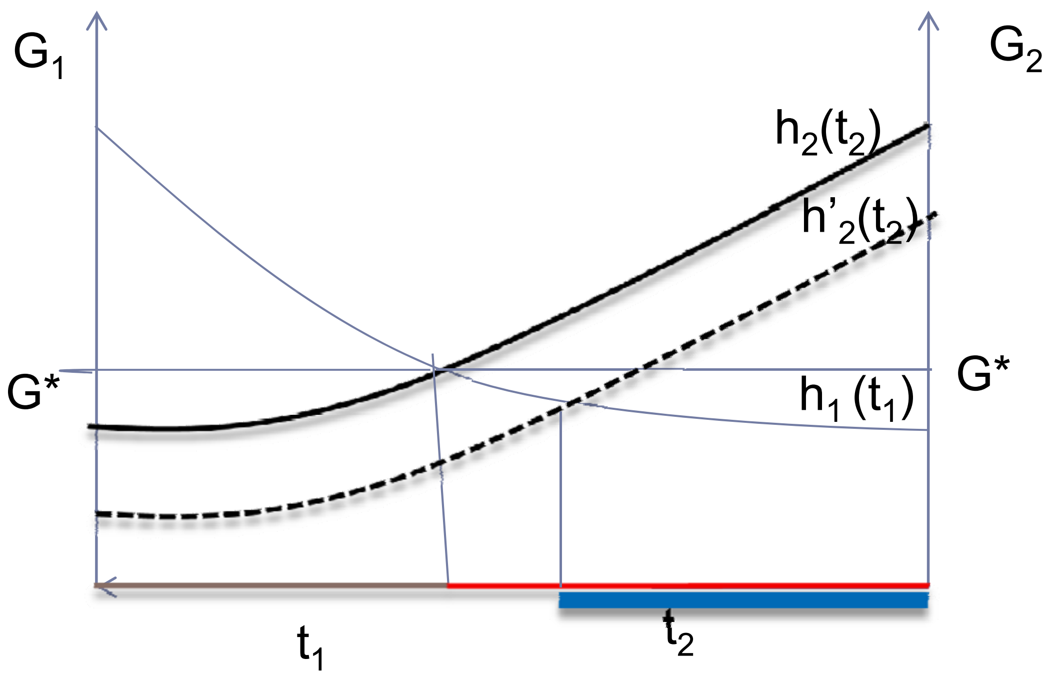

By underreporting preference for G individual 2 secures a lower tax

Lindahl Equilibrium

- Since individuals have an incentive to underreport the Lindahl mechanism is not efficient
- If the government is to provide the public goods efficiently, it needs to have information on preferences/valuations/demands.
- If that information is privately held, then the government must setup an incentive system that induces individuals to reveal preferences, valuations, and demands.
- The goal of mechanism design is to determine how individual preferences can be elicited and aggregated into desirable social or collective decisions.
- Do there exist rules for the provision and financing of public goods that lead individuals to report true information and allow the government to implement efficient allocations?

- Look at examples of mechanisms that do secure efficient provision of the public good

- We say that a mechanism Γ implements f if there is an equilibrium of Γ that yields the same outcome as f for each possible profile (θ_1, θ_2,…, θ_n)

- Three main equilibrium concepts:
  - Dominant Strategy Equilibrium
  - Nash Equilibrium
  - Bayesian Nash Equilibrium

2:Dominant Strategy Implementation

- We say that a strategy profile s*() is a dominant strategy equilibrium of the mechanism if for all i and all θ_i

  $u_i\ (g(s_i^*(\theta_i), s_{-i}), \theta_i) \geq u_i\ (g(s'_i, s_{-i}), \theta_i)$ for all $s'_i \in S_i$ and all $s_{-i} \in S_{-I}$

- Independent of messages sent by others i always reveals her type

- The mechanism $\Gamma = (S_1, S_2, \ldots, S_n, g(\,))$ implements the social choice function f () in dominant strategies if there exists a dominant strategy equilibrium of Γ, s*() such that $g(s^*(\theta)) = f(\theta)$ for all $\theta \in \Theta$

- The Revelation Principle for Dominant Strategies: Suppose that there exists a mechanism $\Gamma = (S_1, S_2, \ldots, S_n, g(\,))$ that implements the social choice function f () in dominant strategies. Then f is truthfully implementable in dominant strategies

