PS2703 Practice Problems
Friday October 19, 2007

Mixed Strategies

Find all of the mixed strategy equilibria of the game described by:

Player 2

Player 1		D	E	F
	A	0,0	1,0	0,1
	B	1,-1	0,0	-1,1
	C	-1,1	-1,1	1,-1

Bayesian Nash equilibrium

Consider an incomplete information version of a modified two person Stag Hunt in which the players choose to either *hunt* or *forage*. Formally, $A_i = \{H,F\}$. As in the original game, capturing the stag requires both players to hunt, and Player 1 prefers capturing the stag to any profile in which he forages to the profile where he hunts and Player 2 does not. Player 2, however, may be one of three possible types: an omnivore, a carnivore, or an herbivore: $\theta_2 \in \{O, C, H\}$. Assume that $Pr(\theta_2 = O) = 1 - \pi$, $Pr(\theta_2 = C) = \pi/2$, and $Pr(\theta_2 = H) = \pi/2$.

If Player 2 is an omnivore, then the game in matrix form is:

Player 2

Player 1		H	F
	H	s,s	0,1
	F	1,0	1,1

If Player 2 is a carnivore, the payoffs are:

Player 2

Player 1		H	F
	H	s,s	0,0
	F	1,0	1,0

And if Player 2 is an herbivore, the payoffs are:

Player 2

Player 1		H	F
	H	s,0	0,1
	F	1,0	1,1

Find all of the Bayesian Nash equilibria of the game.

Bayes' Rule

The Decider is uncertain whether his nemesis Satan possesses weapons of mass destruction. As the head of a team of UN weapons inspectors, you must visit the Underworld to look for evidence of a nuclear weapons program. Given your observations, you must then determine the likelihood that Satan actually possesses a nuclear weapon.

Suppose that there are three possible states of the world: Satan has weapons ($\omega = W$), he has a peaceful civilian energy program ($\omega = E$), or he has no nuclear capability whatsoever ($\omega = N$). Your prior beliefs (the probabilities of each state) are $\Pr(\omega = W) = \pi_W$, $\Pr(\omega = E) = \pi_E$, and $\Pr((\omega = N) = 1 - \pi_{W} - \pi_E$.

The conditional probabilities that you uncover some limited evidence ($\theta = E$) given each state are $\Pr(\theta = E \mid \omega = W) = e + p$, $\Pr(\theta = E \mid \omega = E) = e$, $(\theta = E \mid \omega = N) = e - p$ such that $0 < e - p < e < e + p < 1$.

Find the posterior probabilities of each state if you find evidence and if you don't find evidence. Specifically, use Bayes' Rule to derive the following conditional probabilities:

$\Pr(\omega = W \mid \theta = E)$
$\Pr(\omega = E \mid \theta = E)$
$\Pr(\omega = N \mid \theta = E)$

$\Pr(\omega = W \mid \theta = N)$
$\Pr(\omega = E \mid \theta = N)$
$\Pr(\omega = N \mid \theta = N)$