Problem Set 8
PS 2703

Provide complete explanations for all of your answers.

Problem 1: Alternating vetoes

Suppose that Tim and Nolan decide where to eat together by taking turns vetoing restaurants from a list and then choose the last restaurant that is not vetoed. Suppose the restaurants are Fuel & Fuddle, Mad Mex, and Primanti's. Formally, let R = $\{f, m, p\}$. Tim's strict preferences for restaurants are m P p P f while Nolan's preferences are f P p P m. Suppose that Tim always vetoes first.

(a) Draw a game tree that fully characterizes the extensive form game.

(b) Depict the equivalent normal form of the game as a matrix. Make sure that you properly specify each players' set of strategies.

(c) Find the set of Nash equilibria and the set of subgame perfect Nash equilibria of the extensive form game. Are there any Nash equilibria that are not subgame perfect? Are any of the *outcomes* associated with the Nash equilibria different from the ones associated with the subgame perfect equilibria?

(d) Consider two versions of the game: one where Tim vetoes first and another where Nolan vetoes first. Holding Tim's preference ordering fixed, is there a (strict) preference ordering for Nolan such that the subgame perfect Nash equilibrium outcome of each game is different? If so, what is it? In other words, is there a preference profile where the order of the players' vetoes matters?

Problem 2: Burning a bridge

Consider the model described in Osborne, Exercise 173.4.

(a) First consider the model where Army 2 does not have the option of burning the bridge. Draw a game tree that fully characterizes the extensive form game, then find all of the game's Nash equilibria and subgame perfect Nash equilibria. (It might help to also use the matrix version of the equivalent normal form game to find the Nash equilibria.)

(b) Now extend the model so that Army 2 first chooses whether or not to burn the bridge. Draw a game tree that fully characterizes the extensive form game in such a way that each player has at least two actions at each decision node (if an army has only one action available at a decision node, assume they choose that action). Find all of the Nash equilibria and subgame perfect Nash equilibria. (Again, it might help to use the matrix version of the equivalent normal form game.)

(c) Discuss what burning the bridge means in terms of Army 2's "commitment to fight."

Problem 3: Dollar auction

Osborne, Exercise 176.1

If there is a decision node where a player has only one action available and it is followed by a terminal node, treat the decision node itself as a terminal node (e.g. model the situation in such a way that every decision node has at least two actions). Only model the case where $v = 2$ and $w = 3$.

Problem 4: Firm-union bargaining

Osborne, Exercise 177.1, parts (a) and (b) only.

This is a game where the players have continuous actions. The union's strategy is a wage demand, and the firm's strategy is to choose an amount of labor L. (Assume that L = 0 is the same as rejecting the demand.)

Problem 5: Two-period bargaining

The ultimatum game is an extreme form of bargaining in that Player 2 can only accept or reject and cannot make a counter-proposal, and it may seem that this may lead to the lop-sidedness of the subgame perfect Nash equilibrium outcome. Consider the following variation where Player 2 makes a counter-offer when rejecting Player 1's initial offer.

At the start of the game, the value of the "pie" (total amount of money) is c. Player 1 chooses an amount $x \leq c$ to give Player 2 and keeps c-x for himself (just as in the original ultimatum game). Assume that x is continuous. If Player 2 accepts the offer, then the game ends and the payoffs are as before (c-x for Player 1 and x for Player 2). If Player 2 rejects the offer, then the value of the pie is multiplied by a factor of δ, and Player 2 makes a counter proposal to give y to Player 1 and keep the remainder for herself. Assume that $0 < \delta < 1$, so that the pie "shrinks" after it is initially rejected. Thus, it must be that $y \leq \delta c$ and that the remainder that Player 2 keeps is $\delta c - y$. If Player 1 rejects Player 2's counter-offer, both players get 0. Assume that utility is equal to the amount of money a player gets when the game ends.

(a) What are the subgame perfect Nash equilibria of this two-period bargaining game? Player 1's strategy must specify his offer at the beginning of the game *and* what he would do *for each* possible counteroffer by Player 2. Player 2's strategy must specify what he would in response to *each* offer by Player 1.

(b) What is each player's equilibrium payoff? If we interpret low values of δ to mean that players are impatient and high values to mean that they are patient, what can you say about how the equilibrium payoffs change with players' level of patience? (In other words, interpret how the equilibrium payoffs change when $\delta \to 1$ and when $\delta \to 0$.)