

Summary of Classes of Games and Equilibrium Concepts

Classes of Games

<u>Class of Games</u>	<u>Comments</u>	<u>Preferences</u>	<u>NE</u>	<u>BNE</u>	<u>SPNE</u>	<u>WSE</u>
Strategic games, complete information						
Pure strategies	Players each choose a single action simultaneously (not knowing what the other players choose).	Ordinal	X			
Mixed strategies	Players randomize over actions but choose the likelihood that each pure action is selected. Uncertainty is created “endogenously” by the choices of the players. Action profiles are evaluated in terms of expected payoffs.	vN-M (EU)	X			
Static games, incomplete information	Players choose actions given uncertainty about other players’ types/preferences. Nature chooses a state of the world and/or players’ signals. Players evaluate expected payoffs given a strategy profile.	vN-M (EU)			X	
Extensive games, perfect information						
Without uncertainty	Players may make many decisions in sequence and observe the exact history of play before making a decision. Analysis involves <i>strategies</i> which specify actions for every choice a player must make.	Ordinal	X			X
Repeated games	A strategic game (<i>stage game</i>) that is played multiple times. Analysis involves evaluating discounted sum of stream of payoffs.	Time discounting	X			X

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Exogenous uncertainty	Randomness is given as a feature of the model (represented by “Nature”) and probabilities do not depend on players’ choices. When Nature moves after a player’s decision, it represents uncertainty <i>about the future</i> .	vN-M (EU)	X		X	
Extensive games, imperfect information	Players may not know the exact history of play when making a decision. An <i>information set</i> models <i>uncertainty about the past</i> . Analysis involves explicitly specifying players’ <i>beliefs</i> .	vN-M (EU)	X		X	X
Signaling games	Nature chooses <i>private information</i> revealed to a Sender, the Sender chooses a message, and the Receiver observes only the message (but not the private information) and chooses an action that affects both players’ payoffs. In a <i>separating</i> equilibrium, different types send different messages which reveals information. In a <i>pooling</i> equilibrium, different types send the same message which does not provide any new information.	vN-M (EU)	X	X	X	X

Nash Equilibrium

An equilibrium is something that is stable. The concept of Nash equilibrium identifies two fundamental components of stable behavior. The first is the *rationality* (optimality) of actions *given expectations*. The second is that players' expectations about what others will do are *correct* because they are consistent with others' actions. (One shortcoming of the Nash equilibrium concept is that it does not necessarily tell you *how* an equilibrium comes about, only that once players choose equilibrium strategies that the behavior will be stable.)

Class of Games

Definition and Comments

Strategic games

Pure strategies

An *action profile* in which each player's action is optimal (a best response) to the other players' actions (best responses). Payoffs from the optimal action must be greater than or equal to the payoffs from any other action. Equivalently, no player has a unilateral incentive to deviate from the action profile. Two basic methods for solving: check each profile to see whether any player will deviate or find the intersection the players' best response functions.

Mixed strategies

A *mixed strategy profile* where no player has an incentive to deviate. A player using a mixed strategy must be indifferent between the pure strategies that are used with positive probability. Solving for a MSNE involves finding a mixed strategy for player *i* that makes player *j* indifferent between pure strategies. (The pure strategies must still be best responses.)

Extensive games, Perfect and Imperfect Information

A *strategy profile* in which no player has an incentive to unilaterally deviate to a different strategy. May involve non-credible threats for decision nodes (histories) that are not reached when players use the strategies. Solve by finding each player's entire set of strategies to construct the equivalent strategic form and then find its NE.

Bayesian Nash Equilibrium

The Bayesian Nash equilibrium concept extends Nash equilibrium to static games of incomplete information by incorporating the concept of a *strategy* (an action for each type) and evaluating payoffs in terms of expected utility given the distribution of types and/or information that Nature selects.

Subgame Perfect Nash Equilibrium

In extensive form games, Nash equilibrium does not take into account how the sequence of actions will affect players' decisions or expectations about what others will do. The concept of subgame perfection is a *refinement* of Nash equilibrium because it strengthens the rationality requirement. This strengthening also narrows players' expectations about others' behavior.

Class of Games

Extensive games, Perfect Information

Definition and Comments

A subset of NE in which a strategy profile is a Nash equilibrium in every *subgame*. A subgame consists of a decision node and every decision node or terminal node that follows it. SPNE requires every action within a strategy to be *sequentially rational*—optimal at the decision node or history given that future decisions will also be optimal—even if the strategy profile implies that a history will never be reached in the actual play of the game. Backwards induction will identify *all* SPNE strategy profiles.

Extensive games, Imperfect Information

May be used to rule out NE in which strategies are not sequentially rational but does not help when the best response at an information set is ambiguous.

Weak Sequential Equilibrium

Since perfect information means that players know the exact history of play when making a decision, their beliefs are always correct (by definition) and we did not need to explicitly model them. When players are uncertain about the history, however, sequential rationality depends crucially on what players' beliefs are. At a minimum, beliefs should be consistent with players' strategies.

Class of Games

Extensive games, Imperfect Information

Definition and Comments

An *assessment* (consisting of a strategy profile and belief system) in which the strategy profile is *sequentially rational* (the action at every information set is optimal given beliefs) and beliefs satisfy *weak consistency* (determined by Bayes' Rule when the strategy profile implies that an information set is reached with positive probability). Solve by finding NE (pure and mixed strategy) of the equivalent strategic form, finding beliefs consistent with Bayes' Rule when applicable, and finding beliefs that make the actions sequentially rational when Bayes' Rule is inapplicable, and checking the sequential rationality of each action.