Problem Set 12
PS 2703
Due December 10, 2007

Provide explanations for all of your answers.

Problem 1: Selten's Horse

Osborne, Exercise 331.1

Problem 2: Nuclear Deterrence with Mixed Strategies

The following model of nuclear deterrence captures Schelling's idea of the reciprocal fear of surprise attack between two nuclear powers.[1] Both countries have second strike capabilities, but in the event of nuclear war, it is better to strike first than to strike second. This suggests that each country will consider launching a first strike not because it will gain anything by attacking but because if it thinks that the other side is preparing to attack it will want to capture the first strike advantage for itself.

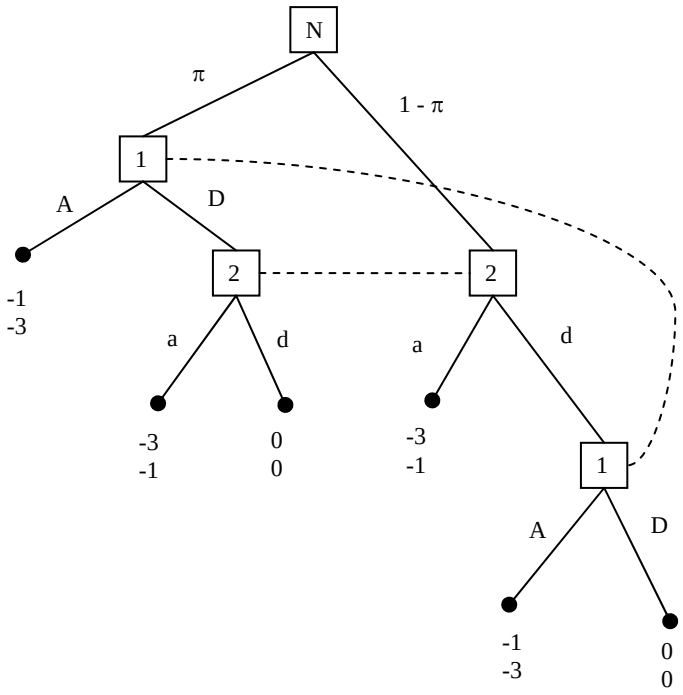
In modeling this, we need to allow the decisions to be made sequentially but each side must not know whether the other side has already launched a first strike. This clearly suggests a game of imperfect information. To represent this uncertainty about whether or not a country has already chosen to launch a first strike, we assume that Nature first chooses which country moves first. Each country has two possible actions: attack or delay. If Country A moves first and chooses to attack then the outcome of the game is that Country A launches its missiles first and Country B launches its remaining missiles in retaliation. If Country A chooses to delay, then Country B can choose whether to attack or delay. In this case, if Country B chooses to attack then the outcome is that Country B launches a first strike and Country A launches a second strike in retaliation. If Country B chooses not to attack, then the game ends in relative peace (i.e., no nuclear war). The situation is symmetric if Nature chose Country B to move first. The game tree is shown below.

Assume that Nature chooses Country A to move first with probability π and chooses Country B to move first with probability $1 - \pi$. Assume that the payoff from the status quo (i.e., no nuclear war) when neither side chooses to attack is 0. Nuclear war is worse than the status quo (but not completely devastating) and striking first is better than striking second, so assume the payoff from striking first is -1 and from striking second is -3.

(a) Find the equivalent matrix form of the game and its mixed strategy Nash equilibrium. Be careful when specifying payoffs for the equivalent normal form, as they should be the *expected payoffs* from each strategy profile.

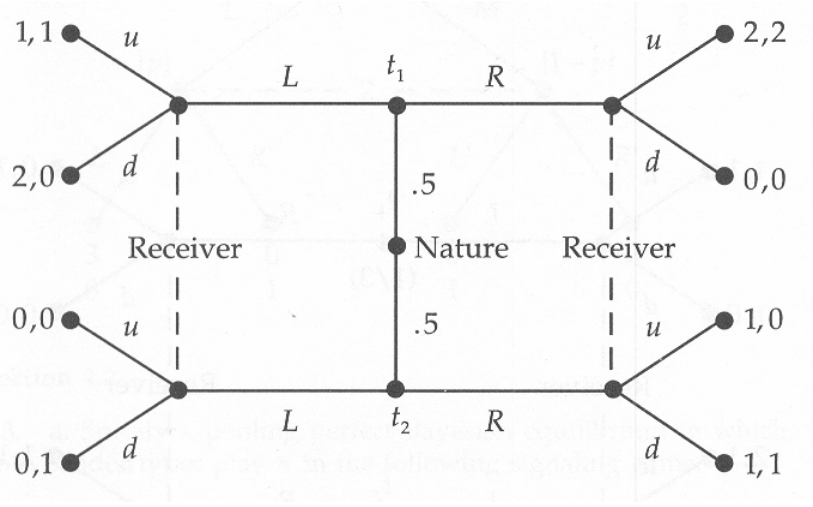
(b) Find the weak sequential equilibrium of the game involving mixed strategies.

[1] Schelling, *The Strategy of Conflict* (1960). The simplified model is discussed in Morrow, *Game Theory for Political Scientists* (1994, pp. 180-186), which in turn draws from Powell, *Nuclear Deterrence Theory: The Search for Credibility* (1990).

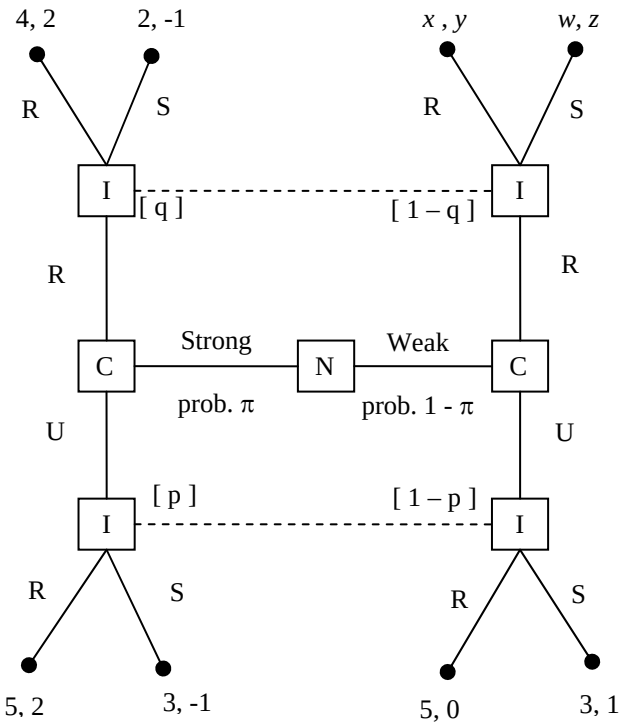


Problem 3: Signaling

(a) Find all of the pure strategy pooling and separating weak sequential equilibria of the following signaling game.



(b) Consider the entry signaling game depicted below in which Nature determines whether the Challenger is Strong or Weak, the Challenger chooses whether to be ready (R) or unready (U), and the Incumbent chooses whether to stay (S) or retire (R) after observing the Challenger's action but not the Challenger's type.



Find inequalities that guarantee that the only weak sequential equilibrium of the game is a separating equilibrium where the Strong type Challenger chooses R and the Weak type Challenger chooses U. Make sure that you fully specify the weak sequential equilibrium in terms of both strategies and beliefs. Also be sure to demonstrate that the equilibrium is unique. (If you have trouble finding the appropriate inequalities, then give specific values for *w*, *x*, *y*, and *z*.)