Problem Set 9
PS 2703
Due November 12, 2007

Provide complete explanations for all of your answers.

Problem 1: Parenting 101

(a) Suppose that a parent and child play the following game. First, the child takes an action that affects both her own and her parent's income. Suppose the action *a* is any real number and that her own income is given by the function *c*(*a*) and her parent's income by the function *p*(*a*) where *c*(*a*) < *p*(*a*). After observing the child's action, the parent chooses an amount of his income *t* to transfer to his child. The child is selfish and wishes to maximize the total amount of money she has (her income plus the transfer from her parent), which is $u_c(a,t) = c(a) + t$. The parent is altruistic and his utility is equal to the minimum of the amount of money he has and the amount of money his child has after the transfer. Specifically,

$$u_p(a,t) = \begin{cases} c(a) + t & if \quad c(a) + t \leq p(a) - t \\ p(a) - t & if \quad c(a) + t > p(a) - t \end{cases}$$

Prove that in a subgame perfect Nash equilibrium that the child chooses *a* to maximize *c*(*a*) + *p*(*a*). This result is known as the "Rotten Kid Theorem"—even though the child is selfish, she maximizes the family's total wealth. (If you find it necessary, assume that *a* must be in a closed interval and that *c*(*a*) and *p*(*a*) are both continuous.)

(b) Now suppose that the parent and child play a different game. In the first period, the child has a fixed amount of money *c* and chooses to save *s* for the future. The parent observes how much the child saves and bequeaths an amount *b* from his own savings *p*. The child spends what she doesn't save in the first period and the sum of her savings and the bequest in the second period. Specifically, her utility function is $u_c(s,b) = \ln(c-s) + \ln(s+b)$. The parent's utility function is the sum of his own total consumption plus his child's consumption in both periods, $u_p(s,b) = \ln(p-b) + \ln(c-s) + \ln(s+b)$. Find the subgame perfect Nash equilibrium of this game. Verify that in the SPNE outcome h = (*s**, *b**(*s**)) the child saves too little in order to induce the parent to leave a larger bequest. In other words, both parent and child would be better off if the child saves *s'* > *s** and the parent leaves *b'* < *b**. (For the last part of the problem, a numerical example suffices.)

(c) Relate one or both of the parent-child models to a situation in politics or public policy and explain what insights it provides.

Problem 2: Models of Legislative Politics

Legislative scholars have engaged in fierce debates regarding which features of American national institutions are most relevant for understanding how laws and policies are made in the contemporary United States. This problem asks you to analyze the comparative statics of two leading models of lawmaking. To simplify the analysis, we will assume that the policy space is the set of real numbers (so it is one-dimensional), the legislature is unicameral, and that all players have single-peaked and symmetric utility functions over final policy outcomes. For the models described in parts (a) and (b), do the following:

- Draw a figure that represents the sequence of events for the extensive form game.
- Characterize the subgame perfect Nash equilibrium for *any* value of the status quo q. (Hint: You can partition the set of possible status quo points into different intervals and characterize the SPNE for each interval.)
- Plot the SPNE *policy outcome* as a function of the status quo q. (The status quo q is in the x-axis while SPNE outcome is in the y-axis.)

(a) The *cartel agenda* model assumes that the majority party in Congress has negative agenda power (also known as “gatekeeping” power)—whether a bill will be considered or if the status quo will remain in place. The relevant players in this model are the majority party leader (whose ideal point is L) and the floor median (whose ideal point is M). Assume that $L > M$. In terms of the sequence of actions, the majority party leader first decides whether or not to allow a bill to be considered. If the bill is not considered, then the game ends and the policy is the status quo q. If he allows a bill to be considered, then the floor median chooses a bill x, which passes and becomes the new policy.

(b) The *pivotal politics* model assumes that supermajority rules are the relevant features of lawmaking. There are two sources of supermajority voting requirements. The first is the 2/3 vote required to override a presidential veto. The second source is the cloture rule in the Senate which requires 60 votes to end a debate. Consider a simplified model where the relevant players are the floor median (whose ideal point is M, as above), the filibuster pivot (whose ideal point is F), and the veto override pivot (whose ideal point is V). Assume that $V > M > F$. In this model, the floor median first proposes a bill b and then the filibuster and veto pivots sequentially choose whether to accept or reject the bill. Suppose that the filibuster pivot chooses first. If she rejects then the game ends and the outcome is the status quo, while if she accepts, then the veto pivot gets to choose. If the veto pivot accepts, then the game ends and the outcome is the bill b while if she rejects, the game ends and the outcome is the status quo.

(c) Assume that $M = 50$, $L = 60$, $F = 35$, $V = 67$, and $0 \leq q$ 100. For what values of the status quo do the cartel agenda and pivotal politics models predict the same SPNE policy outcome? For what values do they predict different outcomes?

Problem 3: Variations of the Ultimatum Game

(a) Consider a variation of the ultimatum game where $c = 1$ in which the second player cares about the *distribution* of payoffs in addition to his/her own payoffs. To describe the utility function, let x_1 and x_2 be the shares that the players receive at the end of the game. If Player 1's offer is y and if they agree, then $x_1 = c - y$ and $x_2 = y$. Otherwise $x_1 = 0$ and $x_2 = 0$. Player 1 only cares about only her own share so that $u_1(x_1, x_2) = x_1$. Player 2's utility is her own share minus a penalty for the difference between the two shares so that $u_2(x_1, x_2) = x_2 - | x_1 - x_2 |$. Find the subgame perfect Nash equilibrium of this game.

(b) Now consider a variation where Player 2 might be one of two possible types. With probability p, he is a selfish type as in the original game and his utility function is $u_2(x_1, x_2) = x_2$. However, with probability $1 - p$, he is an egalitarian type and his utility function is the same as in part (a). Assume that Nature selects Player 2's type *after* Player 1 makes an offer. What is the subgame perfect Nash equilibrium of this game? For what values of p is Player 1's offer the same as in the original game and for what values does it differ? (To make things easier, you may consider only strategies where both types of Player 2 accept the offer when indifferent between accepting and rejecting.)