Problem Set 5
PS 2703
Due October 8, 2007

Provide explanations for your answers and provide proofs or sketches of proofs where appropriate.

1. Osborne, Exercise 79.2 (Variant of war of attrition)

2. Osborne, Exercise 80.2 (A fight)

   Use the suggested value function $f(y_1,y_2) = 2 - y_1 - y_2$.

3. In Cournot's classic model of oligopoly, firms seek to maximize profits and compete by choosing the quantity of goods to produce. (The model is explained in detail in Section 3.1 of Osborne.) Consider the following strategic game:

   Players: Two firms, N = {1,2}

   Actions: Quantity of output, $q_i \geq 0$ for $i \in N$

   Preferences: Profits,

$$\pi_i(q_i, q_j) = (p(q_1, q_2) - c_i)q_i$$

   where the price of each good is determined by the market and is

$$p(q_1, q_2) = \begin{cases} \alpha - q_1 - q_2 & \text{if} \quad q_1 + q_2 \leq \alpha \\ 0 & \text{if} \quad q_1 + q_2 > \alpha \end{cases}$$

   Note that in the utility function $\pi(q_i,q_j)$, $p(q_1,q_2)-c_i$ is the per unit profit, so the total profit is the per unit profit times the number of units q_i. Also assume that the two firms have different unit costs where $\alpha > c_1 > c_2$.

   For each of the assumptions below, solve for the best response functions using the first order conditions and verify that the second order conditions are met. Then solve for the Nash equilibrium using the best response functions. Ensure that the inequality $q1 + q2 \leq \alpha$ is satisfied (this ensures that price will be positive). If it is not satisfied, then look for a Nash equilibrium in which one or both firms produce zero output.

a) First assume that $c_1 \leq \frac{\alpha + c_2}{2}$.

b) Next assume that $c_1 > \frac{\alpha + c_2}{2}$.

c) Discuss the properties of the Nash equilibrium. Which firm produces more output in equilibrium? Perform comparative statics analysis to see how lowering c_2 affects the equilibrium quantities q_1, q_2, q_1+q_2, and $p(q_1,q_2)$ while holding c_1 constant. Graph each of these quantities for c_2 between 0 and c_1.

4. McCarty and Meirowitz, Exercise 3.1 (progressive ambition under uncertainty)

5. McCarty and Meirowitz, Exercise 3.3 (computing expected payoffs)

Assume that the coin is fair so that there is an equal probability of heads or tails.