Problem Set 11
PS 2703
Due December 3, 2007

Follow these instructions carefully:

- Provide complete and thorough explanations for all of your answers.
- Make sure that strategies are properly specified.
- Do as much as possible on your own before consulting with others.
- Check, proofread and edit your answers before turning them in.

<u>Problem 1: Bureaucratic Power</u>

In this problem, you will analyze the effect of delegating policy-making authority to an administrative agency. Consider an extensive form game where the Agency first sets a policy *p* (from the set of real numbers greater than 0) which takes effect immediately. You can think of the Agency's policy as either an administrative rule or as an enforcement policy. Congress can force the agency to change the policy, but only by statute. Suppose that after observing the administrative policy *p*, Congress can propose a bill *b*. The bill is then presented to the President who can either sign or veto the bill. The bill becomes the new policy only if the President signs it. (To keep the model simple, assume the legislature is unicameral, there is no veto override requirement and no filibuster. Congress can therefore be represented by the median legislator.) Assume that the Agency, Congress, and President all have single-peaked and symmetric Euclidean preferences and that their ideal points are A, C, and P, respectively. Suppose that 0 < C < P. Also suppose that the status quo policy is *q*.

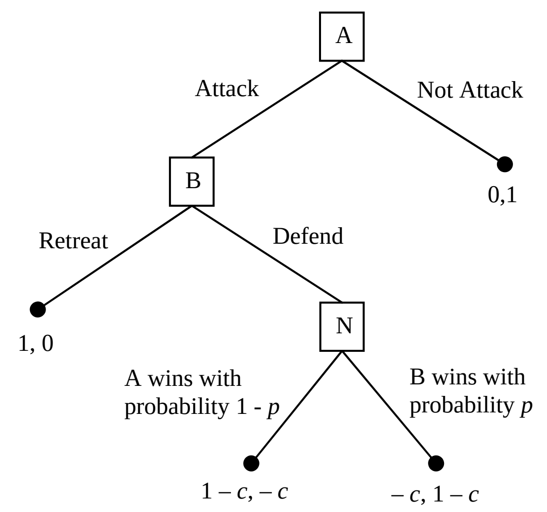
(a) Find a subgame perfect Nash equilibrium assuming the Agency's ideal point is to the right of the President's (i.e., 0 < C < P < A). Ensure that you characterize *complete* strategies for each player (i.e., the strategies specify actions for *every* possible history the player may move).

(b) Consider the effect of varying the Agency's ideal point while holding all of the other parameters fixed (i.e., this is a comparative statics exercise). More precisely, holding the ideal points of Congress and the President constant (still assuming 0 < C < P), plot the SPNE *outcome* as a function of the agency's ideal point A and provide a verbal explanation for your results. (If necessary, you can also choose and fix a value for *q*.)

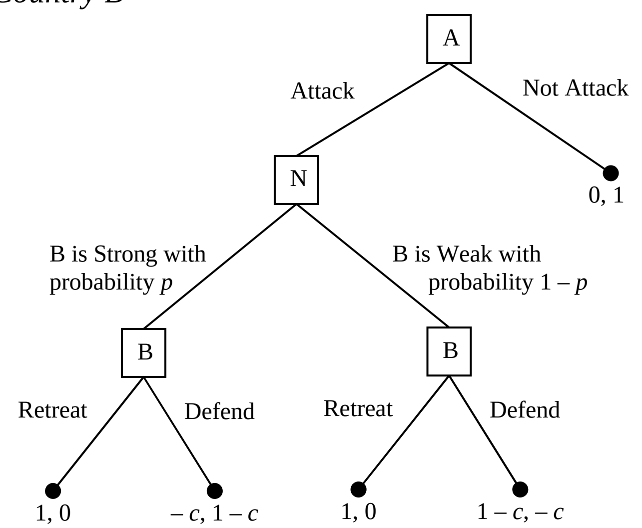
Problem 2: An Island Dispute with Uncertainty and Indestructible Bridges

Two countries, A and B, dispute the ownership of an island between the mainland of each country, but this time the bridges from each country to the island are made out of concrete and cannot be burned with the primitive weapons possessed by either side. Country B currently occupies the island. Country A first chooses to attack or not attack. Country B can defend or retreat. The value of the island is normalized to 1 and fighting costs each country c if Country A attacks *and* Country B defends. We will consider two models involving uncertainty to see how varying the timing of Nature's move affects the outcome. In the first model, Nature moves after both countries choose their actions, while in the second model, Nature moves after Country A but before Country B. Game trees that fully characterize each model are shown below. (N denotes Nature's move.)

Model 1: Nature moves after both countries



Model 2: Nature moves after Country A and before Country B



(a) Provide a substantive justification (i.e., make up a reasonable story) for each model and describe the difference between them in terms of the uncertainty that each player faces.

