Problem Set 7 (Revised)
PS 2703
Due October 29, 2007

Problem 1 (Bayes' Rule)

The Decider is uncertain whether his nemesis Satan possesses weapons of mass destruction. As the head of a team of UN weapons inspectors, you must visit the Underworld to look for evidence of a nuclear weapons program. Given your observations, you must then determine the likelihood that Satan actually possesses a nuclear weapon.

Suppose that there are three possible states of the world: Satan has weapons (ω = W), he has a peaceful civilian energy program (ω = E), or he has no nuclear capability whatsoever (ω = N). Your prior beliefs (the probabilities of each state) are $Pr(\omega = W) = \pi_W$, $Pr(\omega = E) = \pi_E$, and $Pr((\omega = N) = 1 - \pi_{W} - \pi_E$.

The conditional probabilities that you uncover some limited evidence (θ = E) given each state are $Pr(\theta = E \mid \omega = W) = e + p$, $Pr(\theta = E \mid \omega = E) = e$, $(\theta = E \mid \omega = N) = e - p$ such that $0 < e - p < e < e + p < 1$.

(a) Find the posterior probabilities of each state if you find evidence and if you don't find evidence. Specifically, use Bayes' Rule to derive the following conditional probabilities:

$Pr(\omega = W \mid \theta = E)$
$Pr(\omega = E \mid \theta = E)$
$Pr(\omega = N \mid \theta = E)$

$Pr(\omega = W \mid \theta = N)$
$Pr(\omega = E \mid \theta = N)$
$Pr(\omega = N \mid \theta = N)$

(b) What parameters guarantee that $Pr(\omega = W \mid \theta = E) > Pr(\omega = W)$? Substantively, what does this inequality mean?

Problem 2 (Jury Voting)

Consider the jury voting model developed in class (and in McCarty and Meirowitz 6.3).

(a) Assume that the jury votes instead by unanimity rule (convict if and only if every juror votes to convict; otherwise the defendant is acquitted). Find conditions (inequalities) for p such that all players voting sincerely is a Bayesian Nash equilibrium. Compare your results from unanimity rule with simple majority rule. What substantive conclusions can you draw?

(b) Assume $p = ¾$, $\pi = 2/3$ and that the jury decides by simple majority rule but now each juror's Bernoulli utility function is 0 if the correct decision is made, -z if an innocent defendant is convicted, and z-1 if a guilty defendant is acquitted. Also assume $0 < z < 1$. How should you interpret this new utility function? Find an inequality or set of inequalities in terms of z that guarantees a Bayesian Nash equilibrium where each juror votes sincerely.

<u>Problem 3 (Extensive form games of perfect information)</u>

(a-c) Osborne, Exercise 156.2

(d) For the games in part (a) and (b) of Osborne, Exercise 156.2, describe each player's set of strategies, the equivalent (matrix) normal form, and the set of Nash equilibrium strategy profiles.

<u>Problem 4 (Extensive form game of imperfect information)</u>

(a) Depict the game described in Osborne, Exercise 211.2 for T = 2 as a game tree with non-singelton information sets (i.e. an extensive form game with imperfect information.

(b) Describe the game formally in terms of players, histories, a player function, action sets, information sets, and utility functions.

(c) Find each player's set of strategies, the equivalent matrix normal form, and the set of Nash equilibrium strategy profiles.