Dominant strategy implementation: Groves-Clark Mechanism

- Discrete public good: $g \in \{0,1\}$
- Suppose
  - q: the cost of producing the public project.
  - r_i: the maximum willingness to pay by i
  - t_i: i's tax ($t_i \in R$)

- Efficient outcome provide public good ?
- if $\sum r_i > q$

- Social choice function f:
  - $g = 1$ if $\sum r_i > q$
  - $g = 0$ if $\sum r_i \leq q$

Groves-Clark Mechanism

- How do we learn r_i?
- If no effect on tax overstate
- If effect on tax understate

- Groves-Clark mechanism
  - Message: s_i {report wtp}
  - Allocation Rule:
    - $g = 1, t_i = q - \Sigma_{j\neq i} s_j$ if $\Sigma_i s_i > q$
    - $g = 0, t_i = 0$ if $\Sigma_i s_i > q$

- Can Γ_{GL} implement f?

Groves-Clark Mechanism

- Allocation Rule:
  - $g = 1,\ t_i = q - \Sigma_{j\neq i}\, s_j$ if $\Sigma_i\, s_i > q$
  - $g = 0,\ t_i = 0$ if $\Sigma_i\, s_i > q$

- Can Γ_{GL} implement f? Is it optimal for agent i to report $s_i = r_i$?

  - $U_i = w_i – (q - \Sigma_{j\neq i}\, s_j\ ) + r_i$ if $g=1$
  - $U_i = w_i$ if $g=0$

- When does i want the project?
  - $w_i – (q - \Sigma_{j\neq i}\, s_j\ ) + r_i\ \ > w_i$
  - $r_i\ \ > (q - \Sigma_{j\neq i}\, s_j\ )$
- Truth telling is a dominant strategy

Groves-Clark

- g implements f in dominant strategies. Independent of reports by others individuals prefer to reveal their type
- Why does it work?
  - s_i has no effect on t_i except when i is pivotal
  - E.g.
    - $q > \Sigma_{j \neq i} s_j$ and $q < s_i + \Sigma_{j \neq i} s_j$
    - $q < \Sigma_{j \neq i} s_j$ and $q > s_i + \Sigma_{j \neq i} s_j$
    - When pivotal the tax is such that the individual internalizes the externality of the decision
    - $dt_i = 0 – (q - \Sigma_{j \neq i} s_j) = \Sigma_{j \neq i} s_j - q = \Sigma_{j \neq i} r_j - q$ (net cost to society)

Groves Clark

- Example
  - $q = \$300$
  - $(r_A, r_B, r_C) = (0\ ,\ 150, 200)$
  - Efficient to build
- Taxes if built?
- $(t_A, t_B, t_C) = (-50\ ,\ 100, 150)$
- Efficient provision

- Budget
  - Revenue =$200, cost = $300
  - Alternative; absent being pivotal the cost is split equally
  - $t_i = q - \Sigma_{j\neq i}\, s_j + \max\{0, \Sigma_{j\neq i}\, s_j - q\,(n-1)/\,n\}$

Groves Clark

- Alternative tax rule
- $t_i = q - \Sigma_{j\neq i}\, s_j + \max\{0, \Sigma_{j\neq i}\, s_j - q\,(n-1)/\, n\}$
- $t_i = [q + (n-1)q]/n - \Sigma_{j\neq i}\, s_j + \max\{0, \Sigma_{j\neq i}\, s_j - q\,(n-1)/\, n\}$
- $t_i = q/n - \{ \Sigma_{j\neq i}\, s_j - q\,(n-1)\,/n \} + \max\{0, \Sigma_{j\neq i}\, s_j - q\,(n-1)/\, n\}$

- If average valuation by each of the (n-1) exceed q/n, then pay q/n
- If pivotal pay $q/n - \{ \Sigma_{j\neq i}\, s_j - q\,(n-1)\,/n \}$ accounting for the externaliy
- $t_i \geq q/n \Rightarrow \Sigma_i\, t_i \geq q$ thus cost covered

- Is it efficient?

