

Problem Set 3
PS 2703
Due September 24, 2007

Provide full explanations for your answers (e.g. provide proofs or sketches of proofs where appropriate).

1. In the simplest possible strategic situation there are two players and each player has two possible actions (such as the Prisoner's Dilemma, BoS, Stag Hunt, and Matching Pennies). Consider the general game represented by the following matrix:

		Column	
		Y	Z
Row	W	a	c
	X	b	d

For convenience, each action profile (outcome) is denoted by a letter ($a = (U,L)$, etc). Since we have already specified the players and actions, in order to complete the description of a game, we need to specify each player's preferences over the four outcomes. For example, if Row's ordinal preferences are $b P a P d P c$ and Column's are $c P a P d P b$, then the game has the identical incentive structure as the Prisoner's Dilemma (PD). That is, if we re-label the actions so that $W = C$, $X = D$, $Y = C$ and $Z = D$, then both players have an incentive to individually choose $X = D$ or $Z = D$ regardless of what the other player does but they prefer the action profile $(W,Y) = (C,C)$ to $(X,Z) = (D,D)$ because both have $a P d$.

- a. If Row's preferences are instead $a P b P c P d$, then are the incentives still the same as in the PD? If so, re-label the actions to match the PD so that each player chooses either C = cooperate or D = defect. Otherwise, explain why it is not the same as the PD.
- b. What if Row's preferences are $a P b P c P d$ and Column's preferences are $d P b P c P a$? Then does this have the same incentives as the PD? Re-label as above or explain why not.
- c. Specify preferences for Row and Column such that there is *no conflict* between them.
- d. Specify ordinal preferences for Row and Column (possibly including indifference) such that the incentive structure of the game is the same as matching pennies.

2. Osborne, Exercise 27.2 (Variant of *Prisoner's Dilemma* with altruistic preferences)

(more on reverse)

HINT: For the following problems, you are asked to analyze the games for arbitrary parameters k , n , and/or m . First work through the problems using specific values, then prove or sketch proofs for your answers to the general cases.

3. Osborne, Exercise 33.1 (Contributing to a public good)

4. Osborne, Exercise 34.2 (Voter participation)