PS 2703 Practice Problems
November 2, 2007

Constraining the King

Suppose that there is a King who chooses whether or not to impose a tax on one of two social groups: Merchants or Landowners. The King's army can collect the tax at a cost e, which transfers an amount of wealth x from the group to the King, but the army is only strong enough to collect from one of the groups. Assume $x > e$ and that the King also has the option of doing nothing.

If the King selects a group to tax, the Merchants and Landowners next simultaneously decide whether or not to challenge him. Challenging the King costs each group c. If both groups challenge, then the result is a written constitution that constrains the King from imposing taxes without both groups' consent, from which each group receives a benefit b and the King pays a cost k. If only one or neither group challenges, then the King's army collects the tax and x is transferred from the selected group to the King.

Is there a subgame perfect Nash equilibrium in which the King successfully taxes one of the groups? Is there a subgame perfect Nash equilibrium in which both groups successfully challenge the King?

An Extensive Game with Continuous Actions

Assume that two people play a game where they choose numbers sequentially. The first player chooses x. The second player observes x then chooses y. Assume that both x and y can be any real number. Player 1's payoff depends only on the number that Player 2 chooses, while Player 2's payoff depends on both numbers. Specifically, assume the utility functions are

$$u_1(x, y) = 10 - (4 - y)^2$$

$$u_2(x, y) = y(x - y)$$

What is the subgame perfect Nash equilibrium of this game?

Ultimatum Game

(a) Consider the basic ultimatum game where $c > 0$ and both players get nothing if the offer is rejected. Is there a Nash equilibrium (that is not subgame perfect) where Player 1 offers $x > c$ and Player 2 accepts?

(b) Consider a variation of the ultimatum game where $c = 1$ in which the second player cares about the *distribution* of payoffs in addition to his/her own payoffs. To describe the utility function, let x_1 and x_2 be the shares that the players receive at the end of the game. If Player 1's offer is *y* and if they agree, then $x_1 = c - y$ and $x_2 = y$. Otherwise $x_1 = 0$ and $x_2 = 0$. Player 1 only cares about only her own share so that $u_1(x_1, x_2) = x_1$. Player 2's utility is her own share minus a penalty for the difference between the two shares so that $u_2(x_1, x_2) = x_2 - | x_1 - x_2 |$. Find the subgame perfect Nash equilibrium of this game.