Groves Clark

- $t_i = q - \Sigma_{j\neq i} s_j + \max\{0, \Sigma_{j\neq i} s_j - q (n-1)/ n\}$
- Example
  - q = $300
  - $(r_A, r_B, r_C) = (0 , 150, 200)$
  - Efficient to build
- Taxes
- $(t_A, t_B, t_C) = (100 ,100, 150)$

- Budget
  - Revenue =$350, cost = $300
  - $50 budget surplus
  - Individual rationality

Budget balance

- So far, we have studied whether we can implement in dominant strategies a social choice function that results in an efficient allocation
- With continuous provision: In quasi-linear preference environments, the Groves-Clark Mechanism implements the efficient level of the public good with true reporting as a dominant strategy equilibrium.
- Problem: the preference revelation mechanism may require very large side-payments. It may be very costly to induce the agents to tell the truth.
- Efficiency does not only require efficient provision of the public good but also balanced budget
- A mechanism Γ implements efficient allocations if in equilibrium

  (i) Allocation is efficient (Samuelson condition satisfied)

  (ii) Allocation is feasible (Balanced government budget)

Implementation in dominant strategies

- Impossibility

- Proposition (Green and Laffont/Hurwicz) : There is no social choice function that is truthfully implementable in dominant strategies and secures efficient provision and budget-balance.

3. Nash Implementation

- Cannot secure efficiency in dominant strategies – need to relax the dominant strategy implementation
- Groves-Ledyard Mechanism (1977, Econometrica) was the first to give a specific mechanism that Nash implements Pareto efficient allocations for public goods economies.

Groves-Ledyard Mechanism

- Efficiency:
  - $\Sigma_i\, t(s_i) = g(s) \cdot q$
  - $\Sigma_i\, MRS_i = q$

- where
  - s_i – message by i
  - q – unit cost of the public good
  - g(s) - provision of the public good given s
  - $t(s_i)$ - tax on i given s_i

Groves-Ledyard Mechanism

- Mechanism:
  - Message:
    - $S = (s_1, s_2, \ldots, s_n)$
    - s_i = Agent i sends a message that reports the amount i would like the government to add or subtract from amounts requested by others
  - Allocation rule:
    - Provision: $g(s) = \Sigma_i s_i$
    - Tax: $t_i(s_i) = q \cdot g(s)/n + [\gamma/2] \cdot [\,(n-1)/n \cdot (s_i - \mu(s_j))^2 - \sigma(s_i)]$
    - where
      - $\mu(s_j)$ mean of $s_{j\neq i}$, $\mu(s_j) = \Sigma_j s_j / (n-1)$
      - $\sigma(s_j)$ variance of $\sigma(s_{j\neq i}) = \Sigma_{j\neq i} [s_j - \mu(s_j)]^2 / (n-2)$
      - $\gamma>0$ penalty for deviating from the mean

Groves-Ledyard Mechanism

- Allocation rule:
  - Provision: $g(s) = \Sigma_i s_i$
  - Tax: $t_i(s_i) = q \cdot g(s)/n + q \cdot g(s)/n + [\gamma/2] \cdot [\ (n-1)/n \ \cdot (s_i - \mu(s_j))^2 - \sigma(s_i)]$

- Suppose everyone reports same s_i
- $t(s_i) = q \cdot g(s)/n$

- Heterogenous preferences: punished (γ) for deviating form the mean of others but less so if others are very dispersed.

Groves-Ledyard Mechanism

- Show efficient
  - $\Sigma_i\, t(s) = g(s) \cdot q$
  - $\Sigma_i\, MRS_i = q$

- Consider quasi linear preferences
  - Max $V_i(g(s_i)) – t(s_1, s_2,\ldots, s_n)$
  - Max $V_i(g(s_i)) – q{\cdot}g(s)/n + [\gamma/2] \cdot [\,(n-1)/n \;\; \cdot(s_i - \mu(s_j))^2 - \sigma(s_i)]$
  - Max $V_i(g(s_i)) –q{\cdot}g(s)/n + [\gamma/2] \cdot [\,(n-1)/n \;\; \cdot(s_i - \mu(s_j))^2 - \sigma(s_i)]$

