

Problem Set 6
PS 2703
Due October 22, 2007

Problem 1 (Mixed strategy equilibria of 2x2 games)

(a) The game of “chicken” is another 2x2 game with a “cute” but politically incorrect story from the 1950s. Consider two teenage males who are competing for the affections of a teenage girl by engaging in a game of “chicken.” Specifically, the two males drive their cars toward a cliff. The one who swerves and “chickens out” first loses, while the one who swerves last wins. If neither chickens out first, then they both end up going off the cliff—clearly a bad outcome. Both players chickening out first results in the second-best outcome (neither winning the girl). (This game is the same as Hawk-Dove and is a simplified version of the War of Attrition.) The following matrix describes the (Bernoulli) payoffs for the game:

	S	C
S	c, c	10,0
C	0,10	5,5

Find the mixed strategy Nash equilibrium for $c = -5$. Given the equilibrium mixed strategy profile, what is the probability of each action profile actually occurring? (For example, if player 1 chooses S with probability q and player 2 chooses S with probability p , then the probability that the outcome is (S,S) is pq .) Then find the mixed strategy Nash equilibrium and the probability of each action profile for any $c < 0$. Perform comparative statics analysis in terms of the value of c (discuss how the equilibrium changes as c increases).

(b) Find the mixed strategy Nash equilibrium for the Stag Hunt first for $m = 4$ then for any value of $m > 1$. As in part a, also find the equilibrium probabilities of each action profile. Discuss the comparative statics of the equilibrium in terms of the value of m .

	S	H
S	m, m	0,1
H	1,0	1,1

Problem 2 (modeling uncertainty in payoffs)

In this problem you will consider how uncertainty about the state of the world can be incorporated into a (non-Bayesian) normal form game through the players’ utility functions.

Suppose that policymakers are uncertain whether terrorism or global warming poses the greatest danger to the country. Formally, let $\theta \in \{T,G\}$ denote this unknown state of the world. With probability π the true state is $\theta = T$ and with probability $1 - \pi$ the true state is $\theta = G$.

Suppose that there are two government agencies, A and B, and each must decide independently which problem it should prepare for. If both agencies prepare for the correct problem (i.e., the

true state is $\theta = T$ and both prepare for a terrorist attack or the true state is $\theta = G$ and both prepare for global warming) then the number of deaths that occurs is 0. If both agencies prepare for the wrong problem, then the number of deaths is g if the true problem is global warming and t if the true problem is terrorism. If each agency prepares for a different problem, then $p < g$ deaths occur if the true problem is global warming and $m < t$ deaths occur if the true problem is terrorism.

(a) For each pair of actions by the government agencies, what is the expected number of deaths that occurs?

For parts (b) through (d), consider the normal form game where if d is the expected number of deaths, each agency's Bernoulli utility function is $u_i(a_1, a_2) = -d$.

(b) Find an inequality (or set of inequalities) that guarantees a Nash equilibrium exists where both agencies prepare for a terrorist attack.

(c) Find an inequality (or set of inequalities) that guarantees a Nash equilibrium exists where both agencies prepare for global warming.

(d) Find an inequality (or set of inequalities) that guarantees a Nash equilibrium exists where each agency prepares for a different problem.

Problem 3

Osborne, Exercise 282.1

Problem 4

McCarty and Meirowitz, Exercise 6.1