PS 2703 Formal Political Theory
September 5, 2007

Practice Problems: Logic, Proofs, and Sets

1. Formalize the following statements using the notation of logic (letters and symbols). Be sure to identify the English sentence associated with each letter that you use.

    a. Either John went to the store, or we're out of eggs.
    b. Either bill is at work and Jane isn't, or Jane is at work and Bill isn't.
    c. The lecture will be given if at least ten people are there.
    d. If John went to the store then we have some eggs, and if he didn't then we don't.

---

2. Consider the following argument:

    "During exam periods, cookie sales in the college cafeteria are high. Cookies are selling well in the cafeteria today, so it must be exam time."

    Identify the premises and the conclusion, then formalize the argument by defining a set of statements and rewriting it using letters and symbols so that it resembles the form of argument we used in class:

    Premise 1
    Premise 2
    Therefore, Conclusion

    Is the argument valid?

---

3. Formalize the following two arguments using the same structure as in problem 2 and identify which has the same structure as the argument in problem 2.

    a. A computer programmer must be able to think logically. Rob is a very logical person, so he must be a programmer.

    b. Computer programmers are always able to solve logic problems. None of the students can solve logic problems, so none of them are computer programmers.

    Is either argument valid?

---

4. Prove that $(R \vee S) \Rightarrow C$ is equivalent to $(R \Rightarrow C) \wedge (S \Rightarrow C)$. (Use the equivalences on the logic cheat sheet.)

---

5. Assume: (1) $P \vee Q$, (2) $Q \Rightarrow (R \wedge W)$, (3) $(R \vee P) \Rightarrow Z$. Use proof by contradiction to show that Z must be true.

---

6. Suppose $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8, 10\}$. List the elements of the following sets:

   a. $A \cap B$
   b. $A \cup B$
   c. $A \setminus B$
   d. $(A \cup B) \setminus (A \cap B)$
   e. $(A \setminus B) \cup (B \setminus A)$

---

7. Rewrite the following statements using only logical connectives ($\neg, \wedge, \vee$) and $\in$ instead of set operations ($\cap$, $\cup$, $\setminus$):

   a. $x \in A \cap (B \cup C)$
   b. $x \in A \setminus (B \cap C)$
   c. $x \in (A \cap B) \cup (A \cap C)$

---

8. Prove the distributive law $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

   (Hints: Use the fact that a set equality is equivalent to two subset relationships, and use logic operators in the definitions of the sets.)