  - F.O.C.
    - $V'_i(\,) = t'(s_1, s_2,\ldots, s_n) \;= q/n + [\gamma/2] \cdot [\,(n-1)/n\,] \cdot 2 \cdot[s_i - \mu(s_i)]$
    - $V'_i(\,) = t'(s_1, s_2,\ldots, s_n) \;= q/n + [\gamma] \cdot [\,(n-1)/n\,] \cdot [s_i - \mu(s_i)]$

Groves-Ledyard Mechanism

- $V'_i(\ ) = q/n + [\gamma] \cdot [\ (n-1)/n\ ] \cdot [s_i - \mu(s_i)]$

- Check Samuelson condition
- $\Sigma_i\, MRS_i = q$
- $\Sigma_i\, V'_i(\ ) \quad = \Sigma_i \{q/n + [\gamma] \cdot [\ (n-1)/n\ ] \cdot \cdot [s_i - \mu(s_i)]\}$
- $= \Sigma_i \{q/n + [\gamma] \cdot [\ (n-1)/n\ ] \cdot [s_i - [g(s)\text{-}s_i]/(n\text{-}1)]\}$
- $= q + [\gamma \cdot (n-1)/n\ ]\ \Sigma_i\, [s_i - [g(s)\text{-}s_i]/(n\text{-}1)]\}$
- $= q + [\gamma \cdot (n-1)/n\ ]\ [g(s) - [ng(s) - g(s)]/(n\text{-}1)$
- $= q$

Groves-Ledyard Mechanism

- Recall tax $t_i(s) = q\cdot g(s)/n + q\cdot g(s)/n + [\gamma/2] \cdot [(n-1)/n \cdot(s_i - \mu(s_j))^2 - \sigma(s_i)]$
- Example:
  - $N=2$, $\gamma = 1$, $q = 2$
  - $U_1= w_1 - t_1 + V_1 = w_1 - t_1 + 2\cdot g(s) - 0.5 \cdot g(s)^2$
  - $U_2= w_2 - t_2 + V_2 = w_2 - t_2 + 3\cdot g(s) - g(s)^2$
- Solution
  - $G=1$
  - $s_1 = ½ = s_2$
  - $t_1 = 1 = t_2$
- Budget balance a coincidence generally only works with $n>2$, as $\sigma(s_i)^2$ is what balances the budget

Groves-Ledyard Mechanism

- Recall tax $T_i(s) = q \cdot g(s)/n + [\gamma/2] \cdot [\ (n-1)/n \ \cdot (s_i - \mu(s_j))^2 - \sigma(s_i)]$
- Example:
  - $N=3$, $\gamma = 1$, $q = 3$
  - $V_i = A_i \cdot g(s) - B_i \cdot g(s)^2$

	A_i	B_i	s_i	T_i
Al	2	½	0	0.125
Betty	3	1	-1/2	1.25
Charlie	4	½	2	3.125
sum			1.5	4.5

- Pareto efficient and budget balanced?

Groves-Ledyard Mechanism

- Recall tax $T_i(s) = q \cdot g(s)/n + [\gamma/2] \cdot [\ (n-1)/n \ \ \cdot(s_i - \mu(s_j))^2 - \sigma(s_i)]$
- Example:
  - $N=3$, $\gamma = 1$, $q = 3$
  - $V_i = A_i \cdot g\ (s) - B_i \cdot g\ (s)^2$

	A_i	B_i	s_i	T_i
Al	2	½	1/3	1
Betty	3	1	1/3	1
Charlie	4	3/2	1/3	1
sum			1	3

  - Pareto efficient and budget balanced?

Groves-Ledyard Mechanism

- Proposition: The Groves-Ledyard mechanism implements Pareto efficient allocations in Nash equilibrium.

- Properties of GL
  - PAS
  - Budget balanced (if n>2)

- Weakness
  - Implementable as Nash equilibrium – why a weakness?
  - Absent quasilinear preferences there may be multiple equilibria may give individuals an incentive to misrepresent preferences to secure the most preferred equilibrium
  - Does not generally satisfy individual rationality

Next:

- Experimental
  - Chen and Plot, The Groves-Ledyard mechanism: An experimental study of institutional design, *Journal of Public Economics*, 1996
  - Yan Chen and Robert Gazzale. "When Does Learning in Games Generate Convergence to Nash Equilibria? The Role of Supermodularity in an Experimental Setting." *American Economic Review,* 2004