Midterm Exam

PS 2703: Formal Political Theory
October 12, 2007

Instructions. You have 24 hours to complete this exam. It is due at exactly 3:00 PM on Saturday, October 13, 2007. Ideally, your answers should be typed.

The exam is restricted open book. You may consult both textbooks (McCarty and Meirowitz or Osborne), any of your notes, and any course materials handed out in class or on the course web page (http://www.pitt.edu/~woon/courses/ps2703.html). You are prohibited from using the Internet for exam-related purposes, and you may not work with nor consult any other person.

Do all of the assigned problems. Almost all of the questions are suitable for an in-class exam. Partial credit is available if you attempt a problem while you will automatically receive no points for missing answers. There are eight required questions worth a total of 120 points. There are also three bonus questions worth a total of 30 points.

Explanations are only required if a question explicitly asks for one. Where an explanation is required, your answer should be brief but complete. There are no penalties for long-winded answers, but in the interest of time, you should focus on getting the answers right.

I will be available by email if you have questions, but cannot guarantee how long it will take to respond. If any question seems unclear to you, then write in your own words what you think the question means and any additional assumptions you think are required to solve the problem.

Good luck!

1. Rational Voting (10 pts)

Suppose that George is indifferent between the Democrats or Republicans controlling the government but voted for the Democratic candidate in the last presidential election. Is his vote consistent or inconsistent with the theory of rational choice? Provide a brief explanation.

2. Logic and Game Theory (10 pts)

Which of the following statements are logically equivalent? (It might be helpful to express each statement in its logical form.) Given what you know about game theory, which ones are true?

S1: If the action profile (a_1, a_2) is a Nash equilibrium then a_1 is a best response to a_2.

S2: If a_1 is a best response to a_2 then (a_1, a_2) is a Nash equilibrium.

S3: If a_1 is not a best response to a_2 then (a_1, a_2) is not a Nash equilibrium.

3. Revolution (10 pts)

Consider the following simple model of revolution. There are two citizens, Jane and Kane. Each citizen can choose to revolt or stay home. If they both revolt, then the government is overthrown and each citizen receives a payoff of g. If only one citizen revolts, then the citizen that revolted is put in prison and receives a payoff of p while the other citizen remains free and receives the payoff f. If neither citizen revolts, then they both remain free and receive the payoff f. Suppose that $g > f > p$.

		Citizen Kane: Revolt	Citizen Kane: Stay home
Citizen Jane	Revolt	g, g	p, f
	Stay home	f, p	f, f

(a) Does this game resemble any of the 2x2 games we have seen in class? If so, which one?

(b) Explain what is wrong with the following argument:

> Both citizens staying home is not a Nash equilibrium because if they both revolted they would be able to overthrow the government and each would obtain a higher payoff $g > f$.

4. Social Choice in a Bicameral Legislature (25 pts)

Let the set of policy alternatives be X = {a,b,c,d} and suppose the set of agents N is partitioned into two subsets C = {1,2,3} and L = {4,5,6}. Consider the *bicameral* preference aggregation rule where the social preference between an arbitrary pair of alternatives $x, y \in X$ is xPy if and only if at least a majority of C and at least a majority of L strictly prefer x to y. Assume that the rule is complete and reflexive.

For parts (a) through (d) below consider only the following preference profile ρ in which individual preferences are strict:

1	2	3	4	5	6
a	a	a	b	b	b
b	c	c	a	c	c
c	d	d	c	a	a
d	b	b	d	d	d

(a) Given the preference profile ρ and the bicameral preference aggregation rule described above, what is the social preference between each pair of alternatives?

(b) Are the social preferences you found transitive or quasi-transitive? Does this tell you anything about the existence of a core?

(c) What is the core for the bicameral rule given the profile ρ? Why is it important if the core exists?

(d) Does the bicameral preference aggregation rule satisfy the weak Pareto property for the profile ρ? Is it non-dictatorial? Given what you know about this aggregation rule, does it illustrate Arrow's Theorem or does it contradict it?

(e) Suppose there are only three alternatives, X = {a,b,c}. Construct a preference profile such that the bicameral preference aggregation rule produces a social preference cycle.

5. Dominated Strategies (20 pts)

Consider the following matrix representation of a normal form game:

	E	F	G	H
A	1, 3	2, x	0,2	0, 4
B	3, x	1, 1	7,11	2, 3
C	2, 2	3, 2	4,7	1, 10
D	1, 4	0, 1	6, 3	3, 1

(a) For what values of x does the game have multiple Nash equilibria?

(b) For what values of x does E either weakly or strictly dominate F? Is the domination weak or strict?

(c) For what values of x is F strictly dominated? Which strategy strictly dominates it?

(d) What strategy profiles survive the iterated elimination of strictly dominated strategies? Does the set of surviving profiles depend on the value of x? If so, how? (Do not make any assumptions about x until *after* you finish eliminating strategies.)

6. Healthy Forests (15 pts)

Suppose that there is a group of 5 people. Each person owns one-fifth of a forest that is worth $100 if used privately by its owner but earns $150 if used for the "common good." However, any money earned "for the common good" is divided equally between *all* members of the group (regardless of whether or not they contributed their resource). Money that is earned privately is not shared. Assume that each person prefers having more money to less.

(a) Is there a Nash equilibrium where every member of the group contributes?

