Problem Set 2
PS 2703
Due Monday, September 17

Problem 1

Consider a simple version of *approval voting*. There are three alternatives X = {*a*,*b*,*c*} and each agent has strict preferences and casts one vote for each her top two alternatives. The social preference is determined by the number of total votes each alternative receives. That is, let v(*x*) be the number of votes for any alternative *x* so that for any two alternatives, *xPy* if and only if v(*x*) > v(*y*) and *xIy* if and only if v(*x*) = v(*y*).

Suppose that there are 61 voters with the preference ordering *aPbPc*, 20 voters with preferences *bPaPc*, and 20 voters with preferences *cPbPa*.

a. For each pair of alternatives, what is the social preference?

b. Is the core non-empty (i.e. is there a Condorcet winner)? If so, what is it?

c. Which conditions of Arrow's Theorem does approval voting satisfy and which does it violate? (Assume unrestricted domain, so also consider preference profiles other than the one above.)

Problem 2

Let the set of alternatives be X = {*a*,*b*,*c*} and the set of agents be N = {1,2,3}. Suppose that society adopts a preference aggregation rule that works in the following way. First, there is a simple majority rule vote between *a* and *b*. Second, there is a simple majority rule vote between the winner of the first round and *c*. Let x_1 be the winner of the second round, x_2 be the loser of the second round, and x_3 be the loser of the first round. The social preference relation is then defined as $x_1 P x_2$, $x_2 P x_3$, and $x_1 P x_3$. (This procedure is a version of an *amendment agenda* where the first round is a vote between a bill and an amendment, and the second round pits the winner of the first round against the status quo.)

a. Construct a preference profile that shows this preference aggregation rule is not weakly Paretion.

b. Construct a preference profile that shows this preference aggregation rule is not independent of irrelevant alternatives.

Problem 3

Let X = {a,b,c,d,e,f,g,h} and let N = {1,2,3}. Suppose the agents have the following preference rankings:

1	2	3
a	d	h
b	e	g
c	c	f
d	f	e
e	g	c
f	h	d
g	b	a
h	a	b

a. What is the simple majority rule core given these preferences?

b. Is majority rule transitive given these preferences?

c. Are these preferences single-peaked? If so, give the ordering of the alternatives for which preferences are single-peaked. Otherwise, provide a proof that they are not (without enumerating all possible orderings).

Problem 4

Suppose that voters have the following preference orderings (two alternatives on the same line for a voter indicates indifference):

1	2	3
x	z	y
y,z	x	z
	y	x

a. Find an ordering of the alternatives such that preferences are *weakly* single-peaked. That is, if t is an agent's ideal point and q( ) is the ordering function, then:
    - $q(x) < q(t) \Rightarrow tRx$
    - $q(x) < q(y) < q(t) \Rightarrow tRyRx$
    - $q(t) < q(x) < \Rightarrow tRx$
    - $q(t) < q(y) < q(x) \Rightarrow tRyRx$

b. What are social preferences according to simple majority rule? Is majority rule transitive for this preference profile? Is it quasi-transitive? Is it acyclic?

c. Discuss how weakly single-peaked preferences differ from the definition of single-peakedness in McCarty and Meirowitz's Definition 4.7 and how your results about majority rule differ from Theorem 4.2.

Problem 5

McCarty and Meirowitz, Exercise 4.5