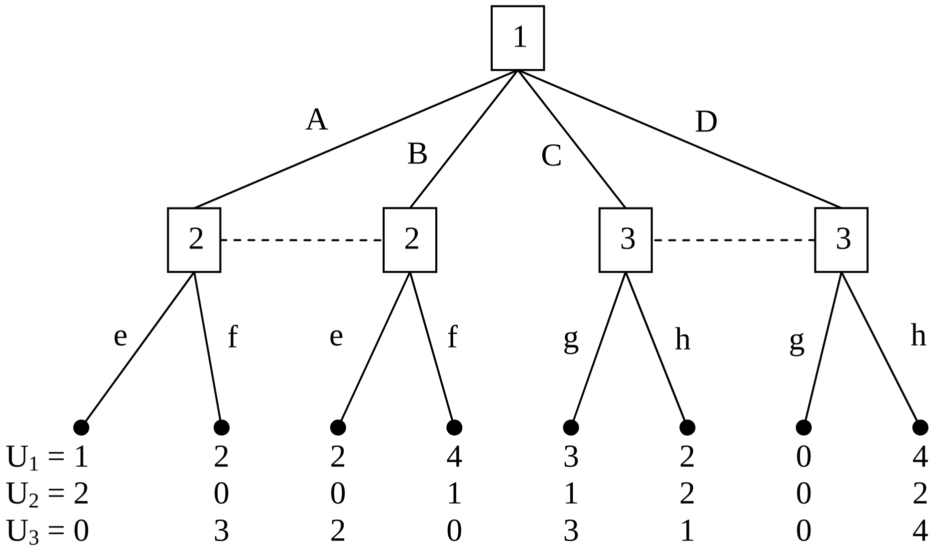
(b) For $p = ½$ and $c = ¼$, fully characterize the subgame Perfect Nash equilibrium *strategy profile* for each model.

(c) For each model, what are the values of p and c such that Country A does not attack in the SPNE? For what values of p and c does Country B always retreat in the SPNE? For what values of p and c is there some chance that the countries will fight (A attacks and B defends)? Provide a graph for each model that illustrates your answers (put p on the horizontal axis and c on vertical axis and assume both are between 0 and 1).

(d) Interpret your results. Which model appears to favor Country A and which favors Country B, and why? In which model does there appear to be a greater propensity for a fight to break out?

Problem 3: Beliefs and Sequential Rationality

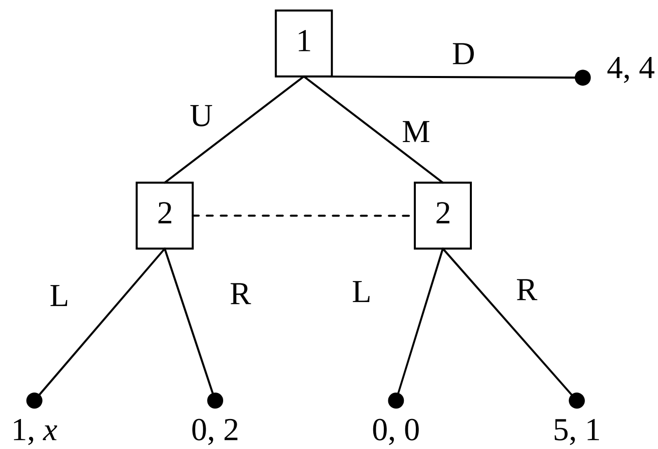
For this problem, consider the following extensive game of imperfect information:



(a) Suppose that Player 1 uses a mixed strategy where she chooses A with probability 3/8, B with probability 1/8, C with probability 1/3, and D with the remaining probability. Let p be Player 2's belief that Player 1 played A, and let q be Player 3's belief that Player 1 played C. What are the beliefs (values of p and q) for Players 2 and 3 implied by Bayes' Rule? Given those beliefs, what are the sequentially rational actions for Players 2 and 3?

(b) Suppose that Player 1 instead uses the generic mixed strategy where A is played with probability $\alpha/2$, B is played with probability $(1 - \alpha)/2$, and C and D are each played with probability ¼. Now what are the beliefs (values of p and q) implied by Bayes' Rule? Find an inequality for α that guarantees that choosing f is sequentially rational for Player 2.

Problem 4: Weak sequential equilibrium



(a) Find the equivalent normal form representation, Nash equilibria, and subgame perfect Nash equilibria of the above game of imperfect information for $x = 1$. (Only consider pure strategies.)

(b) Find a (pure strategy) weak sequential equilibria of the game for $x = 1$? Be sure to specify the equilibrium strategies *and* beliefs. Is the equilibrium belief unique or is there a range of possible equilibrium beliefs?

(c) When $x = 3$ is there a (pure strategy) weak sequential equilibrium with a different strategy profile than when $x = 1$? Again, be sure to specify the equilibrium strategies *and* beliefs. Is the equilibrium belief unique or is there a range of possible equilibrium beliefs?