(b) Is there a Nash equilibrium where no member contributes?

(c) Would the members of the group prefer that everyone contributes or no one contributes? What kind of a social situation is this?

7. Colonel Blotto (15 pts)

The lean and mean Colonel Blotto must defend the Steel City from being attacked by the greasy Colonel Sanders. Colonel Blotto (the defender) has three river boats and must decide how to allocate them to each of the three rivers (A, M, and O). Colonel Sanders (the attacker) has two swift boats that he can allocate between the three rivers. Formally, let $b = (b_A, b_{M,} b_O)$ denote Colonel Blotto's allocation of ships such that each b_i is an integer from 0 to 3 and $b_A + b_M + b_O = 3$. For example, the allocation $b = (1,0,2)$ means that 1 ship is located in river A, no ships are in river M, and 2 ships are in river O. Similarly, let $s = (s_A, s_M, s_O)$ be Colonel Sanders' allocation such that each s_i is an integer from 0 to 2 and $s_A + s_M + s_O = 2$.

Each of Colonel Blotto's river boats can successfully defend the Steel City against one of Colonel Sanders' swift boats. If Colonel Blotto allocates his boats so that he successfully defends against both of Colonel Sanders' boats, then the city is safe. If one swift boat is undefended, then Colonel Sanders steals steel from the city but is unable to occupy it. If two swift boats are undefended, then Colonel Sanders is victorious and occupies the city. For example, if $b = (1,0,2)$ and $s = (1,1,0)$, then Colonel Blotto successfully defends against the swift boat on river A but the city is undefended against the swift boat on river M, and the outcome is that Colonel Sanders steals some steel.

Colonel Blotto's most-preferred outcome is to save the city, his middle-preferred outcome is for Colonel Sanders to steal some steel, and his least-preferred outcome is for the city to be occupied. Colonel Sanders has the exact opposite preference ordering.

(a) Specify the matrix form of a strategic game that represents this situation.

(b) Does the game have any pure strategy Nash equilibria? If so, what are they? Explain the intuition behind your answer.

(c) Suppose that Colonel Blotto can obtain more river boats to defend the Steel City. If he has 4 ships, is there a pure strategy Nash equilibrium? What if he has 6 ships? Explain the intuition behind your answer. (You should do this *without* specifying the entire game.)

8. Candidate Competition with a Valence Advantage (15 pts)

Consider a variation of the Hotelling two-candidate competition model in which one of the candidates has a "valence advantage." Specifically, suppose that some fraction δ of voters *always* vote for candidate 1 regardless of the positions each candidate takes. Assume that $0 < \delta < \frac{1}{2}$. The remaining fraction $1 - \delta$ of voters have ideal points distributed uniformly between 0 and 1 and vote for the candidate whose position is closest to their own ideal point.

Formally, the game has two candidates, $C = \{1,2\}$ and each candidate chooses a position in the unit interval, $x_i \in [0,1]$ for $i \in C$. Let the vote share of candidate 1 be described by the equation:

$$v(x_1,x_2) = \begin{cases} \left(\frac{x_1 + x_2}{2}\right)(1-\delta)+\delta & \text{if} \quad x_1 < x_2 \\ \frac{1+\delta}{2} & \text{if} \quad x_1 = x_2 \\ \left(1-\frac{x_1 + x_2}{2}\right)(1-\delta)+\delta & \text{if} \quad x_1 > x_2 \end{cases}$$

Assume that candidates only care about winning: a win is preferred to a tie, which is preferred to a loss. The candidates' utility functions can therefore be described by:

$$u_1(x_1,x_2) = \begin{cases} 1 & \text{if} \quad v(x_1,x_2) > \frac{1}{2} \\ 0 & \text{if} \quad v(x_1,x_2) = \frac{1}{2} \\ -1 & \text{if} \quad v(x_1,x_2) < \frac{1}{2} \end{cases}$$

$$u_2(x_1,x_2) = -u_1(x_1,x_2)$$

(a) Is $(x_1, x_2) = (\frac{1}{2}, \frac{1}{2})$ a Nash equilibrium? Explain your answer using words. (You can use some math if it helps, but it is optional.)

(b) Is there a Nash equilibrium where the candidates do not adopt the same position? State the Nash equilibrium and explain your answer using words. (You can use some math if it helps, but it is optional.)

(c) Discuss how the valence advantage model compares to the standard model of candidate competition (without a valence advantage).

Bonus Problems (10 pts each)

B1. Explain why it might be a weakly dominant strategy to attempt all of the bonus problems. (If you do not think it is weakly dominant, then explain why you think that is the case.)

B2. Suppose that $A \wedge \neg B \Rightarrow C$ is true. Prove that $A \wedge \neg C \Rightarrow B$ must also be true.

B3. Two social scientists collaborate on a research project. Each can put in a level of effort x_i such that $0 \le x_i \le 1$. The project is worth $f(x_1, x_2)$ in terms of scholarly fame and recognition (but not fortune), which is split equally between the two. The cost to each social scientist is $c(x_i)$. For each set of assumptions below, find all of the Nash equilibria.

(a) $f(x_1, x_2) = 3x_1x_2$ and $c(x_i) = x_i^2$ for $i \in \{1,2\}$

(b) $f(x_1, x_2) = 4x_1x_2$ and $c(x_i) = x_i$ for $i \in \{1,2\}$