Problem Set 10
PS 2703
Due November 19, 2007

Provide complete explanations for all of your answers.

Problem 1: Twice Repeated Games

For this problem, assume there is no discounting so that each player's payoff at each terminal history is the sum of the player's per period payoffs.

(a) Consider the twice repeated Prisoner's Dilemma with the following payoffs:

	C	D
C	c, c	y, x
D	x, y	0, 0

In order for the game to be the PD, x > c > 0 > y. Find additional inequalities for x and y that guarantees there is a Nash equilibrium where both players choose D in the first period, D in the second period only if they played (D,D) in the first period, and (C,C) in the second period after any other history.

(b) Consider the stage game with payoffs:

	L	C	R
U	3,1	0,0	5,0
M	2,1	1,2	3,1
D	1,2	0,1	4,4

Is there a subgame perfect Nash equilibrium where the first period action profile is (B,R)? If so, state the strategies and show that it is a SPNE. If not, prove that there is no such SPNE.

Problem 2: Alternative Strategies for the Infinitely Repeated Prisoner's Dilemma

(This is a reworded version of Osborne, Exercise 431.2.)

Consider the infinitely repeated Prisoner's Dilemma with the following stage game payoffs:

	C	D
C	2,2	0,3
D	3,0	1,1

For each of the strategies described below, determine if there is a range of the discount factor δ such that both players using the strategy is a Nash equilibrium. If there is such a range, what is it? If the strategy pair is not a Nash equilibrium for any value of δ for the given payoffs, determine if there are any other PD payoffs such that there is some discount factor δ that makes the strategy pair a Nash equilibrium. Remember that the generic version of the PD, as in the following table, must satisfy T > R > P > S

	C	D
C	R,R	S,T
D	T,S	P,P

(a) *Delayed Grim Trigger*. In the delayed grim trigger, players wait one period before implementing the punishment part of the strategy. Specifically, the strategy says to choose C in period 1 and any history in which (C,C) was played in every previous period. If (C,C) is not played in period *t*, then choose C in period *t* + 1, and then play D in period *t* + 2 and every subsequent period.

(b) *Forgiving Grim Trigger*. Suppose instead that a single defection is "forgiven" so that it takes a second defection to trigger the punishment. The defections do not have to be consecutive, but punishment begins immediately after the second defection occurs. Specifically, the strategy says to choose C in period 1 and after any history in which (C,C) was played in every previous period *or* after any history in which (C,C) is played in all but one period. After any history in which (C,C) was not played in at least two periods, play D.

(c) *Pavlov*. In this strategy, except for the first period, the choice of action in period *t* + 1 depends only on the actions in the last period (period *t*). The strategy is to play C in the first period, then play C if the actions in the last period were (C,C) or (D,D) and to play D if the actions in the last period were either (C,D) or (D,C). Osborne also describes this strategy as "win-stay, lose-shift"—in addition to answering the questions above, can you explain why that is an appropriate description of this strategy?

<u>Problem 3: Repeated Chicken</u>

(a) Consider the stage game of Chicken with the following payoffs:

	Chicken	Tough
Chicken	5, 5	0, 10
Tough	10, 0	-5, -5

If the game is repeated three times and there is no time discounting (i.e., δ = 1), is there a subgame perfect Nash equilibrium strategy profile where both players choose Chicken in the first period? If so, what is it?

(b) Now suppose the payoffs in Chicken are:

	Chicken	Tough
Chicken	0, 0	k, -1
Tough	-1, k	-5, -5

Is there a Nash equilibrium where both players choose (Chicken, Chicken) on the equilibrium path? What about an equilibrium where they alternate between (Chicken, Tough) and (Tough, Chicken)? Use the folk theorem to justify your answer.