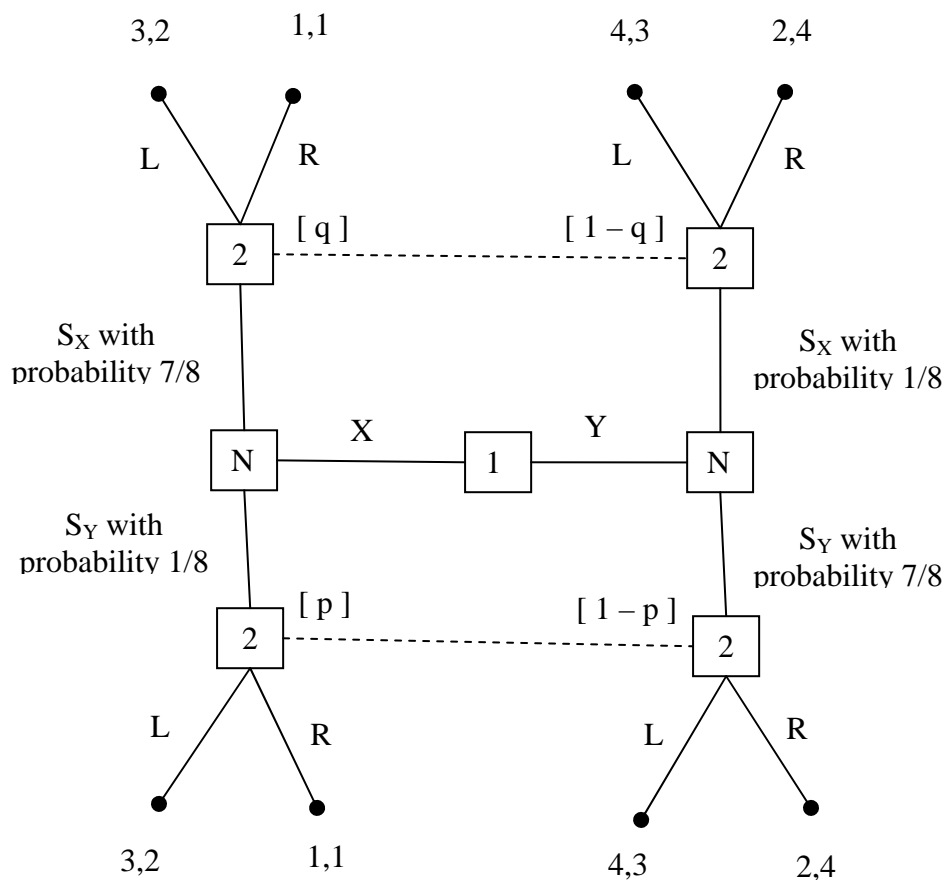


PS 2703 Practice Problems
November 30, 2007

Beliefs and Imperfect Observability

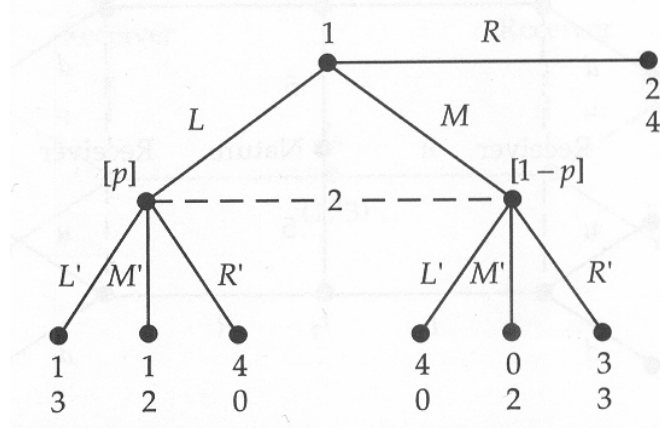


(a) Suppose that Player 1 uses a mixed strategy where he chooses X with probability $1/4$ and Y with probability $3/4$. What are Player 2's beliefs p and q ? What actions are sequentially rational for Player 2? Is Player 1's mixed strategy part of a weak sequential equilibrium?

(b) Suppose that Player 1 uses a mixed strategy where he chooses X with probability $1/8$ and Y with probability $7/8$. What are Player 2's beliefs p and q ? What actions are sequentially rational for Player 2? Could Player 1's mixed strategy be part of a weak sequential equilibrium?

Weak Sequential Equilibrium

Find the pure strategy weak sequential equilibria of the following game.



Nuclear Deterrence

The following model of nuclear deterrence captures Schelling's idea of the reciprocal fear of surprise attack between two nuclear powers.¹ Both countries have second strike capabilities, but in the event of nuclear war, it is better to strike first than to strike second. This suggests that each country will consider launching a first strike not because it will gain anything by attacking but because if it thinks that the other side is preparing to attack it will want to capture the first strike advantage for itself.

In modeling this, we need to allow the decisions to be made sequentially but each side must not know whether the other side has already launched a first strike. This clearly suggests a game of imperfect information. To represent this uncertainty about whether or not a country has already chosen to launch a first strike, we assume that Nature first chooses which country moves first with equal probability. Each country has two possible actions: attack or delay. If Country A moves first and chooses to attack then the outcome of the game is that Country A launches its missiles first and Country B launches its remaining missiles in retaliation. If Country A chooses to delay, then Country B can choose whether to attack or delay. In this case, if Country B chooses to attack then the outcome is that Country B launches a first strike and Country A launches a second strike in retaliation. If Country B chooses not to attack, then the game ends in relative peace (i.e., no nuclear war). The situation is symmetric if Nature chose Country B to move first.

Assume that the payoff from the status quo (i.e., no nuclear war) when neither side chooses to attack is 0. Nuclear war is worse than the status quo (but not completely devastating) and striking first is better than striking second, so assume the payoff from striking first is -1 and from striking second is -2.

¹ Schelling, *The Strategy of Conflict* (1960). The simplified model is discussed in Morrow, *Game Theory for Political Scientists* (1994, pp. 180-186), which in turn draws from Powell, *Nuclear Deterrence Theory: The Search for Credibility* (1990).